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How do planets find their way?
Laws of nature and the transformations of knowledge in the Scientific Revolution

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To Uğur

In his own handwriting, he set down a concise synthesis of the studies by Monk Hermann, which he left José Arcadio so that he would be able to make use of the astrolabe, the compass, and the sextant. José Arcadio Buendía spent the long months of the rainy season shut up in a small room that he had built in the rear of the house so that no one would disturb his experiments. Having completely abandoned his domestic obligations, he spent entire nights in the courtyard watching the course of the stars and he almost contracted sunstroke from trying to establish an exact method to ascertain noon. When he became an expert in the use and manipulation of his instruments, he conceived a notion of space that allowed him to navigate across unknown seas, to visit uninhabited territories, and to establish relations with splendid beings without having to leave his study. That was the period in which he acquired the habit of talking to himself, of walking through the house without paying attention to anyone, as Úrsula and the children broke their backs in the garden, growing banana and caladium, cassava and yams, ahuyama roots and eggplants. Suddenly, without warning, his feverish activity was interrupted and was replaced by a kind of fascination. He spent several days as if he were bewitched, softly repeating to himself a string of fearful conjectures without giving credit to his own understanding. Finally, one Tuesday in December, at lunchtime, all at once he released the whole weight of his torment. The children would remember for the rest of their lives the august solemnity with which their father, devastated by his prolonged vigil and by the wrath of his imagination, revealed his discovery to them:

“The earth is round, like an orange.”

Úrsula lost her patience. “If you have to go crazy, please go crazy all by yourself!” she shouted. “But don’t try to put your gypsy ideas into the heads of the children.” José Arcadio Buendía, impassive, did not let himself be frightened by the desperation of his wife, who, in a seizure of rage, mashed the astrolabe against the floor. He built another one, he gathered the men of the village in his little room, and he demonstrated to them, with theories that none of them could understand, the possibility of returning to where one had set out by consistently sailing east.

Gabriel García Márquez, *One hundred years of solitude*.

But this deviation from the Law, which the Law took into account, this violation of the rule did not make the marvel any less marvelous.

Umberto Eco, *Foucault’s pendulum*.

Objectivity and rationality must be things that we forge for ourselves as we construct a form of collective life. So the work of Copernicus is undone. Human beings are back in the centre of the picture. Things that had seemed distant become close; product is replaced by process. Apparent universals become variable and relative. The things we had seen ourselves as answerable *to*, we are now answerable *for*. So the body of work that we are about to examine redraws the boundaries of responsibility; it is a subtle attempt to change our cultural self-consciousness.

David Bloor, *Wittgenstein: A social theory of knowledge*.

Abstract

Laws of nature are perceived as playing a central role in modern science. This thesis investigates the introduction of laws of nature into natural philosophy in the seventeenth century, from which modern science arguably evolved. Previous work has indicated that René Descartes was responsible for single-handedly introducing a mathematical concept of laws into physics under the form of ‘laws of nature’.

However, there is less agreement on the originality, causes and aftermath of this manoeuvre. This thesis is sensitive to the circumstance that the introduction of ‘laws of nature’ in the seventeenth century is a problem for *us* given our hindsight perspective of the origins of modern science, not an explicit concern of the actors; ‘laws of nature’ emerged as part of a network of problems and possibilities converging in Descartes’ reform of natural philosophy. Then, the appropriation of his laws was not an assessment of isolated statements on nature, but a process bounded by critical stances towards the Cartesian enterprise involving theological and social underpinnings. Accordingly, this thesis approaches ‘laws of nature’ as *by-products* of the changing boundaries between mechanics, mathematics and natural philosophy in the seventeenth century and interprets them as embedded within the circumstances and interactions among the practitioners of these disciplines in which these laws were introduced, criticised and appropriated.

Based on this approach, this thesis tracks the background of Descartes’s project of reform of physics from the sixteenth-century fascination for machines that led to codifications of mechanics as a mixed-mathematical science, generating quantitative ways to design and fabricate physical (artificial) objects (Chapter 1). This approach was picked up by Galileo, who transformed it to include natural motion. In so doing, Galileo developed a mathematical approach to natural philosophy—a mathematical science of motion—which ultimately relied on the physical assumption of the motion of the Earth (Chapter 2). An alternative reorganization of mathematics and natural philosophy was put forward by the Lutheran theologian Kepler, who considered that the natural knowledge of the world may be founded *a priori* by deciphering the archetypes that God followed when creating the world. His archetypal cosmology provided a link between geometry and natural philosophy, involving mechanics

(Chapter 3). However, Descartes moved in a different direction. Instead of connecting mathematics to natural philosophy, he tried to anchor both mathematics and natural philosophy on certainty, claiming that matter is but extension and that a few principles codified all possible interactions among parts of this geometrical matter. These principles were three ‘laws of nature’ erected as foundations of an *a priori* physics (Chapter 4). These ‘laws of nature’ received considerable attention in England. Informed by local traditions, English writers rejected the causal role attributed to laws but reworked their contents in laws of motion that were moved to mechanics and extended to astronomy, in line with the local practices of the ‘elliptical astronomy’ (Chapter 5). The relocation of ‘laws of nature’ from physics to mechanics was connected with English debates concerning the role of motion in geometry. These discussions drew different consequences for the connections between mathematics and nature (Chapter 6). In line with the English appropriation of Descartes, the young Newton assumed laws of motion as mathematical explanations in mechanics. When asked by Halley about orbital motion, his answer displayed characteristics of the English disciplinary setting. However, in connection with his historical studies, Newton realised that his laws of motion were capable of accounting for the true system of the world and then they were transformed into mathematical principles of natural philosophy, redrawing the contours of mathematics, natural philosophy and mechanics. The most important outcome of this reorganization—the law of gravitation—raised suspicions for going beyond the boundaries of established practices in the Continent (Chapter 7).

The thesis concludes that ‘laws of nature’ did not emerge as a generic label to denominate findings in science. On the contrary, they appeared as concrete achievements with an operative function within Descartes’ reform of natural philosophy and consequently embedded within a network of assumptions, traditions and practices that were central to the appropriation of ‘laws of nature’. English natural philosophers and mathematicians reworked these ‘laws of nature’ within different disciplinary settings and put forward alternative ‘laws of motion’ in ways not previously noticed. The picture that emerges is not that of an amalgamation of previous meanings into a more complex one that was subsequently disseminated.

Instead of a unified concept of 'laws of nature', Descartes' project triggered reactions framed within local traditions and therefore it is hard to claim that at the end of the seventeenth century there was any agreement on the meaning of 'laws of nature' or even laws of motion beyond the narrow circles that shared disciplinary commitments and values. It was during the appropriation of Newton in the eighteenth century that his achievements and those honoured as his peers were labelled with a non-Newtonian concept of 'laws of nature', creating a foundational myth of the origins of modern science that reached up to the twentieth century.

Lay summary

The idea that one of the main tasks of scientists is to find laws of nature explaining the occurrence of natural phenomena is part of our contemporary understanding of science. The expression ‘laws of nature’ has different meanings and these have been associated to the natural world since ancient times. For example, it was used as a synonym of the general order and regularity of nature or, in mathematics, to indicate that two things were related without the intention of clarifying why. However, the idea that some singular statements denominated ‘laws of nature’ explained the occurrence of natural phenomena, that is, stood for the reasons why things occur the way they do and not otherwise, was first articulated by René Descartes (1596-1650). This thesis investigates Descartes’ introduction of ‘laws of nature’ as singular claims explaining the natural world and the immediate impact of this strategy in England’s local traditions of enquiry up to the formulation of Isaac Newton’s ‘laws of motion’ (1687-1713).

Previous attempts to shed light on the origins of ‘laws of nature’ in this specific sense have focused on the meaning of the expression and how it was related to previous, different meanings. However, preceding uses of one expression do not explain how a term will be used in the future. Therefore, this thesis investigates the specific use of ‘laws of nature’ in connection with Descartes’ plan to reform the knowledge of the natural world. In his *Principles of Philosophy* (1644), Descartes sketched the foundations of a new approach to nature whose purpose was delivering certain and unquestionable explanations resulting from his ‘laws of nature’, ultimately founded upon the divine perfection. In England, this project was critically appropriated by further developing the content of Descartes’ laws and by integrating them into local agendas of reform of knowledge, such as those converging at the Royal Society of London (1660-1680). Nevertheless, English writers were suspicious of the theological implications of Descartes’ project and claimed that some of the ‘laws of nature’ did not match the outcome of experiments. Consequently, they reworked the ‘laws of nature’ into mathematical ‘laws of motion’. These laws of motion did not derive from God’s attributes but were conceived as mathematical statements describing how things happen in nature. Although these laws of motion did not

uncover the ultimate cause behind the nature, they enabled a high degree of accuracy in predicting the occurrence of natural phenomena. In this way, the laws of motion transformed the values and practices implied in the knowledge of the natural world that the 'laws of nature' of Descartes had attempted to make universal.

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Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification.

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Introduction

I have often wondered, why the planets should move about the sun according to Copernicus's supposition, being not included in any solid orbs (which the antients possibly for this reason may embrace) nor tied to it, as their center, by any visible strings; and neither depart from it beyond such a degree, nor yet move in a strait line, as all bodies, that have but one single impulse, ought to do (...) The cause of inflecting a direct motion into a curve may be from an attractive property of the body placed in the center; whereby it continually endeavours to attract or to draw it to itself.

Robert Hooke.¹

Problems

Writing in 1666 to the Royal Society, Hooke conjectured how the planets find their way according to the Copernican arrangement. Hooke's problem was physical: why is it that the planets instead of moving in 'strait' lines were continually deviated 'into a curve' if there was no strings or celestial orbs keeping them in this path? A few years later, Newton argued that planets find their way because, according to the mathematical law of gravitation, they were 'continually drawn away from rectilinear motions and are maintained in their respective orbits'.² The idea that planets—and all bodies in general—follow specific, mathematical laws and that these laws may constitute a sufficient explanation of natural phenomena was elaborated in the seventeenth century. Although the expression laws of nature was hanging in the air since ancient times as a vague reference to order in the natural world, it acquired a different meaning in the hands of René Descartes; a meaning that was critically transformed by English natural philosophers, including Hooke and Newton. This thesis investigates the introduction of laws of nature into natural philosophy and their immediate appropriation in seventeenth-century England.

The idea that laws of nature were central to the seventeenth century is not new. It was a commonplace in the foundational studies on the Scientific Revolution—on which the discipline of the history of science was erected—that laws of nature were the basic unit of the new quantitative science. Alexandre Koyré characterised the

¹ Birch, *The History of the Royal Society*, 1756, 2:91.

² Newton, *The Principia*, 802; 943.

Scientific Revolution as the disappearance of the hierarchically organised cosmos and the advent of an infinite universe, united not by its immanent structure ‘but only by the identity of its fundamental contents and laws’.³ Rupert Hall filled the period from Galileo to Newton with laws. In his view, the most important innovation of the seventeenth century was the universality of motion as change of place, that is, that the same kind of motion occurred everywhere and that it was ‘invariably subject to the same laws’. Consequently, Hall described the most important achievements in terms of laws: the law of inertia, Galileo’s law of free fall, Kepler’s laws of planetary motion and Newton’s law of universal gravitation.⁴ However, these uses of laws in the history of science were subordinated to more ambitious attempts to ‘capture the nature’ of the Scientific Revolution: the mathematisation of nature, the rise of experimental sciences or the emergence of a new metaphysics.⁵

This ‘grand narrative’ was set aside. Distrust towards the generalisations stating the essence of the Scientific Revolution, the concentration on a few canonical figures and the emphasis on some disciplines in detriment of others considered ‘irrational’ (alchemy, astrology) led to question the very idea of a ‘Scientific Revolution’ as a ‘change that is sudden, radical, and complete’.⁶ Maybe ‘the Scientific Revolution’ was to a large extent a construct of previous historians who reduced the scope of their investigations and thus presented this heroic narrative of accumulative progress based on individual findings. Accordingly, revisionist tendencies made imperative the reassessment of textual evidence, the introduction of new methodological approaches, the study of social and institutional settings in which knowledge was produced and used, not to mention the consideration of previously neglected practices, figures and locations beyond Europe.⁷ These changes generated a

³ Koyré, *Newtonian Studies*, 7; Koyré, *Études Galiléennes*; Koyré, *Études d'histoire de la pensée scientifique*.

⁴ Hall, *From Galileo to Newton, 1630-1720*, 37.

⁵ Hall, 13; Koyré, *Newtonian Studies*, 6; Kuhn, ‘Mathematical vs. Experimental Traditions in the Development of Physical Science’.

⁶ Cohen, *The Revolution in Science*, 51–90.

⁷ All relevant references on revisionism and the Scientific Revolution cannot be included in one footnote. However, see Shapin, *The Scientific Revolution*; Lüthy, ‘What To Do With Seventeenth-Century Natural Philosophy? A Taxonomic Problem’; Henry, *The Scientific Revolution and the Origins of Modern Science*; Dear, ‘What Is the History of Science the History Of?: Early Modern Roots of the Ideology of Modern Science’; Osler, *Rethinking the Scientific Revolution*; Roux, ‘Forms

spectacular enlargement in the scope and depth of our understanding of the origins of modern science. Nevertheless, the idea of laws of nature seems so attached to the ‘grand narrative’ that the revisionist studies on laws of nature are limited either to fragmentary aspects—such as small-scale analysis of specific authors—⁸ or to approaches that, by avoiding the manners now considered outdated, dissolved the specific character of the question at hand:⁹ how and why ‘laws of nature’ were introduced in natural philosophy in the seventeenth century and how they were immediately appropriated.

Approach

My solution to this problem is that ‘laws of nature’ emerged as a *by-product* of the transformations of the disciplinary boundaries between mathematics, mechanics and natural philosophy during the seventeenth century. Put otherwise, I read ‘laws of nature’ as a *function* of the redrawing of the boundaries of knowledge in the period. The key point here is that ‘laws of nature’ emerged and varied in connection with the transformations of the bounds of these disciplines but, at the same time, these disciplines were evolving in function of other factors. Therefore, detaching ‘laws of nature’ from this fluctuating, disciplinary setting amounts to disconnecting the solution from the problem that they were intended to solve. This approach requires some clarifications.

1. Hindsight. A significant source of confusion in dealing with ‘laws of nature’ has been the failure in recognising the extent to which this is a problem for *us*, not for the

of Mathematization (14-17 Centuries)’; Cohen, *The Rise of Modern Science Explained: A Comparative History*; Henry, ‘The Scientific Revolution: Five Books about It’.

⁸ Steinle, ‘The Amalgamation of a Concept. Laws of Nature in the New Sciences’; Steinle, ‘Negotiating Experiment, Reason and Theology: The Concept of Laws of Nature in the Early Royal Society’; Steinle, ‘From Principles to Regularities: Tracing “Laws of Nature” in Early Modern England and France’; Graßhoff, ‘Natural Law and Celestial Regularities from Copernicus to Kepler’; Gingerich, ‘Kepler and the Laws of Nature’; Garber, ‘God, Laws, and the Order of Nature: Descartes and Leibniz, Hobbes and Spinoza’; Hattab, ‘Early Modern Roots of the Philosophical Concept of a Law of Nature’; Jalobeanu, ‘The Cartesians of the Royal Society’; Psillos, ‘Laws and Powers in the Frame of Nature’; Domski, ‘Laws of Nature and the Divine Order’.

⁹ Ruby, ‘The Origins of Scientific Law’; Roux, ‘Les lois de la nature à l’âge classique la question terminologique’; Lehoux, ‘Laws of Nature and Natural Laws’; Ott, *Causation and Laws of Nature in Early Modern Philosophy*; Jalobeanu and Anstey, *Vanishing Matter and the Laws of Motion Descartes and Beyond*.

seventeenth-century actors. We are interested in the origin and transformation of ‘laws of nature’ because during the eighteenth and nineteenth centuries the investigation of the natural world became a ‘quest for laws’ and in an important sense, the idea still prevails in contemporary science.¹⁰ Arguably, laws became a component of current definitions of science.¹¹ Therefore, scholars interested in science—philosophers, historians, sociologists—look for answers to questions such as ‘what is it to be a law’. But in the seventeenth century, ‘laws of nature’ appeared in Descartes as part of an incipient endeavour to outline a systematic way to account for natural phenomena. This solution was tied to Descartes’ singular formulation of what he called an *a priori* physics and to its contents. Therefore, projecting ‘laws of nature’ beyond Descartes without considering other aspects runs the risk of imposing our questions in the place of the actors’ concerns and, instead of clarifying the historical development of science, we misconstrue it. The term ‘law’ was not common in *natural philosophy* before 1660.¹² Galileo did not name ‘law’ his ‘law of free fall of bodies’. The most similar concept in Kepler—both in meaning and use—to Descartes’ ‘laws of nature’ is ‘archetype’, not the geometrical propositions that he denominated ‘laws’ or his references to laws of nature as synonyms of the natural order following Lutheran teachings. Neither did Kepler refer to his ‘laws of planetary motion’ as ‘laws’. Furthermore, English writers after 1650s were reluctant to adopt the terminology of ‘laws of nature’ in *natural philosophy*; the stipulations of Descartes’ ‘laws of nature’ were debated in the *mathematical* science of *mechanics* and were called ‘laws of motion’. Newton barely used the expression laws of nature and his scattered uses of the term in print were controversial. The expression laws of nature responds to a particular context in which it was seen as a solution, so not attending the specific meanings and circumstances runs the risk of falling into the nominalist fallacy of assuming as equal different problems because they are named with the same words.

¹⁰ Chartrak, *Contingence et nécessité des lois de la nature au XVIIIe siècle*; Stan, ‘Newton’s Concepts of Force among the Leibnizians’; Shank, *Before Voltaire*.

¹¹ Lange, *Laws and Lawmakers*; Massimi, ‘Prescribing Laws to Nature. Part I. Newton, the Pre-Critical Kant, and Three Problems about the Lawfulness of Nature’.

¹² Roux, ‘Les lois de la nature à l’âge classique la question terminologique’, 554–55.

2. *Terminology.* The expression laws of nature had been around since the Ancients with different meanings. One initial, extended sense was vague and general, as a synonym of order or regularity in nature.¹³ In the Judeo-Christian traditions, this notion of order became associated to God as a ‘law-giver’, as the one imposing order on nature.¹⁴ In the context of *scientiæ*, the term ‘law’ was used since the Middle Ages in mathematics, especially in optics, to denote a natural regularity; this regularity, formulated in abstract terms, pointed to a connection between elements. For example, the law of refraction—denominated as a ‘law’ by Roger Bacon in the twelfth century—related the angle of incidence and the angle of refraction, but the law did not contain any causal explanation of this ‘constant conjunction’.¹⁵ The specific use of ‘laws of nature’ as quantifiable, necessary statements accounting for the regularity of phenomena—operating as secondary causes—was introduced by Descartes.¹⁶ Because this is the most important meaning for my argument, I mark it with inverted commas; so by ‘laws of nature’ I make reference to Descartes’ ‘principal rules according to which it must be thought (*il faut penser*) that God causes the nature of this new world to operate’, in order to differentiate it from other uses.¹⁷

Descartes’ use of the expression is innovative, but the term was not created *ex nihilo*: he borrowed it from the mixed-mathematical sciences and thus it was associated with the idea of regularity. At the same time, Descartes’ conception of God guaranteed the necessity and universality of these ‘laws of nature’ in a way that cannot be detached from the Catholic context of France.¹⁸ However, Descartes’ use cannot be entirely

¹³ Zilsel, ‘The Genesis of the Concept of Physical Law’; Needham, ‘Human Laws and Laws of Nature in China and the West (I)’; Needham, ‘Human Laws and Laws of Nature in China and the West (II): Chinese Civilization and the Laws of Nature’; Lehoux, ‘Laws of Nature and Natural Laws’; Henry, ‘The Theological Origins of Laws of Nature and Its Subsequent Secularisation’.

¹⁴ Milton, ‘Laws of Nature’; Roux, ‘Les lois de la nature à l’âge classique la question terminologique’; Steinle, ‘From Principles to Regularities: Tracing “Laws of Nature” in Early Modern England and France’; Harrison, ‘The Development of the Concept of Laws of Nature’.

¹⁵ Ruby, ‘The Origins of Scientific “Law”’; Roux, ‘Les lois de la nature à l’âge classique la question terminologique’, 547–50.

¹⁶ Henry, ‘Metaphysics and the Origins of Modern Science’; See also Psillos, ‘Laws and Powers in the Frame of Nature’.

¹⁷ Descartes, *AT*, 1909, XI:38.

¹⁸ Henry, ‘Metaphysics and the Origins of Modern Science’; Cf. Funkenstein, *Theology and the Scientific Imagination*; Cook, *The Young Descartes. Nobility, Rumor and War*.

explained in terms of the previous occurrences of laws; Descartes' use was not prefigured or foreshadowed in them.¹⁹ Therefore, we need to analyse the conditions under which the term acquired different meanings and functions: Descartes' reform of natural philosophy. Additionally, Descartes' introduction of 'laws of nature' did not banish other meanings of law. His contemporaries had access to mathematical, theological and metaphysical meanings and read Descartes' laws against these backgrounds. When Boyle complained that he could not conceive 'how a Body, devoid of understanding and sense ... does properly act by Laws',²⁰ he was criticising from a theological angle the conception of laws as immanent in nature, although he enthusiastically embraced Descartes' 'laws of nature' with modifications. Boyle's criticism of 'laws of nature' was based on the rejection of the world implied by Descartes' physics, not on other meanings of laws available to him.

Finally, by using inverted commas, I also highlight that in this thesis 'laws of nature' and its immediate appropriations are assumed as historical terms whose meaning has to be investigated as any other particular term in the history of science, such as Newton's gravity, Priestley's phlogiston or Higgs' boson. During the eighteenth and nineteenth centuries, laws of nature became a label to denominate *any* scientific achievement within fields that came to form the modern sciences. The expression lost its initial connection with specific statements about motion and particles of matter, founded upon divine immutability. However, for the seventeenth century actors 'laws of nature' were not a label but a technical term, with defined contents, associated to Cartesianism—and later to Leibniz. The historian faces the temptation of studying 'laws of nature' as other terms that operated as labels in the seventeenth century, such as 'axiom', 'law', 'proposition' or 'demonstration' and then to address the history of 'laws of nature' as an investigation on scientific demonstration or on causation in science. But history rarely follows the order of our philosophical anxieties. Although 'laws of nature' operated as principles of explanation of the natural world and thus as causes, they were introduced and appropriated as specific statements regulating the motion of matter as extended. The disciplinary approach

¹⁹ Bloor, Barnes, and Henry, *Scientific Knowledge: A Sociological Analysis*, 46–80; Kusch, 'Meaning Finitism and Truth'.

²⁰ Boyle, *A Free Enquiry into the Vulgarly Receiv'd Notion of Nature*, 42–43.

makes clear the function that ‘laws of nature’ played in Descartes and how their contents and assumptions were pivotal in their subsequent appropriation in England.

3. *Disciplinary boundaries.* A central component of my claim is the redefinition of disciplinary boundaries between mathematics, mechanics and natural philosophy. Historians of science have widely explained that the crisis of the Scholastic philosophy was related to important transformations in the way in which disciplines were organised and practised during the sixteenth and seventeenth centuries.²¹ The natural philosophies of the Renaissance and the reception of Copernican astronomy are examples of how the strict organisation of knowledge that Scholastic philosophers had carefully crafted was in crisis at the end of the sixteenth century.²² However, the emergence of early modern science did not displace from one day to the next the institutional and intellectual settings created over centuries by the Scholastics.²³ On the contrary, all the authors referred to in this thesis were educated in, or somewhat engaged with Scholastic traditions and some of them developed their activities in universities, where Aristotle was the dominant authority. It falls beyond the scope of this (and of any) thesis to present a full-scale panorama of such transformations and it is not necessary for my present purpose. My interest has been to examine the aspects of the redefinition of mechanics, mathematics and natural philosophy as far as it is related to the introduction of ‘laws of nature’ into natural philosophy and their immediate appropriations in England.

²¹ Jardine, ‘Problems of Knowledge and Action: Epistemology of the Sciences’; Wallace, ‘Natural Philosophy: Traditional Natural Philosophy’; Lohr, ‘The Sixteenth-Century Transformation of the Aristotelian Division of the Speculative Sciences’; DesChene, *Physiologia*; Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*; Barker and Goldstein, ‘Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal’; Rutkin, ‘Astrology’; Andersen and Bos, ‘Pure Mathematics’; Westman, *The Copernican Question : Prognostication, Skepticism, and Celestial Order*.

²² Westman, ‘The Astronomer’s Role’; Westman, ‘The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory’; Barker and Goldstein, ‘Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal’; Henry, ‘The Fragmentation of Renaissance Occultism’.

²³ Feingold, *The Mathematicians’ Apprenticeship. Science, Universities and Society in England, 1560-1640.*; Wallace, *Galileo and His Sources*; Gascoigne, *Cambridge in the Age of the Enlightenment*; Ariew, *Descartes and the Last Scholastics*; Levitin, *Ancient Wisdom in the Age of the New Science*; Levitin, ‘Newton and Scholastic Philosophy’.

Consequently, I deal with mechanics, mathematics and natural philosophy as practices historically situated in contexts where their very conditions, scope and ways of proceeding were at question. This means that, instead of assuming these fields as stable platforms on which seventeenth-century writers developed their achievements, I approach these author's enquiries as endeavours to transform the traditions in which they were involved, particularly the organisation of *scientiæ*. It may be argued that something similar occurs in contemporary sciences, in the sense that findings reconfigure the field at some level; but scientific research today exhibits levels of standardisation that makes it hardly comparable to the thin layer of shared assumptions in the enquiries of the natural world in the early modern period.²⁴ A common feeling of the actors studied in this thesis is that the *overall* organisation of knowledge in which they were initiated and the work of their contemporaries required some reform, amendment or even a complete subversion in order to introduce new disciplines or to restore forms of lost knowledge. In this sense, these authors developed their work with an eye on the transformations of knowledge that they were consciously pushing beyond customary boundaries. Therefore, I assume as subject of investigation the elements that they used to shape their intellectual practices: from the technical elaborations to the varying representations of them as part of the strategies deployed in order to persuade their contemporaries of the validity both of their findings and of their ways of finding them. These elements converge in the disciplinary identities that these authors elaborated, that is, their self-representations as mechanicians, mathematicians, natural philosophers or 'experimental philosophers' and in the way in which these identities limited or motivated interactions within different communities—including virtual communities through correspondence networks. 'Laws of nature' and the transformed 'laws of motion' in England were central to the formation of individual, disciplinary identities; but also they became part of the shared assumptions of groups, such as the early Royal Society or circles associated with Cartesianism.

²⁴ Bloor, Barnes, and Henry, *Scientific Knowledge: A Sociological Analysis*, 140–68; Dear, 'What Is the History of Science the History Of?: Early Modern Roots of the Ideology of Modern Science'; Shapin, *A Social History of Truth*, 3–41.

Finally, approaching mechanics, mathematics and natural philosophy as practices that went under important reconfigurations during the seventeenth century implies that ‘mechanics’, ‘mathematics’ and ‘natural philosophy’ are terms whose meaning is changing and even contradictory during the period. For Guidobaldo, mechanics was concerned with the material aspects of machines that could be approached geometrically only in terms of equilibrium; a mathematical science of motion was impossible. Galileo faced Guidobaldo’s challenge and applied mathematical procedures of mechanics to motion in general. Meanwhile, Descartes considered that mechanics was just a branch of natural philosophy that took shelter in mathematics when ancient philosophers confused the knowledge of nature. Newton claimed that his achievements in the *Principia* were part of a ‘rational mechanics’ that investigated with mathematical principles connecting forces and motion in general, including natural motions and forces generating them.

4. *The Scientific Revolution.* A full discussion of the problems related to the notion of ‘the Scientific Revolution’ falls beyond my expectations here. However, this thesis contributes to revisit traditional topics, authors and studies. I find useful a non-naïve use of the notion of ‘the Scientific Revolution’, as a historiographical term to denote some processes during the seventeenth century. It is useful because it locates my discussion in a period recognisable for the reader and, at the same time, points to a rich historiographic tradition that since nineteenth century has gathered considerable evidence and offered a wide-range of perspectives to better understand the origins of modern science. The term is also desirable because it works as a short-hand to make reference both to this historiographic tradition but also to some historical phenomena which validate the use of the term. In the particular case of ‘laws of nature’, I have mentioned that the authors I revisit *felt* that the structures of knowledge dominant in their contexts required amendment or radical transformations. The term ‘Scientific Revolution’ refers then to these historical phenomena. By ‘non-naïve’ I mean that we already know that these reforms were not a radical, sudden break with the past but rather complex reorganisation of practices trying to answer to a variety of problems, from the encounter with the ‘New world’ to the rediscovery of ancient authorities.

The ‘Scientific Revolution’ cannot be reduced to any of these aspects and arguably none of them constitute something like its essence.

Another consequence of this is that the alternative projects to transform knowledge are not approached in this thesis as fixed in texts that passed from one author to the next and whose meaning we should try to clarify by textual comparison. I have read the textual evidence as produced by and playing a role in intellectual, social and material settings which were complex and changing, as we have learnt from other studies on the Scientific Revolution. In this sense, I have dealt with the emergence and appropriation of ‘laws of nature’ as embedded within the possibilities and limitations of a wide range of circumstances: the casual finding of a book, the accidental meeting with an old friends, specific social and political circumstances that forced writers to move from places, the threat of wars, the access to a tradition based on some specific version of a document, the strategies deployed to defend a position.

Overview

The idea that planets—and any particle of matter in the universe—find their way following mathematical laws and that this constituted a sufficient explanation in natural philosophy was an elaboration of the seventeenth century. The problem of the physical explanation of the motion of the planets, as Hooke stated it, was connected to Copernicanism. Indeed, it was vividly formulated by Tycho Brahe who, after observing the comet of 1577 and calculating that it appeared ‘far above the moon’, concluded that ‘the celestial machine is not a hard and impenetrable body, crammed full of various real orbs, as was heretofore believed by most people’.²⁵ How the ‘dissolution of celestial orbs’ gave way to an astronomy based in laws has been widely studied in the history of science since its early days.²⁶

²⁵ Rosen, ‘The Dissolution of the Solid Celestial Spheres’, 21–22.

²⁶ Koyré, *The Astronomical Revolution. Copernicus, Kepler, Borelli.*; Dreyer, *A History of Astronomy from Thales to Kepler*; Aiton, *The Vortex Theory of Planetary Motions*; Rosen, ‘The Dissolution of the Solid Celestial Spheres’; Gingerich, ‘Kepler, Galileo and the Birth of Modern Astronomy’; Westman, *The Copernican Question : Prognostication, Skepticism, and Celestial Order*; Vesel, *Copernicus: Platonist Astronomer-Philosopher: Cosmic Order, the Movement of the Earth, and the Scientific Revolution.*

The problem was originated in astronomy, but one of the most important resources for its solution—the explanation based in laws of nature—has its roots in the transformations of mechanics in the sixteenth century (Chapter 1). The loose set of practices then denominated as mechanical were assimilated by ecclesiastic, political and military elites under the form of a mixed-mathematical science. At the end of the century, this organisation took diverse forms of axiomatisation. In this context, machines were defined as artificial objects whose crucial characteristic was their configuration or design: the disposition of their parts in order to produce some desired effect. Therefore, the explanation and production of machines relied on the *reductio* of any complex configuration to the form of the lever—the simplest machine—, whose properties were explained by the geometry of the circle. In so doing, mathematicians practicing mechanics elaborated a new way to account for some physical, material objects in geometrical terms.

The explanation based on *reductio* assumed that mathematics dealt with machines in terms of equilibrium, not of motion, and that there had to be a perfect fit between the material condition of the machine and its design. Given the innumerable variables involved in motion and the essential differences between natural and artificial, mechanics could not account for the defining traits of the natural world. However, Galileo, educated in the Scholastic tradition of Pisa and in the study of mechanics broadly construed, attempted to extend the *reductio* in order to include natural phenomena (Chapter 2). In so doing, Galileo developed a mathematical approach to natural philosophy—a mathematical science of motion—that he considered sufficient to reject Scholastic explanations and the alternative natural philosophies of the Renaissance based on occult qualities. Galileo’s explanations ultimately relied on the assumption of an Earth in motion. Thus, Galileo’s axioms, insofar as they are mathematical are related to actual motion of this world if the Earth is considered in motion; from it, all mathematical constructions relevant to natural philosophy obtained empirical significance. Based on these assumptions, Galileo projected a ‘system of the world’.

Galileo's practice of natural philosophy quickly attracted attention of mathematicians and natural philosophers all over Europe, but it was not the only alternative to refer mathematics to the study of natural phenomena. Inspired by Lutheranism, the young Kepler ended up teaching mathematics and discovered that by revealing the geometric structure of the planetary system he was able to account for the natural world (Chapter 3). However, instead of relying on mathematical techniques of mechanics, Kepler considered to have found the archetypes that God followed when creating the world. His archetypal cosmology provided a link between geometry and natural philosophy which also encompassed mechanics. But the point is that Kepler's discovery moved him to put forward a reform of astronomy in order to include natural philosophical considerations, such as the central position of the Sun, in astronomical calculations. This reform was underpinned by the archetypes (geometric and harmonic) instantiated in the quantitative nature of matter, imprinted in the human soul by God and acting as formal cause of nature. Ultimately, this structure provided the model for Descartes' reform of natural philosophy.

These initial chapters set the scene for the history of 'laws of nature'. They constitute the immediate context for Descartes' redrawing of disciplinary boundaries in which 'laws of nature' emerged. However, they should not be considered as a mere theatrical scenery disposable after the first act but as an integral part of the entire dramatic action, so to speak. Descartes and also the English natural philosophers appropriating his 'laws of nature' interacted with themes, practices and values introduced in these chapters.

In 1630 Descartes formulated a plan to establish an *a priori* physics: an explanation of the effects of the world from their causes that would replace Scholastic natural philosophy. This project was inspired by his acquaintance with Kepler from Beeckman's journal (Chapter 4). Instead of connecting mathematics and natural philosophy, Descartes conceived his *a priori* physics as founded upon the claim that matter was but (geometric) extension and that its motion, from which all observable phenomena emerged, depended on three 'laws of nature' that codified all possible interactions among parts of matter. In this sense, 'laws of nature' did not describe

motion but acted as causes rendering it possible. These laws were founded upon divine immutability, so they could not be otherwise. The knowledge of the world, ultimately explained by ‘laws of nature’, was modelled after the way in which ‘men experienced with machinery’ accounted for machines: ‘conjecturing’ about the design of their parts.

The Cartesian philosophy provided a systematic substitute for Scholastic philosophy with an approach that contemporary philosophers and mathematicians perceived as intelligible, although critics appeared everywhere. In England, ‘laws of nature’ were integrated into philosophies involving different assumptions such as active principles, experimental practices and a local tradition of astronomy underpinned by physical considerations—the ‘elliptical astronomy’ inspired by Gilbert and Kepler (Chapter 5). Instead of initiating the mechanical philosophy in England, ‘laws of nature’ were reworked in the light of local programmes already ongoing. Descartes’ view of nature as governed by immanent laws was considered as untenable when compared with experiments and as suspicious for theological reasons. For the *virtuosi*, the idea of matter deprived of principles of motion excluded God from the operation of the universe and therefore may lead to atheism. Their experimental approach had provided evidence of *foci* of activity in nature, from the motion of animals to the growing of metals and plants, and therefore they postulated active principles superadded in matter as responsible for these phenomena, although their cause remained unknown. Some English writers rejected Descartes’ laws *tout court*. Others considered that Descartes had provided ‘the outer shell’ of nature, the rules explaining motion generated by collision, but that he had missed ‘the nucleus’, the sources of activity in matter. The ‘elliptical astronomers’ elaborated a conception of circular motion as produced by a balance between a reworked version of Descartes’ first two laws and an attractive force to the centre. In England, I identified a tendency to transform ‘laws of nature’ into laws of motion indicating the move of laws from natural philosophy back to mathematical sciences—such as mechanics and astronomy—concerned with quantitative correlations between forces and motions.

English *virtuosi* reworked the stipulations of ‘laws of nature’ into mathematical laws of motion. The denial of the causal role of Descartes’ laws entailed a rejection of his solution to an *a priori* physics and then the revised laws became principles of explanation in mechanics—not of motion in natural philosophy. This move occurred when the mathematical nature of this new field was under discussion. Two representative positions of the spectrum of these debates, John Wallis and Isaac Barrow, discussed the nature of mechanics, particularly divergent conceptions of the role of motion in mathematics (Chapter 6). While Wallis considered that geometry measured what was physically performed, Barrow claimed that geometrical figures were generated by motion and, therefore, geometry and nature were coextensive. The source of this disagreement stemmed from divergent positions on the nature of quantity and consequently on the legitimate mathematical methods and their applicability to the natural world.

One of the English writers that reworked the ‘laws of nature’ into laws of motion was Isaac Newton (Chapter 7). Since his early years, Newton was critical of Descartes’ philosophy in general, and developed laws of motion as axioms of mechanics which applied also to astronomical computations, in line with the local traditions. When Halley visited Newton in 1684 asking about the orbit of planets, Newton’s initial solution moved within the boundaries of mechanics. His answer was based on the reworked axioms assumed as mathematical hypotheses. However, in revising his initial ‘De motu’, Newton realised that he had in his hands not only a mechanical puzzle but an insight into the true system of the world known to the ancients. Then, the mathematical hypotheses were reworked as ‘laws of motion’ and became mathematical principles of natural philosophy. This reorganisation led Newton to portray a system of the world in terms of three passive ‘laws of motion’ arising from the force of inertia—accepted by everyone—and at least one law arising from the active principle of gravity acting at a distance—discovered or recovered by him. The main outcome of Newton’s reordering of the disciplines was the law of gravity. Reactions against the force of gravity did not wait. Leibniz accused Newton of reintroducing occult qualities into natural philosophy, for the *Principia* did not specify how the force of gravity operated and thus Newton had crossed the

boundaries of natural philosophy. Newton replied with intricate strategies; one of these involved comparing active principles—not the laws of motion—to ‘*universal laws of nature*’ in the *Optice*.

In the closing Scholium I point to some aspects drawn from tracing the introduction of ‘laws of nature’ into natural philosophy and their immediate appropriation in England. The emergence of ‘laws of nature’ and their subsequent reworking into laws of motion was not a progressive accumulation of pieces that were finally put together in a meaning passed on to the Enlightenment. On the contrary, ‘laws of nature’ were a built-in component of the transformation of natural philosophy as science capable of *a priori* demonstration in Descartes and therefore their appropriation was done against the background of alternative endeavours to connect natural philosophy to mathematics or to set them definitively apart. The distinction between ‘laws of nature’ and laws of motion reveal unexpected aspects of the disciplinary transformations of mechanics, mathematics and natural philosophy. Instead of a collaborative effort towards the ‘mathematisation of nature’, the rearrangement of disciplines from which ‘laws of nature’ emerged was the outcome of competitive views whose origin goes back to the sixteenth-century assimilation of mechanics as a mixed-mathematical science. It was in the eighteenth century that the expression laws of nature became dominant to unify and make coherent the immediate past of the newly born modern science.

1. Machines and the course of nature

During the sixteenth century, the loose set of practices and techniques widely denominated mechanics became organised under the form of a mathematical mid-science inspired by Scholastic divisions of knowledge. One important consequence of this transformation was the elaboration of a new way to explain and produce machines by relying on geometry. This way consisted in the reduction of the configuration of any complex machine to the simplest one, the lever, whose properties were explained by the geometry of the circle. Thus, uncovering the geometric design of machines was thought to make possible, in principle, machines performing *any* task.

The importance of this new way of explaining physical objects may be appreciated by two points. First, the shaping of mechanics as a mixed-mathematical science stimulated discussions on the differences between machines and nature that were interpreted as an opposition between human actions and the workings of nature. This latter was repeatedly referred to as the laws of nature, usually meaning the unaided course of natural events as different from its altered course as a consequence of using machines. However, this opposition is in fact a distinction, for the laws of nature were the starting point of the construction of machines. Machines were objects ultimately made from (natural) elements disposed according to human intentions, modifying their accidental characteristics and not their essential properties. Mechanics dealt with machines *qua* configurations or proportions of natural elements arranged to generate effects that unaided nature would never produce. Then, the capacity of producing desired effects depends on the configuration of the machine (the *exact* disposition of their parts), crafted according to geometrical proportions. In this sense, mechanics became predominantly a mathematical science, for mathematics supplied the formative principle of the machines. In contrast with other middle-sciences such as astronomy and optics, mechanics could deliver the formal cause of its object. Second, the developments in mechanics during the sixteenth century and the discussions about its status converged in efforts to codify and arrange mechanical knowledge in systematic ways at the end of the century. A significant number of these efforts claimed to be restoring mechanics to ‘its former glory’

following the steps of Archimedes, Euclid or some other ancient authority. In so doing, these reorganisations incorporated diverse mathematical traditions accounting for machines.

These topics are developed in three sections. The first presents some transformations of the mechanicians and mathematicians during the sixteenth century. Mechanics became central for holding political power and social orders; this transformed the boundaries and social places of mechanics and its practitioners. Largely, this increasing importance of mechanics was assimilated in the institutional forms of knowledge—universities, the humanists’ circles and ultimately the courts—by translating and circulating the *Mechanical Problems* and other ancient works. The second section analyses some topics from *Mechanical Problems*. Of particular interests is the way of explaining machines by reducing them to the geometry of the circle. Finally, the last section focuses on the systematization of mechanics occurring at the end of the century. This sheds light both on the changes of mechanics now dressed as a mathematical *science* and on the explanation of physical objects by mathematical principles in terms of design or configuration.

1.1.Beyond the shoe

At the turn of the sixteenth century, mechanical arts were indiscernible from practical mathematics. Practical mathematics included activities employing machines and instruments such as architecture, engineering, gunnery, surveying and bookkeeping.¹ Notwithstanding the increasing importance of practical mathematics in Renaissance societies, the social status of mathematicians was determined by the medieval social distinction between practical mathematicians and the astrologer-physicians.² Despite sharing a significant portion of mathematical knowledge, the practitioners of these groups remained in social disparity. Within practical mathematicians there were also internal distinctions, for those closer to craftsmanship were regarded as considerably inferior. However, as the century ran, the distinction became blurry and more

¹ Bennett, ‘The Mechanical Arts’, 673; Jones, ‘Improvement for Profit: Calculating Machines and the Prehistory of Intellectual Property’.

² Biagioli, ‘The Social Status’, 42; Bennett, ‘The Mechanical Arts’; Clagett, *The Science of Mechanics in the Middle Ages*, 164.

complex social forms emerged—such as court mathematicians or nobles with military-mathematical interests.³

Practical mathematicians were usually trained in the *scuole di abaco* (schools of practical mathematics). These were institutions of varying complexity, ranging from single, private tutors paid by the lesson to guilds established in schools and endorsed by wealthy merchants with influence on politics. The tradition of these *scuole* was founded on the study of Fibonacci's *Liber abacci* ('The book of calculation' or 'the book of practical mathematics'), a thirteenth-century compendium of practical mathematics adapted to the needs of merchants. The *Liber* introduced the Hindu-Arabic numeral system in the West. In subsequent centuries, practical mathematics became an integral part of the vernacular schooling. However, Latin schools, in which elites were educated, omitted most kinds of mathematics and completely rejected practical mathematics, because it added nothing to the social status of their students.⁴ The links among guilds of practitioners trained in practical mathematics vary considerably depending on the place: in some cities, the abacists and land surveyors had their own guild, but in other towns they were united with masons or with elementary-level teachers. Lucca, for example, merged its *abbaco* school with the communal elementary reading and writing school in the sixteenth century. It was customary as well that *abbachisti*, teaching basic arithmetic and geometry, bookkeeping and usually serving as accountants of the commune were paid less than grammarians, though these were in a similar social position.⁵

The first significant sixteenth-century development in the tradition of the abacus did not change the social status of these mathematicians. The discovery of the solution to the third and fourth degree equations gave much visibility to those involved in the controversy—Tartaglia, Ferrari and Cardano—, but neither they nor the field improved their social status.⁶ Tartaglia's works reveal some mid-century perceptions of the social status of mechanics from the perspective of a mathematical practitioner

³ Biagioli, 'The Social Status', 42–44.

⁴ Grendler, *Schooling in Renaissance Italy*, 311.

⁵ Biagioli, 'The Social Status', 43; Grendler, *Schooling in Renaissance Italy*, 22–23.

⁶ Biagioli, 'The Social Status', 56–67.

belonging to the abacus tradition. Tartaglia not only was of ‘humble birth’ but also had an unfavourable childhood; it is said that he only learned half of the alphabet, because by the time he reached ‘k’ he ran out of funds to pay his private tutor. In his later works, he relates how he pieced together his own education. During his lifetime, Tartaglia was a teacher of the abacus in Verona before moving to the Venice as engineer and surveyor. Irrespective of his achievements in mathematics, he is reported to have passed away ‘poor and alone’.⁷

In the dedicatory letter to Henry VIII of *Quesiti e inventioni diverse* (1546), Tartaglia sketched a conception of mechanics mainly concerned with war. Tartaglia mentions that Richard Wentworth, his disciple and a nobleman, referred to the King’s ‘great delight in all matters pertaining to war’.⁸ Because of this, he thought that ‘new things naturally gratify the human intellect’. In his view, these things were the first fruits of his ‘season’ and were presented ‘to gentlemen and to persons of high place, not for their quality, but for their novelty’.⁹ Indeed, the questions and inventions contained in the book ‘are mechanical things, plebeian, and written as spoken, in rough and low style’. ‘Mechanical’ is presented in the company of ‘plebeian’, ‘rough’, ‘low’. Furthermore, Tartaglia explains that his book was written in the form of a dialogue between an expert and a layman. The writing was so close to oral expression (‘written as spoken’) that Baldi in his *Cronica* (1617) remarked how the language ‘brings a smile to the face of those who read his works’.¹⁰ The form of dialogue, later fuelled by the *Mechanical Problems*’ structure of question-answer, dominated the genre until the end of the century, when the axiomatic style of Euclid largely supplanted it.

A different perspective emerges from Robert Norman’s presentation of his experimental enquiries on magnets. Norman was an elite craftsman whose reputation

⁷ Masotti, ‘Tartaglia, Niccolò’, 258, 259; Pisano and Capecchi, *Tartaglia’s Science of Weights and Mechanics in the Sixteenth Century*, 3–20.

⁸ On the puzzling question of Tartaglia’s dedicatory in the context of patronage and social classes see Pisano and Capecchi, *Tartaglia’s Science of Weights and Mechanics in the Sixteenth Century*, 57.

⁹ Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 100.

¹⁰ ‘Attese nondimeno così poco alla bontà della lingua, che move à risa talhora chi legge le cose sue’ Baldi, *Cronica de Matematici*, 133.

was based on his experience as a seaman that later settled down in Ratcliff to make nautical instruments; his sea-compass was widely recognised as a key improvement for navigation.¹¹ In his *Neue attractiue* (1581), Norman exposed the findings on the properties of the magnets ‘grounding my arguments onely upon experience, reason, and demonstration, which are the grounds of Arts’. Norman is aware that not only his way of enquiry but also his conclusions run against the traditional way to ‘discover the secrets of nature’ by the ‘learned or ancient writers’. These learned are ‘the Mathematicalles’ considering ‘that this is no question or matter for a Mechanitian or Mariner to meddle with, no more then is the finding of the Longitude’. The reason behind this exclusion is that the matter ‘must bee handled exquisitely by Geometricall demonstration, and Arithmetickall Calculation; in which Artes, they would have all Mechanitians and Sea-men to be ignorant, or at least insufficientlie furnished to performe such a matter’. Norman quoted the proverb of Apelles that summarised, in his view, the attitude of ‘the Mathematicalles’ towards the ‘Mechanitians’ crossing boundaries: ‘Ne sutor ultra crepidam’ (‘Shoemaker, not beyond the shoe’).¹²

Nevertheless, the distinction between mechanics and mathematicians is not based on the knowledge of mathematics. Norman emphasised that mechanics ‘have not the use of the *Greek* and *Latin* Tongues, to search the varietie of Authors in those [mathematical] Artes’. However, ‘they have in *English* for Geometrie, *Euclides* Elements, with absolute demonstrations: and for Arithmeticke, *Records* works ... which books are sufficient to the industrious mechanician to make him pearfect and ready in those Sciences, but especially to apply them same to the Art and faculty which he chiefly professeth’.¹³ In other words, mechanics were as well trained in mathematics as other mathematicians, according to him. But this presentation also provides some insights into other changes related to mechanics: the allusion to mathematical works in ‘vulgar languages’ is a sign of the growing importance of mechanics and the improvement of mechanics for military and civil purposes (trade, navigation, infrastructure). The fact that a craftsman published a book (a

¹¹ Taylor, *The Mathematical Practitioners of Tudor and Stuart England*, 172–73.

¹² Norman, *The Newe Attractiue Containyng a Short Discourse of the Magnes or Lodestone*, i–iii.

¹³ Norman, ii–iii.

rather expensive enterprise) implies that he considered his enquiries of public interest. The *Neue Attractiue* originally published in 1581, was enlarged with a presentation by William Borough—Treasurer of Queen’s Ships and Master of Trinity House—and reprinted in 1585, 1590 and 1592. In this preface, Barlow, an Oxford educated clergyman with interest in mathematics and mechanics, said of Norman that ‘although he was not learned, yet was a very expert mechanician’.¹⁴

Some perspectives of the mathematicians on mechanicians highlight further struggles for social recognition. During the development of mathematics in the second half of the sixteenth century in England some mathematicians, especially those perfecting instruments, raised up their social status by stressing the errors of the mechanicians, accusing them of being uneducated and vulgar because of their poor knowledge of mathematics. Instead of promoting the study of geometry to improve the practice of manual workers, as was frequent in Italy, the theme of the ‘vulgar errors’ was to turn on the advantage of these mathematicians encouraging the use of sophisticated instruments to improve their social status at the expense of the craftsmen’s.¹⁵ These mathematicians portrayed their carefully crafted instruments and their abstract knowledge as a solution to the ‘errors common in [the] daily practice’ of the ‘ignorant of Arithmetike and Geometrie’. Books and pamphlets disapproving the practices of craftsmen circulated in London.¹⁶

In the higher range of the social spectrum of mathematics we find the ‘celestial’ mathematicians, that is, the astrologer-physicians.¹⁷ These were mostly related to universities and this safe positioning implied that they sustained conservative role alignments.¹⁸ Indeed, mathematics held a safe place in the universities given the links between astrology, a mathematical discipline, and medicine. While there was no degree or licensing in mathematics—a sign of its ancillary position, the mathematical

¹⁴ Taylor, *The Mathematical Practitioners of Tudor and Stuart England*, 172–74.

¹⁵ ‘Yet the errors do not come from the art but from those who practice the art’. Newton, *The Principia*, 381.

¹⁶ Johnston, ‘Mathematical Practitioners and Instruments in Elizabethan England’, 325–26. The quotations come from the title of Edward Worsop’s *A Discoverie of sundrie errours and faults daily committed by Landmeaters, ignorant of Arithmetike and Geometrie* (1582)

¹⁷ Biagioli, ‘The Social Status’.

¹⁸ Westman, ‘The Astronomer’s Role’, 117.

education of physicians was customary and provided mathematics a position beyond the *quadrivium*.¹⁹ Widely recognised astronomers and mathematicians obtained their degrees in law and medicine, such as Copernicus, or in theology, such as Kepler, and usually developed their careers in mathematics outside the universities.²⁰ In fact, universities were oriented towards three professions: law, theology and medicine. Law and canon law were central as preparation for the clergy, and consequently these faculties were powerful and influential. Medicine was prestigious not only intellectually, for its close connection with natural philosophy, but also socially for its acclaimed utility. The connection between medicine and the preservation of life positioning physicians in royal courts dates back to legendary times.

Meanwhile, the connection between astrology and medicine has a long history going back at least to Ptolemy's *Tetrabiblos*. However, a new impetus was given to their connection in the Renaissance after the recovery of Neo-Platonic and Stoic sources by Marsilio Ficino and Pico della Mirandola. Celestial bodies influenced life on Earth. A collection of theories rooted in ancient times postulated diverse entities responsible for these interactions. Astrology provided calculations to diagnose and forecast the development of diseases and plagues, the outbreak of wars or the fate of the harvest based on the application of mathematical calculations and instruments. In the case of medicine, the reformed natural philosophies and medical theories challenging Aristotle and Galen provided causal accounts of the entities responsible for diseases—Paracelsus, Jean Fernel and Girolamo Fracastoro were the most influential. On the other hand, astrology accounted for the conjunctions of the stars against the background of fixed stars and the angular separations of heavenly bodies. These conjunctions were meaningful to diagnose or even to predict when someone reached the point of no return in their way to death.²¹ This alliance between the

¹⁹ Maclean, *Logic, Signs and Nature in the Renaissance: The Case of Learned Medicine*; Andersen and Bos, 'Pure Mathematics'; Westman, 'The Astronomer's Role'; Henry, 'Why Jean Fernel'.

²⁰ Rose, *The Italian Renaissance of Mathematics*, 118–20; Westman, 'The Astronomer's Role', 117–18.

²¹ Henry, 'Why Jean Fernel'; Henry, 'The Fragmentation of Renaissance Occultism'; Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order*, 43–48; 62–66.; Thomas, *Religion and the Decline of Magic: Studies in Popular Beliefs in Sixteenth and Seventeenth Century England*; Rutkin, 'Astrology'.

explanatory role of the (reformed) natural philosophies and the predictive function of the astronomer in the person of the physician was a conservative one.²²

The connection between astrology and medicine made possible that some mathematicians were not limited to teaching introductory studies; they could have a place as well in Medical Schools, yet significantly inferior in social status (and salary) to professors of medicine. In contrast with the practitioners trained in the abacus school, whose highest position was usually as city's official surveyors, the university teachers were required to compile the city's yearly horoscopes.²³

However, mathematicians connected to universities were in better position when compared to its 'terrestrial' counterparts because of their connection with medicine, not because their mathematical knowledge was highly valued. Interestingly, the link between mathematics and medicine amounted to a social bridge that provided 'celestial' mathematicians with a route to climb up in social status that their counterparts lacked. Crossing the bridge was not a matter of personal choice; mathematics, even celestial, was not a profession standing by itself, lacked social recognition and its practise was poorly paid—when paid. The reasons leading Jean Fernel, a practitioner of mathematics, to become a physician illustrate this point. Fernel tried to make a name for himself and for a living as a mathematical practitioner, before completely abandoning mathematics for medicine.²⁴ Since his formative years, Fernel was more inclined to mathematics than to the established professions—theology, law and medicine—, as it is indicated by his publication of three books on mathematics during his training as physician. Subsequently, Fernel not only published other books on mathematics (remarkably on cosmography), but also accumulated an impressive collection of mathematical books and instruments, a very expensive venture by then. His practice and also his attempts to win patronage for his mathematical work depict someone with 'genuine ambitions to establish himself as a leading cosmographer'. However, after publishing *De proportionibus* in 1528, Fernel fully dedicated to medicine and the publications on the matter quickly established his reputation as a leader in the field. By the time that his 1542 *De*

²² Westman, 'The Astronomer's Role', 118–19.

²³ Biagioli, 'The Social Status', 43.

²⁴ On this, see Henry, 'Why Jean Fernel', 196–206.

naturali parte medicinæ appeared in print, his collection of mathematical books and instrument had been decimated. The reason for this change of direction came from Fernel's family. His father-in-law realised that Fernel was not earning enough to support his family out of mathematics and was using his daughter's dowry to fund his expensive mathematical undertakings. He also complained that medicine, Fernel's early devotion, was no longer of his interest; in his first biography, it is reported that he was so 'clung to mathematics that neither love of his wife, nor the endearments of his children, nor the care of his house, could take him off them'.²⁵ Once Fernel moved to medicine and climbed up to be a respected professor of medicine at the Collège de Cornouailles in Paris, he attracted a huge personal practice and became physician of Henri II. By the time of his death, Fernel had inspired a considerable number of followers who practised medicine all over Europe and left a fortune.²⁶

While 'celestial' mathematicians were in a better position than their 'terrestrial' counterpart, the social status of mathematics was largely considered subsidiary to medicine in the former case and 'plebeian' and vulgar in the latter. These two trends constitute the axes from which the wide-ranging transformations of mechanics spread out. As late as 1577, Guidobaldo dal Monte declared against the bad reputation of mechanics 'as for certain recent manipulators of words who deprecate mechanics, let them go and wipe away their shame, if they have any, and stop falsely charging mechanics with lack of nobility and lack of usefulness'.²⁷ Mechanics was still a second-class practise; not everyone was dazzled by the rigour of the mathematical proofs or the benefits of their application.

In the meantime, the Italian city-states witnessed some of the most impressive developments of mechanics and mathematics, to a large extent, as part of a response to the Italian wars and particularly to the disturbance produced by the invasion of Charles VIII of France which inaugurated the so-called 'cannon age'. Military technologies—arsenals and the improvement of fortifications—came to be seen as

²⁵ See Henry, 207.

²⁶ Forrester and Henry, *On the Hidden Causes of Things*.

²⁷ Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 243.

central to the power of the princes and the political stability of the cities after these had proven to be vulnerable to the King's cannons. In *Il Principe* (1532), Machiavelli assessed the strength of the principalities by their capability to support themselves, stressing a well-fortified town. In his view, a powerful prince is the one 'not hated by his people' and whose 'town [is] well fortified'.²⁸ This view stimulated to the emergence of the *milites*, ex-soldiers from aristocratic origins who became practitioners of mechanics as a contemplative, mathematical science that promoted the improvement of the practice of the manual workers by the development of mathematics. The 'syndrome cannon' and the introduction of the bastion forced *milites*, the professional warriors of aristocratic origins 'to begin to rely less on their horses and more on Euclid for their survival as a distinct social group'.²⁹ Guidobaldo, wealthy son of an ex-soldier turned into nobleman for his services to the Duke of Urbino and soldier himself for a short time in the wars between the Hapsburgs and the Ottomans, not only praised mechanics but practised it, first by getting some training at the University of Padua and later by joining the circle of Federico Commandino.³⁰ But Guidobaldo's father was not an isolated case; soldiers and military engineers were ennobled by their princes in recognition of their services.³¹

The interest in mechanics also made its way into different forms of education. Mathematics was progressively included in the education of young aristocrats. Galileo tutored the young Cosimo II de Medici during some summers and Giovanni Magini—who was appointed chair of mathematics at Bologna over Galileo—tutored Vincenzo I Gonzaga, Duke of Mantua, who later became a major patron of arts and science employing artists like Monteverdi and Rubens.³² By the mid sixteenth-century major Italian universities rearranged the teaching of mathematics sometimes by fusing chairs or by appointing teachers with backgrounds in practical mathematics. Astronomical and astrological teaching concerning medicine was still

²⁸ Machiavelli, *The Prince*, chap. 10.

²⁹ Biagioli, 'The Social Status', 44.

³⁰ Bertoloni Meli, 'Guidobaldo Dal Monte and the Archimedean Revival'; Baldi, *Cronica de Matematici*, 145–47.

³¹ Biagioli, 'The Social Status', 45.

³² Drake, *Galileo at Work: His Scientific Biography*; Heilbron, *Galileo*; Fenlon, *Music and Patronage in Sixteenth Century Mantua*, 119–60.

the centre of the attention, but some teachers with professional background in geography, hydrostatics and fortification began to be appointed chairs of mathematics. The University of Padua was probably the first, and perhaps the only, offering lectures on mechanics based on the *Mechanical Problems*. Leonico and Piccolomini taught there, Guidobaldo attended Catena's lectures on mechanics by 1564, as did Baldi from 1573 to 1575.³³ Three teachers of mathematics lectured on mechanics following the text of the *Mechanical Problems*: Pietro Catena, Giuseppe Moletti and Galileo Galilei. Unfortunately, most of their notes for the lectures are lost, except for some of Moletti's.³⁴ According to the university records, Galileo's duties included publicly teaching mechanics by 1598. It is also known that he had been offering private lessons for which various syllabuses still remain—some of them gathered together and published posthumously as *Le meccaniche*.³⁵

The rising of absolutist states in Italy involving alterations in the structure of sixteenth-century courts opened new avenues for interaction with political power that ended up opening possibilities for the emerging mathematicians. Before these changes (1400-1500), courts were unstructured bodies with bureaucratic and administrative functions not clearly differentiated. In the new structure, the distinction between functions of state management and representation of the prince's power opened a door for mathematicians in court. In practice, this distinction modified the above-mentioned axes of the social status of mathematicians. The administrators, by hiring practical mathematicians for military and civil works, created a new distinction by appointing mathematicians to increase the representation of the power of the prince. After gaining the favour of the prince, these new upper-class mathematicians were responsible for the production of *mirabilia* to honour their patrons such as theatrical machines, or as Galileo did later, the telescope and the Medicean Stars.³⁶ But more often than not, practical mathematicians did not play any important part in this new role and their social status did not change in any considerable way. Another group, that we now identify with 'fine arts' (painters,

³³ Laird, 'The Scope of Renaissance Mechanics', 58–59.

³⁴ Laird, 58–64; Bertoloni Meli, *Thinking with Objects*, 12–15; Rose and Drake, 'The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture', 92–96.

³⁵ Rose and Drake, 'The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture', 94.

³⁶ Biagioli, 'The Social Status', 47.

sculptors and architects) used this path to increase their position that materialised in institutions such as the *Accademia del disegno* in Florence. These artists shaped a fresh conception of visual arts drawn from the same roots of the mechanical arts blended with the recovery of mathematics, but stressed ideas of harmony and order rather than utility. By putting together architecture, sculpture and painting under the idea of *disegno*, the social status of visual artists increased considerably with this new way of patronage, Michelangelo and Vasari being the best examples. Visual artists forged a professional identity and found a rewarding way of glorifying the prince.³⁷ The resemblance with mechanics is striking: Galileo's wrapping up of the satellites of Jupiter with the mythology making 'Jupiter' and 'cosmos' into symbols for the founder of the dynasty is paralleled with Vasari's reform of the Palazzo Vecchio under Cosimo I, portraying the Medici family in a similar narrative. The paintings in the ceilings and the association of mythological names with the uses of rooms hinted at a view of the prince and his family fused together with the genealogy of Roman gods.³⁸

1.2.The most wondrous thing

Social and political factors played important roles in shaping mechanics as a significant practice in the cultural life of the sixteenth century. This change had consequences in the transition of mechanics from a vague set of operational rules into a structured science. Mechanics was becoming more visible, some mathematicians were gaining respected places in society and mathematical education was extending beyond traditional practitioners through several forms of training. In this context, the circulation of the *Mechanical Problems* attributed to Aristotle provided a set of concepts and problems to which humanist and mathematicians referred in order to codify the dispersed set of rules. Other ancient texts dealing with mechanics were translated and circulated during the period, such as Archimedes or Hero. However, *Mechanical Problems* was particularly influential because it addressed the pressing question, coming from the most educated, concerning the

³⁷ Biagioli, 54.

³⁸ Payne, 'Vasari, Architecture, and the Origins of Historicizing Art'; Kirwin, 'Vasari's Tondo of Cosimo I with His Architects Engineers and Sculptors' in the Palazzo Vecchio. Typology and Re-Identification of Portraits'; Rosen, 'A New Chronology of the Construction and Restoration of the Medici Guardaroba in the Palazzo Vecchio, Florence'.

framework to explain in causal terms the visible achievements of mechanics and engineers.³⁹ The *Mechanical Problems*' opening outlined mechanics as a contemplative science and its outcomes as matching in certainty those of traditional middle-sciences, such as astronomy and optics. Mechanical arts may be the practice of mechanics; but the knowledge involved in their practice—explaining the construction and operation of machines—is *scientia*.

Mechanical Problems is made up of an introduction, noticeably discussed in the late Renaissance, and thirty-five problems. Some are daily-life problems, while others are more theoretical in nature.⁴⁰ In this section, I will look at the introduction in order to highlight the aspects of the discussion relevant for the transformation of the discipline. Although the reception of specific problems was influential in the assimilation of the work, the philosophical nature of the opening lines of *Mechanical Problems* and the prestige of its attributed author made it stand out.⁴¹ The main concern of the introduction is to depict the knowledge of the problems called mechanical (τῶν προβλημάτων μηχανικὰ) as a science (ἐπιστήμη), rather than to differentiate what happens by nature (κατὰ φύσιν) and what occurs beyond it (παρὰ φύσιν) or to distinguish between natural and artificial; these are subordinate claims. According to the tradition receiving the text in the Renaissance, the introduction is divided in three parts: general considerations, the lever and the peculiarities of the circle.

The introduction starts off with two kinds of things considered wondrous (θαυμάζεται):⁴² in the domain of things occurring by nature, those whose cause is unknown; among things occurring beyond nature, those that come about by means of

³⁹ Valleriani, 'The Transformation of Aristotle's Mechanical Questions'; Laird, 'The Scope of Renaissance Mechanics'; Rose and Drake, 'The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture'.

⁴⁰ Cf. Schiefsky, 'Art and Nature in Ancient Mechanics'; Schiefsky, 'Structures of Argument and Concepts of Force in the Aristotelian Mechanical Problems'. Leeuwen, *The Aristotelian Mechanics*.

⁴¹ Because sixteenth-century writers did not doubt about the authenticity of the text, I assume for my argument that Aristotle is the author.

⁴² 847a10-11. I have used the Greek text in Aristotle, 'Mechanical Problems'.

art (διὰ τέχνην). In the second case, Aristotle specified that their purpose is the benefit of humankind (πρὸς τὸ συμφέρον τοῖς ἀνθρώποις). The course of nature, acting ‘always in the same way and simply’ (ἀεὶ τὸν αὐτὸν ... τρόπον καὶ ἀπλῶς), is contrasted to what is useful (τὸ δὲ χρήσιμον) that ‘changes in many ways’ (μεταβάλλει πολλαχῶς).⁴³

The distinction between things occurring by nature and those beyond it has been the focus of attention since the early modern period. The complexity of the passage is better understood when it is noted that ‘by nature’ is a central concept in Aristotelian thought encompassing, among others, necessity (ἀνάγκη) and what belongs to something in itself (καθ’αὐτό), not to mention potentiality (δύναμις) and actuality (ἐνέργεια). In fact, the development of the constitutive principle of substances (εἶδος) individuating them—from potentiality to actuality—is said to occur ‘by nature’. On the other hand, Aristotle uses παρὰ φύσιν in inconsistent ways: (1) beyond nature in the sense of supernatural, or something exceeding the scope of natural processes. (2) It can refer to something contrary to nature, for example natural as opposed to violent or forced motion. (3) It is widely used in the *corpus* to signify what is unusual, not as a general rule but as an exception, for example when explaining ‘monstrous births’ (τέρας). Finally, it can signify ‘beyond nature’ not in the sense of something outside the domain of nature but of something that nature does not do without human intervention. This last notion has notable precedents in Hippocratic medicine as is widely referred to by Aristotle when dealing with this art.⁴⁴ When reading the opening passage using the second meaning, as some traditional readings did, art and nature seems to be conflicting.⁴⁵ When this sense is combined with the third, art is reduced to the study of violent motion, being a

⁴³ 847a15-16.

⁴⁴ Schiefsky, ‘Art and Nature in Ancient Mechanics’, 75–76; Leeuwen, *The Aristotelian Mechanics*, 161.

⁴⁵ See Krafft, ‘Die Anfänge einer theoretischen Mechanik und die Wandlung ihrer Stellung zur Wissenschaft von der Natur’.

consequence of differentiating between phenomena occurring κατὰ φύσιν as subject of natural philosophy, and phenomena occurring παρὰ φύσιν as subject of mechanics. But the opposition in this passage is between the constant operation of nature and the changing nature of human needs and desires, not between art and nature.⁴⁶

The changing character of human desire, as opposed to the regularity of nature, implies that ‘whenever, then, it is necessary to do something beyond nature, because of the difficulty we are at a loss and have need of an art’.⁴⁷ The passage is not entirely clear, because the adverb ‘whenever’ (ὅταν) does not specify under which circumstances it is necessary to do something beyond nature. It seems plausible to infer that ‘whenever’ refers to those cases in which the regular nature and the changing desire do not coincide. On the other hand, the passage states that in those cases ‘we are at a loss’ (ἀπορίαν) and ‘have need of an art’. Stones naturally fall, but if we wish to lift them up, because of the difficulty—the one-way course of nature—, we need to *know how*. This situation is described as ‘being at a loss’ (ἀπορία), which is the state previous to knowledge in which we are perplexed. Because of this, we need an art. In *Nichomachean Ethics*, a text widely used in the sixteenth-century universities, Aristotle claims that

Art is identical (ταὐτὸν) with a state of capacity to make (τέχνη καὶ ἔξις), involving a true course of reasoning. All art is concerned with coming into being (γένεσιν), i.e., with contriving and considering how (τεχνάζειν καὶ θεωρεῖν) something may come into being (...) art is concerned neither with things that are, or come into being, by necessity (ἐξ ἀνάγκης), nor with things that do so in accordance with nature (κατὰ φύσιν).⁴⁸

⁴⁶ Schiefsky, ‘Art and Nature in Ancient Mechanics’, 77; Micheli, *Le origini del concetto di macchina*, 86.

⁴⁷ 847a18-19.

⁴⁸ *Nic. Eth.* 1140a4-10. For the Greek text I have followed Aristotle, *Aristotle’s Nichomachean Ethics*, ed. J Bywater (Oxford: Clarendon Press, 1984). For the translation, I have followed Aristotle and Johnathan Barnes, *The Complete Works of Aristotle. The Revised Oxford Translation*, 2 vols (Oxford: Oxford University Press, 2005);

In the light of this passage, rather than claiming that mechanicians are required to act against nature, Aristotle claimed that when we want to do something not performed by nature itself we are initially perplexed; in consequence, we require knowledge (ἐξίς) of the know-how (τεχνάζειν καὶ θεωρεῖν), an art. This specific art uses ‘devices’: ‘we also call that part of art that assists in such situations a device (μηχανήν)’.⁴⁹ In other words, this art is the knowledge of bringing into existence devices assisting us to go beyond nature, for human benefit. The passage is summarised by quoting Antiphon: ‘By means of art we gain mastery over things in which we are conquered by nature’.⁵⁰

At this point, Aristotle introduced two examples of the kind of situation in which a device supports us in going beyond nature: when ‘the lesser master the greater’ and when ‘things possessing a small inclination move great weights’. In Aristotle’s view, these are general instances of what he called mechanical problems (τῶν προβλημάτων μηχανικὰ).⁵¹ Although the references are general, the first can be understood in the light of the paragraph immediately following, in which Aristotle claimed that one of the most surprising achievements of mechanics was the movement of a great weight by a small force using a lever. Aristotle considered the lever a paradigm of mechanics, for it is the simplest machines by which ‘a great weight can be moved by a small force’. The second makes references to the inclined plane, a machine widely used in antiquity, yet the knowledge of its law appeared later.⁵²

These general considerations locate mechanical problems between mathematics and physics: ‘These [mechanical problems] are not entirely identical with physical problems nor entirely separate from them, but they have a share in both mathematical

⁴⁹ In translating this central concept, I have followed Schiefsky, ‘Art and Nature in Ancient Mechanics’; Leeuwen, *The Aristotelian Mechanics*. For a philological study of μηχανή see Micheli, *Le origini del concetto di macchina*, 9–20.

⁵⁰ Cf. Schiefsky, ‘Art and Nature in Ancient Mechanics’, 78–79; Micheli, *Le origini del concetto di macchina*, 139, 143.

⁵¹ 847a22–24.

⁵² Roux and Festa, ‘The Engima of the Inclined Plane’.

and physical speculations: for the ‘how’ (ὥς) in them is made clear through mathematics, while the ‘about what’ (περὶ ὅ) is made clear through physics’.⁵³ The passage sums up an Aristotelian commonplace: that some sciences take their object from one discipline and their way to explain it from other. However, this should not be taken as any kind of mathematical physics. On the contrary, the idea that every science has its proper object and method lies at the heart of Aristotelian thought (μετάβασις εἰς ἄλλο γένος).⁵⁴ Mixed or middle-sciences, as they came to be known in the Middle Ages, are not the exception. Aristotle’s definition of science implies that knowledge of something is always taken from some perspective: physics is the knowledge of nature per se, that is, of natural things insofar as they are natural. However, natural objects can be considered from other viewpoints—from any of the categories—which are accidental, but also provide some knowledge. Put otherwise, we can consider things from different perspectives: science in its proper sense considers things from what makes them being what they are (εἰδός), but we can also consider them insofar as they are measurable, for example.⁵⁵ The passage, then, clarifies that mechanics belongs to this kind of knowledge: its ‘what about’ comes from physics; its way to deal with it (‘how’) from mathematics. Aristotle claims here that mechanics belongs to the sciences considering physical objects not *qua* natural, but *qua* mathematical: natural objects are measurable, although measurability is not their defining characteristic.⁵⁶ As renaissance commentators highlight, this definition includes astronomy, optics and music.⁵⁷ On the other hand, the objects of mechanics are natural objects modified in order to fulfil human desires and, in consequence, their explanation should include this desire as final end (τέλος).

In the Aristotelian context, to know is to know why and in this sense, the possibility to specify the causes of the phenomenon.⁵⁸ In the kind of mechanics that Aristotle

⁵³ 847a25-30.

⁵⁴ See Funkenstein, *Theology and the Scientific Imagination*, 299–327.

⁵⁵ On this see *Met.* 983a24-984b22; 193b22-30 and *Post. An.* 78b36-39. See also Schiefky, ‘Art and Nature in Ancient Mechanics’, 90–91; Lear, *Aristotle: The Desire to Understand*.

⁵⁶ See *Cat.* 1b25-6a35.

⁵⁷ Laird, ‘The Scope of Renaissance Mechanics’; Rose and Drake, ‘The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture’.

⁵⁸ See, for example, *Met.* 982a20-982b-10.

sketched, the formal cause would be provided by mathematics, because it establishes the principles of explanation (ὥς). The primary cause of all mechanical phenomena is the circle (Πάντων δὲ τῶν ποιούτων ἔχει τῆς αἰτίας τὴν ἀρχὴν ὁ κύκλος).⁵⁹ Because the way of explaining is mathematical, the geometric properties of the circle provide the principles or causes of mechanical phenomena. The circle is described as wondrous, for it encompasses the combination of opposites, ‘the most wondrous thing’. The fundamental opposition is that between ‘the moving and the stationary’ acting at the same time in generating the circle: tracing a circle encompasses both. The mathematical properties of the circle, from which all mechanical phenomena could be explained, include: (1) the presence of the concave and the convex in the circumference. (2) The simultaneous motion in opposite directions—backwards and forwards—of the extremes of a diameter when a circle is rotating on its centre. And finally, (3) the fact that, as it rotates, a point farther from the centre moves faster than one closer to it.

Because of these properties, the argument runs, there is nothing strange in the circle being the origin of all these wondrous things (οὐδὲν ἄτοπον τὸ πάντων εἶναι τῶν θαυμάτων αὐτὸν ἀρχήν).⁶⁰ By origin (ἀρχήν) Aristotle means the principle of explanation of these mechanical wonders that, once made clear, are not marvellous anymore. The circle is the starting point of the mechanical explanation, for the phenomena of the balance can be explained (ἀνάγεται) in terms of the circle and those about the lever in terms of the balance and ‘nearly all other problems of mechanical movement can be explained in terms of the. The core of the passage is the verb ἀνάγεται whose meaning is central not only in terms of how mechanics should proceed in the *Mechanical Problems* but also became relevant to the late renaissance projects of restoring ancient mechanics.⁶¹ The verb has a wide variety of meanings in ancient Greece: in a physical sense, it means leading up (from a lower

⁵⁹ 847b16-17.

⁶⁰ 848a10.

⁶¹ Bertoloni Meli, *Thinking with Objects*, 24; Schiefky, ‘Structures of Argument and Concepts of Force in the Aristotelian Mechanical Problems’, 50; Laird, ‘The Scope of Renaissance Mechanics’; Bertoloni Meli, ‘Mechanics’.

place to a higher); in a more abstract sense, it can be translated as bringing back, referring to (its principles), reducing one thing to another (forms of syllogisms) or finally reckoning or calculating. Commandino systematically rendered it as *reducere* and Guidobaldo assumed this verb as the cornerstone of the procedures in mechanics: all kind of machines can be reduced to most basic principles of geometry in order to demonstrate their properties.⁶² What appears as wondrous at first sight can be explained in terms of the properties of the circle; from the lever to any mechanical problem. Aristotle added that by using these properties of the circle ‘craftsmen make an instrument concealing the original circle, so that the marvel of the machine is alone apparent, while its cause is invisible’.⁶³

1.3. La virtù de la machina

The distinction between natural and artificial lies at the heart of mechanics as a form of knowledge. *Mechanical Problems* sketched a science of machines rather than straightforwardly tackling the general question of what is artificial and how it differs from the natural. However, the characterisation of mechanics and the kind of problems postulated in the work recurrently brought up the distinction. Most Renaissance translators had something to say about this. Some of the most puzzling passages were interpreted in the light of other Aristotelian works particularly by humanist scholars. Also, some practitioners of mathematics attempted to reconcile the *Mechanical Problems* with other ancient authors such as Archimedes, Apollonius, Hero, and Pappus, considering Aristotle’s mechanics as embedded in a more general trend. In so doing, these practitioners began to deal—not always fully aware of it—with incompatible approaches to mechanical problems that either emphasised or rejected the role of motion. These tensions surface in the attempts to codify and axiomatise mechanics.

Mechanical Problems was published for the first time in Greek as part of Aldus Manutius’ edition of Aristotle’s works (1495-1498); there is no record of any

⁶² Bertoloni Meli, ‘Guidobaldo dal Monte and the Archimedean Revival’; Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 241; M. Henninger-Voss, ‘Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge’.

⁶³ 848a36-37.

previous reception. The Venetian humanist Vittore Fausto made the first Latin translation in 1517. However, Niccolò Leonico Tomeo's overshadowed this translation in 1525. This went through successive editions, becoming the standard in the sixteenth century; Galileo's copy, for example, was an original 1525 edition.⁶⁴ Tomeo, a professor of Aristotelian philosophy at Padua and protégé of cardinal Besarion, also translated other works of Aristotle. The translation of *Mechanics* was published together with translations of and commentaries to *De motu animalium*, *De animalium incessu*, and an extract of Proclus' commentary to Plato's *Timaeus*.⁶⁵ His 1525 translation of the *Mechanical Problems* includes commentaries on the Aristotelian text and diagrams that were removed from subsequent editions.⁶⁶ Because Tomeo's interest is mainly philological rather than mathematical, his commentaries were harshly criticised both by philosophers and mathematicians for not extending or developing the conceptual and mathematical aspects of the text.⁶⁷

From Tomeo's philological angle, there are revealing aspects both in his translation of and in his commentary to *Mechanical Problems*. The first sentence of the translation, highlighting the wondrous character of some phenomena, reads: 'Miraculo sunt ea quidem quæ natura contingunt quorum ignorantur causæ: illa vero que preter naturam quæcunque ad hominum utilitatem arte fiunt'.⁶⁸ The commentary omits all reference to natural things whose cause we ignore and focuses on the nature of the productions of art. 'Preter naturam' is referred to the outcomes of art insofar as they repeatedly go beyond the 'laws of nature' (*quoniam sæpenumero ars naturæ transgressa leges*). The main goal behind this transgression is the benefit of man. Tomeo illustrated this with the case of stones and timber used to make the walls and beams of a house, arranged against their natural tendencies, emphasising how they

⁶⁴ Rose and Drake, 'The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture', 80.

⁶⁵ Details on Tomeo's manuscript sources for the translation can be seen in Leeuwen, *The Aristotelian Mechanics*, 165–70. See also Laird, *The Unfinished Mechanics of Giuseppe Moletti*, 10–11; Laird, 'The Scope of Renaissance Mechanics', 48–49; Rose and Drake, 'The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture', 15–16.

⁶⁶ Leeuwen, *The Aristotelian Mechanics*, 167.

⁶⁷ Leeuwen, 167; Laird, 'The Scope of Renaissance Mechanics'.

⁶⁸ Leonico Tomeo, *Opuscula nuper in lucem aedita quorum nomina proxima habentur pagella*, xxiii.

were shaped by art. From here, Tomeo concluded that the simplicity of the natural course contrasts with the complex variety of uses invented by art.⁶⁹

After this, Tomeo claimed that Greeks called this art ‘mechanics’ because it made use of machines to multiply forces in order to overcome the course of nature. Because the use of machines seems to be the distinctive feature of this art, Tomeo made central the definition of machine.⁷⁰ In his view, machines were complex instruments made of the conjunction of matter (*ex continenti materiæ coniunctione*), capable of moving weights (*pondera*) by the arrangement of spheres rotating and revolving (*per orbium quosdam rotates et circuitiones*).⁷¹ Three major issues arise from this definition: machines are not mere devices or instruments, but a specific kind of artificial object composed of instruments; machines are made of matter, i.e., they are ultimately formed by natural elements. Finally, the arrangement or design of the natural elements composing machines is their form and these can be explained in mathematical terms. The geometrical nature of mechanics relied on the unique (mathematical) configuration of its subject.

Tomeo’s definition of machine implies a difference with plain instruments, relying on a tradition going back to Plato.⁷² Both machines and instruments are material objects with particular functions, largely used to simplify human activities. Nonetheless, machines are more complex than instruments, for they incorporate what Greek denominated *μητις* (*ingenium*) that is, the materialisation of design. While instruments are material objects shaped to improve human performance in specific activities (for example, a knife), machines are the result of *ingenium*: they require knowledge and planning. If machines are human constructions after designs (inventions) they presumably have a history: Hero attempted to reconstruct the genesis of the lever.⁷³ The Greek language crystallised this distinction in two

⁶⁹ Leonico Tomeo, xxiii.

⁷⁰ ‘Mechanicam autem edificatoriae artis eam graeci appellaverunt partem quae machinis ad conficienda opera uteretur’. Leonico Tomeo, xxiii.

⁷¹ Leonico Tomeo, xxiii–xxiv.

⁷² Micheli, *Le origini del concetto di macchina*, 10–13.

⁷³ Pappus also reports this in his *Collectiones quae supersunt*. Micheli, 11.

different terms: μηχανή for machines and ὄργανον for instruments.⁷⁴ In the sixteenth century, Vitruvius became the most influential source to define machine. According to him a machine ‘est continens ex materia coniunctio, maximas ad onerum motus habens virtutes’ (a machine is a combination of matter, mainly efficacious in moving great weights). Vitruvius, then, clarified that machines ‘need more workmen and greater power to make them take effect’, whereas instruments ‘accomplish their purpose at the intelligent touch of a single workman’.⁷⁵

A similar set of arguments was further developed in Giuseppe Moletti’s *Dialogue on Mechanics* (1576). Moletti held the professorship of mathematics at the University of Padua, just after Pietro Catena, the first lecturer on the *Mechanical Problems*, and immediately before Galileo. The *Dialogue* pursued two related goals: the presentation of mechanics as a noble discipline worthy of the admiration and dedication of a prince and the foundation of mechanics on Euclid’s geometry, that is, to lay down the Euclidean principles that Moletti considered to underlie Aristotelian mechanics.⁷⁶ In dealing with the subject of mechanics, AN, the character usually asking, prompts debate about the topic by quoting the Vitruvian definition of machine and immediately adding that he did not understand the distinction between machine and instrument. The Prince, PR, the character usually staging Moletti’s points of view, replied that the distinction is such in terms of ‘workmanship’ (*la differenza è manifesta appreso à quello poi che gli la distingue dal lavoro*).⁷⁷ This difference is illustrated with two examples. Machines are represented by the mechanical clock, in which ‘there is much workmanship and gears’, while instruments are exemplified with a lever having ‘little workmanship and no gears’. But Moletti’s argument goes further. In his view, ‘workmanship’ was not enough to distinguish between these two kinds of objects, ‘because more or less workmanship does not make a sensible difference or change the species’. The substantial difference lies on the contrivance: ‘if the contrivance is good or if the craftsman has thought it

⁷⁴ Micheli, 123–24.

⁷⁵ *De architectura*, X, 1.3: 228.

⁷⁶ Laird, *The Unfinished Mechanics of Giuseppe Moletti*, 22; Rose and Drake, ‘The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture’, 93–94; Laird, ‘The Scope of Renaissance Mechanics’, 60–64.

⁷⁷ Laird, *The Unfinished Mechanics of Giuseppe Moletti*, 82.

out well (*Ma si bene il machinamento, ò l'havervi l'artefice pensato su*), then from such contrivance I shall with as much reason call the press with which one presses wine a machine as a crossbow or arquebus'.⁷⁸ Put otherwise, the distinctive characteristic of machines is the complexity of 'machinamento' (*ingenium*) not of 'workmanship'; what truly distinguishes a machine is the materialisation of a design. Moletti was not the only one insisting upon the intellectual character of machines over their material condition.

The emphasis upon machines as products of the mind was a consequence of shaping mechanics as a mathematical science. At the same time, this opened the door to the integration of other mathematical developments into mechanics, put forward by the systematisation projects inspired in the recovery of Euclid and Archimedes, such as Moletti's and Guidobaldo's, and ultimately framed in the idea coming down from Aristotle that mechanics is some kind of theoretical knowledge.

These projects, however, have to be read against the background of other mid-century debates about the disciplinary nature of mechanics leading to divergent conclusions on the balance between mathematics and natural philosophy.

Mechanical Problems put on the table the idea that there was some kind of theoretical knowledge about machines. The kind of problems studied by this science belonged both to mathematics and to natural philosophy, but *Mechanical Problems* hardly specified what this meant. Divergent approaches, coming from humanists, teachers and practitioners of mathematics ended up in mutual disqualifications and bitter disputes. Tartaglia suggested that mechanics might be considered either mathematically—removed from any physical consideration—or else physically.⁷⁹ Guidobaldo condemned this approach for misunderstanding the proper approach of mechanics and the nature of its subject. In his view, Tartaglia's position assumed that

Mechanics could be considered apart from either geometrical demonstrations or actual motion! Surely when that distinction is made, it seems to me ... that all they [Tartaglia and similar mathematicians] accomplish by putting themselves forth alternately as physicists and as mathematicians is simply that they

⁷⁸ Laird, 83.

⁷⁹ Tartaglia, *Quesiti et Inuentione Diverse de Nicolo Tartaglia...*, 78–79.

fall between two stools, as the saying goes. For mechanics can no longer be called mechanics when it is abstracted and separated from machines.⁸⁰

Tartaglia's view articulated a conception deriving from his own practice, from his own way of dealing with mathematical problems coming from different traditions. In fact, Tartaglia embraced the medieval tradition of the science of weights that Commandino and Guidobaldo rejected on the grounds of inadequately appealing to experience and mathematics in their approach. Guidobaldo described the medieval science of weights as 'a thick mist of ignorance'.⁸¹ Guidobaldo insisted on the central character of machines as physical objects, which is one-of-a-piece with his understanding of mechanics as a discipline eminently mathematical. In his view and following the lines of *Mechanical Problems*, the mathematical character of mechanics consists in the reduction of (*reducere*) any machine to simpler forms and all these to the lever, whose mathematical principles constitutes the starting point of mechanics.⁸² Nevertheless, Guidobaldo's insistence on the centrality of machines underlines its material character and offers an interesting nuance of the emerging views of the relationship between mathematics and physical objects. The material character of machines is central to the axiomatisation projects set in by Moletti and Guidobaldo. But this aspect was already present in mechanical trends, at least, since Vitruvius' definition and it utterly found a longer expression in Galileo's science of materials introduced as a *scienza* in his *Discorsi*.⁸³ The main point of the material character of machines is that, while the methods and proof of mechanics are mathematical, its subject comes from physics, i.e., it is ultimately a *natural* phenomenon.

At the end of the century, mechanics begins to acquire an identity as a *science*. The insertion into universities, the publication of textbooks and manuals and the new social forms of practitioners indicate this transformation. Mechanics was considered

⁸⁰ Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 245.

⁸¹ Drake and Drabkin, 244–45. Cf. Bertoloni Meli, 'Guidobaldo Dal Monte and the Archimedean Revival'; Clagett, *The Science of Mechanics in the Middle Ages*. Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 2–60; Rose, *The Italian Renaissance of Mathematics*.

⁸² Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 245–46.

⁸³ See Bertoloni Meli, *Thinking with Objects*, 91–95; Valleriani, 'The Transformation of Aristotle's Mechanical Questions'.

as a theoretical, mixed-science, branch of mathematics, different both from natural philosophy and from the activity of building machines (mechanical arts). The prevailing sense of mechanics circulating before different groups paid attention to the *Mechanical Problems* referred to the practice of craftsmen, as it has been mentioned above.⁸⁴ The theoretical character of mechanics, emerged as a form of assimilation into dominant schemes of knowledge, provided a wide range of points of contact with dissimilar mathematical traditions. The science of mechanics is not only a theoretical, causal explanation of the machines already existing and produced by mechanical arts; it also explains *why* any machine produces the effects it does. Thus, uncovering the principles of mechanics was expected to boost the construction of new machines. In his commentary to *Mechanical Problems*, Piccolomini claims that this work investigates ‘the true reasons for almost all the wonderful machines which not only have already been discovered but which will be invented in the future’.⁸⁵ The study of the *Mechanical Problems* is not limited to the explanation but it is projected into ‘the future’.

The theoretical, causal nature of mechanics is not opposed to the material dimension of its subject. On the contrary, the material character of machines was a defining feature of mechanics and introduced a series of theoretical and operative requirements. This is prominent in Guidobaldo’s mathematical practice. In explaining the balance, Guidobaldo claimed that the weights of the balance act on lines of descent converging toward the centre of the world, not on parallel lines as it was supposed by Tartaglia. The fact that the two weights attached to the sides of a balance were conceived not as parallel but as convergent towards the centre, bore

⁸⁴ On machines before ‘mechanics’ see Eamon, ‘Technology as Magic in the Late Middle Ages and the Renaissance’.

⁸⁵ Piccolomini, *Parafrasi ... sopra Le Mekaniche d’Aristotile*, 11–12; Rose and Drake, ‘The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture’, 82–83.

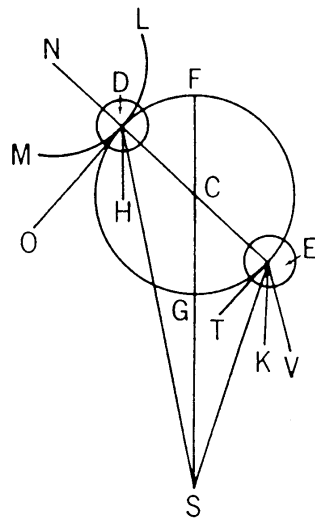


Figure 1. Dal Monte's representation of the lines of descent of a balance.
 Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 265.

important consequences in Guidobaldo's mathematical analysis of this machine—and others—that were criticised since his own times until Duhem.⁸⁶

The diagram represents a balance (DCE) and DS and ES are the lines of descent converging towards the centre of the earth. D and E are the weights. According to Tartaglia, D would be heavier than E because the angle of contact HDG would be smaller than the angle KEG. According to Guidobaldo, E would be heavier, because the angle SEG would be smaller than the angle SDG.⁸⁷ Guidobaldo has been criticised for the excessive rigour of his approach, making virtually impossible to build a balance acting this way. But what underlies this view is not a case of pedantry but a complex relationship between mathematical rigour and the physical world, encompassing the material character of the machines with the mathematical design. The distinction between the geometrical conception and the actual, material machine is not a distinction between theory and practice, but rather between different types of practices. In Guidobaldo's *Mechanicorum liber*, machines are represented under two forms, next to each other, pointing to the dependence of the geometric design on the material configuration.

⁸⁶ Bertoloni Meli, *Thinking with Objects*, 30–32.

⁸⁷ My analysis follows Bertoloni Meli, 30–32.

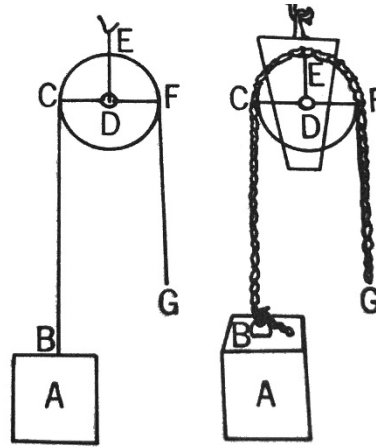


Figure 2. Pulley from *Mechanicorum Liber*. Drake and Drabkin, *Mechanics in Sixteenth-Century*, 305

This connection between mathematical design and material aspects of the machine is also manifest in Guidobaldo's emphasis on the precision of the tools used to build machines out of their geometrical representation. These tools were meant to be a bridge between the geometrical representation and the figure, not between theory and practice. If the right tools were selected, theory will finally agree with experience: Guidobaldo's mechanics is based on equilibrium, not on motion.⁸⁸ Were the approach based in terms of motion, the materials composing the machine would generate disturbances in the mathematical analysis. Because elements have their own natural tendencies, the nature of the body would interfere with the expected mechanical effects. However, in the case of equilibrium, the imperfections of matter would not generate such a disturbance between the geometrical figure and the material conditions, for the geometrical analysis would ultimately provide insights into the counterbalance of these natural tendencies in the production of the machine. A major consequence of this approach is the impossibility of a mathematical science of motion.⁸⁹ For Guidobaldo, the disagreement between machines and geometrical constructions is not a discrepancy between a theoretical prediction and a test or experiment, but a physical limitation in materialising what can be proven as certain of machines in geometry. These limitations may be the lack of skills of the

⁸⁸ Bertoloni Meli, 34.

⁸⁹ Renn and Damerow, *The Equilibrium Controversy*.

instrument maker, the inadequacy of the tools or the contingency of matter but this does not run against the mathematical conclusions.

The relationship between mathematics and the natural world assumed in Guidobaldo's view of mechanics, as different and original when compared to other mixed-sciences, depends on the conception of machines as artificial objects embodying an intentional design—the artificial character of machines and its properties, originated in their mathematical configuration altering the natural tendencies of the elements, in order to accomplish a specific effect. In this sense, machines generate effects transgressing the laws of nature (Tomeo) or in opposition to the laws of nature (Guidobaldo). We have seen, however, that artificial and natural realms were not disconnected. Machines were thought of as natural objects transformed by design in order to redirect the course of nature. This does not create a new kind of object in a strong, metaphysical sense. However, the introduction of a design implies different theoretical resources in order to causally explain its properties.

I have suggested that some sixteenth-century scholars on mechanics held that machines exist as arrangements of natural elements following a complex, mathematical design according to the principles of the circle. Mechanics was primarily concerned with these principles as causes (αἰτία), explaining the generation of effects. The Vitruvian definition of machine highlighted how the properties of the circle are embedded in the machine as their constitutive element. The second part of this definition clarifies that 'a machine is a combination of matter, mainly efficacious in moving great weights. Such a machine was set in motion by art in rotating circles, which the Greeks called (κυκλικὴ κίνησις)' (*ea movetur ex arte in circulatorum rotundationibus*).⁹⁰ The operation of machines is possible by the arrangement of orbs rotating and revolving,⁹¹ that is, the materialisation of the properties of the circle mentioned in *Mechanical Problems*. In clarifying the position of mechanics between mathematics and natural philosophy, Tomeo claimed that machines are made of

⁹⁰ Vitruvius, *De arch.* X, 1. 1, 332.

⁹¹ Leonico Tomeo, *Opuscula nuper in lucem aedita quorum nomina proxima habentur pagella*, xxiv.

natural materials: bars are made of iron and balances and trebuchets out of wood and copper. In this sense and beyond controversy, they are natural things (*rebus quae sine controversia naturales existunt*). However, the mode of knowledge of mechanics (*operandique vim*) is mathematical because starting from the natural materials we search for the circles, diameters and circumferences contained in matter. In Tomeo's view, mechanical knowledge consists of principles of design (*formarum rationes*) abstracted from weights and quantities that naturally exist in matter.⁹² In contrast with the connection between mathematics and the natural world that we have seen in Guidobaldo, Tomeo explained the mode of knowledge of mechanics following the lines of Aristotle's on epagoge, rather than the materialisation of a geometrical configuration. However, Tomeo, as well as Moletti and Guidobaldo, emphasised the design or arrangement of the machine as its distinctive characteristic, discoverable through mathematics. Their differences reveal their philosophical and disciplinary backgrounds.

Moletti's *Dialogue* introduced the matter in a different way. The emphasis here, as in Guidobaldo, was on the useful character of mechanics in order to build machines based on mathematics, instead of the interest of harmonising mechanics with the *corpus*, prevalent in the approach of humanists like Tomeo and Piccolomini. In the *Dialogue*, the prince tells the story that according to Proclus, Hero had built a big ship for Ptolemy, King of Egypt. Once built, the whole population of Syracuse was not enough to launch it into the water. Archimedes said that he wanted him to launch it by himself, to which the King is reported to have laughed. Archimedes made a machine, the King set it in motion and it began to draw the ship into the water.⁹³ Moletti's point seems rather trivial, but it contains the seed of his main argument, for the moral of the story is that 'We shall determine what the cause was that made the ship go into the water.

And we shall find it to have been the machine, not as made of ropes and iron, for one must suppose that those who tried to launch it would have used both ropes and ironworks to do it, but they did

⁹² Leonico Tomeo, xxiv.: 'Pondera quinetiam & mensuras, quae in naturali licet existant material, ab illa certe abstrahere, & sese abducere non immerito videntur, solasq. ostendere & representare formarum rationes'.

⁹³ Laird, *The Unfinished Mechanics of Giuseppe Moletti*, 81.

not succeed. Therefore, we shall say that it was the power of the machine (*la virtù de la machina*), which is none other than its property, and thus it follows that the machine is the subject of which we now seek the property.⁹⁴

Instead of emphasising the material strenght of the machine, Moletti's strategy resembles Guidobaldo's approach in which there is a primacy of *la virtù* over the properties of matter. This property arises from 'the form of the machine (*forma della machina*), which is the same as the principle of the machine, and so when we know the principle, we shall know both the form and the cause of the power of the machine'.⁹⁵ Moletti clarified that by the principle or power of the machine he was not making reference to its capacity to perform effects but to 'its intrinsic power' (*la intrinseca virtù della machina*).

This conception of machines, defined by a design in terms of arrangement, is the foundation of Guidobaldo's programme of axiomatisation of mechanics. Guidobaldo assembled his mechanics out of the historical materials that he gathered from ancient authors, fitting them into the model of certitude and rigour stemming from the geometrical root of mechanics, following lines of Commandino. In Guidobaldo's view, Aristotle, 'the leader of the philosophers', established some physical principles of the science, while Archimedes, who 'alone is to be praised most eloquently, in comparison with all other mathematicians', developed these into mathematical principles.⁹⁶ To illustrate his admiration, Guidobaldo added that Archimedes created a model of the universe enclosed in a sphere so perfect and accurate 'exhibiting the nature of the heavens by their precise motions' that 'the nature herself is thought to have imitated his [Archimedes'] hand'. Also, that Archimedes was able to pull a load of '5000 pecks' using just one hand with the help of a block and tackle. Guidobaldo also brought up the 'engines of war' that Archimedes invented to defend Syracuse against the Romans. The power of mechanics in the hands of Archimedes, remarked Guidobaldo, allowed him to claim that if he could get a place to stand, he shall move

⁹⁴ Laird, 82–83.

⁹⁵ Laird, 83–85.

⁹⁶ In his 'Life of Archimedes', Baldi notes that 'he [dal Monte] shows in the preface of this book [the *Mechanicorum*] that Archimedes followed entirely in the footsteps of Aristotle as to the principles, but added to them the exquisite beauty of his proofs'. Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, 15.

the earth, ‘a statement in such conflict with the laws of nature’ (*eam vocem naturae legibus adeò repugnantem protulerit*). The mathematical genius of Archimedes is praised in function of his ability to build powerful machines (the model of the universe, the tackle and the engines of war), to overcome the laws of nature.

In Guidobaldo’s origin myth of mechanics, Aristotle and Archimedes functioned as supreme authorities laying down the principles on which the science of mechanics was built. Indeed, Aristotle and Archimedes are discussed and reworked in the *Mechanicorum liber*.⁹⁷ Despite their central role, Guidobaldo’s historical conception is not limited to them. Other mechanicians, such as Hero, Ctesibus and Pappus did not reach the pinnacle of mechanics, as did Archimedes, yet ‘they had a remarkable understanding of the subject’, especially Pappus, to whom Guidobaldo declares his deepest admiration. This admiration is based on two features: first, that Pappus never ‘depart[ed] even a nail’s breath from the principles of Archimedes’; and second, because in his works the teachings of mechanics are ‘gathered together, as in an abundant store’. Indeed, Pappus highlighted how ‘all cases of machines’ could be reduced to the five primary machines, whose mathematical principles were ‘brilliantly investigated’. Guidobaldo’s story goes: a ‘mist of ignorance’ which covered ‘all the earth’ (a reference to the medieval science of weights) almost buried the science of mechanics which can now be re-established thanks to the works of Commandino and others who ‘shone like the sun’, dispelling ‘the darkness’. Guidobaldo suggests that Jordanus de Nemore and his followers had taken the place of the true mechanicians, holding ideas that ‘they themselves now declared valid and correct’, but they have to be ‘shaken and overturned’ by the rigour of mathematics. But not only the medieval authors; Tartaglia and other contemporaries are considered by Guidobaldo as still covered by this ‘mist of ignorance’ concealing the ancient mechanics. Guidobaldo’s rejection of the *scientia de ponderibus* runs at odds with most of the leading mathematicians of his age—including Galileo.

Considering himself as part of the recovery of knowledge, Guidobaldo devotes

⁹⁷ Bertoloni Meli, ‘Guidobaldo dal Monte and the Archimedean Revival’; M. Henninger-Voss, ‘Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge’; Valleriani, ‘The Transformation of Aristotle’s Mechanical Questions’.

himself to ‘what a lever is, a pulley, wheel and axle, wedge and screw and how they should be arranged so that weights may be moved’. By establishing ‘the many properties present in those machines by virtue of the lever, properties connecting force and weight’ (*potentia et pondus*), Guidobaldo hoped to rearrange the science of mechanics. The proper way to do this ‘from its foundation to its very top’ is by revealing the properties of the balance. These would explain the functioning of the lever, to which all the remaining simple machines could be reduced. In so doing, Guidobaldo is putting together the pieces of re-emerging mechanical traditions. Indeed, Guidobaldo embraced Pappus’ idea that the science of mechanics was an explanation of any machine by its reduction to the form of one of the five machines. However, Guidobaldo went further and argued that these five machines are to be utterly reduced to the lever, which, in fact, is mathematically explained by the properties of the balance—the central argument of *Mechanical Problems*.⁹⁸ According to Guidobaldo’s *Mechanicorum liber*, Pappus promised ‘to show that the screw is nothing but a wedge used without percussion, which makes its movements by means of a lever, and this is lacking his book, we shall attempt to show this and, moreover, to reduce the screw to the lever and the balance in order that ultimately we shall understand it completely’. The ultimate reduction to the geometrical principle of the lever would be the completion of the science of mechanics.

* * *

This reduction of machines to geometry exhibits a pattern of solving problems by mathematical techniques in which the principles of demonstration constitute the foundation of demonstration and of the construction of machines.⁹⁹ The purpose of this reduction was to uncover the mathematical principles underlying the operation of machines and to open the possibility to build better ones in the future. Guidobaldo’s reductionist approach—encompassing complex material arrangements, mathematical

⁹⁸ Bertoloni Meli, *Thinking with Objects*, 24; Bertoloni Meli, ‘Guidobaldo Dal Monte and the Archimedean Revival’; Bertoloni Meli, ‘Mechanics’; Bennett, ‘The Mechanical Arts’; Schiefsky, ‘Structures of Argument and Concepts of Force in the Aristotelian Mechanical Problems’; Laird, ‘The Scope of Renaissance Mechanics’; M. Henninger-Voss, ‘Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge’.

⁹⁹ Bertoloni Meli, *Thinking with Objects*; Bennett, ‘The Mechanical Arts’; M. Henninger-Voss, ‘Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge’.

principles and the demonstration of the mathematical properties of machines from geometrical representations—reflects the challenges derived from the reorganisation of mechanics as a middle-science of theoretical nature. Guidobaldo's solution to these challenges rearranged mechanics as a science mainly concerned with the equilibrium of bodies. The perfect fit between mathematics and matter entirely depends on this.

2. ‘The dancing whirl of the stars’. Galileo’s *reductio* of motion

Nature first made things in her own way,
and then made human reason skilful enough to be able to understand,
but only by hard work, some part of her secrets.
Galileo, *Dialogue*.¹

It is hard to deny that Galileo formulated the ‘law of free fall’. However, there is no evidence of him using the expression ‘law of free fall’ in the meaning later attributed to it. In the *Excellency of the Mechanical Hypothesis*, Boyle mentioned ‘the laws of acceleration in heavy bodies descending’.² In the *Principia*, Newton suggested that ‘the descent of heavy bodies is in the squared ratio of the time’, and implied that Galileo knew the first two laws of motion and their corollaries.³ Later, Voltaire presented this passage to his readers as the ‘laws of the descent of bodies discovered by Galileo’ (*loix de la chute des corps*).⁴ The attribution of ‘the law of free fall’ to Galileo is common currency in the history of science, including authoritative translations. In the opening lines of the *Dialogue*, the English translator put these words in the mouth of Salviati:

Yesterday we resolved to meet today and discuss as clearly and in as much detail as possible the character and the efficacy of those *laws of nature* which up to the present have been put forth by the partisans of the Aristotelian and Ptolemaic position on the one hand, and by the followers of the Copernican system on the other.⁵

Galileo’s original reads ‘razioni naturali’, alluding to the natural arguments or reasons as opposed to biblical or theological. Interpreting this expression as ‘laws of nature’ rather than as ‘natural arguments’ suggests that Galileo’s aim was the examination of systematic natural philosophies whose outcome was ‘laws of nature’. In this way, both ‘world systems’ are presented as isomorphic. The effect of this approach has played no minor role in the consolidation of traditional narratives of the

¹ Galileo, *Dialogue*, 306.

² Boyle, *Works*, vol. 8:107.

³ Newton, *The Principia*, 424.

⁴ Voltaire, *Elémens de la philosophie de Neuton*, 217.

⁵ Galileo, *Dialogue*, 9.

scientific revolution: Galileo's laws of free fall and the isochronism of pendulums constitute premises of Newton's laws of motion and the universal gravitation.⁶

From the overall majority of translations and scholarship on Galileo, it appears that his achievements can be formulated without distortion in terms similar to Descartes' or Newton's, although he did not conceive them in such a way. These approaches locate the expression laws of nature out of history and make it a general concept that may be applied indiscriminately to any kind of product recognised as knowledge. This is a sophisticated version of a somewhat 'text-book' approach that couches the history of scientific thought in terms of laws. Moreover, an opposite view comes from the terminological studies of laws of nature. Roux suggested that Galileo's part in this story has to do with his use of the term 'rule', which is 'less general' and 'more artificial' than law. However, the term rule, as Roux claims, denotes a numerical relationship indicating a human way to measure rather than the enunciation of the essence or nature of things. The assumption seems to be that Galileo did not use the term law in any significant way.⁷

In this chapter, I present Galileo under a different light. 'Laws of nature' emerged as by-products of transformations taking place in the disciplinary boundaries of mechanics, mathematics and natural philosophy. While Galileo did not rely on the expression laws of nature to organise his achievements and nothing in his thought plays a role comparable to Cartesian 'laws of nature', his work was a sustained effort to reformulate the questions of natural philosophy in a mathematical way, drawing from the traditions outlined in the previous chapter. Thus, Galileo plays a central role in the history of laws of nature because he carried out a significant transformation in the boundaries of disciplines dealing with motion by offering a mathematical explanation of it which was appropriated as a standard of practice by Beeckman and Descartes but also in England. In his formulation, Galileo mobilised concepts and patterns of explanation from the recently fashioned science of mechanics. Galileo

⁶ See, for example, Hall, *From Galileo to Newton, 1630-1720*; Koyré, *Études galiléennes*; Cf. Roux, 'Forms of Mathematization (14-17 Centuries)'; Palmerino, 'The Geometrization of Motion: Galileo's Triangle of Speed and Its Various Transformations'.

⁷ Roux, 'Les lois de la nature à l'âge classique la question terminologique', 546.

rendered plausible a mathematical science of motion and made it available for his contemporaries. By the time Descartes' *Principia Philosophiæ* appeared in print in 1644, most of his readers were familiar with its approach by elements already circulated in Galileo's *Dialogo* and *Discorsi*.⁸ Also, these readers connected Galilean achievements with models similar to those Descartes depicted in his *Principia*. Furthermore, Galileo's extension of mechanics to include phenomena falling exclusively into the domain of natural philosophy a generation before him, prompted a conception of mathematical axioms or principles as causal explanations. Galileo's conception of mathematical principles and causality was informed by the Jesuits' standardisation of Aristotle's teaching. Finally, Galileo felt that his mathematical approach to natural philosophy was enough to reject both Peripatetic explanations and the emerging alternatives to them based on occult qualities.⁹

In elaborating a mathematical science of motion, Galileo developed a view of causal but non-essentialist explanations in natural phenomena, in a way similar to mechanics in connection with machines and different from both the Peripatetic and the naturalist philosophies of the Renaissance. However, these explanations ultimately relied, as I will argue, on the assumption of an Earth in motion. In a sense, Galileo's axioms, rules or principles, insofar as they are *mathematical*, are not comparable to (physical) 'laws of nature'. In other words, the applicability of Galileo's mathematical principles to nature relies on the (physical) postulate of the motion of the Earth. In fact, once the motion of the Earth is assumed, it shall be clear which set of mathematical axioms are relevant to natural philosophy.

This claim is developed in three sections. In the first, I present the terminology that Galileo employed to shape a mathematical science of motion and indicate conflicts emerging with its sources. Galileo's approach to mathematics was ultimately founded on a view of knowledge as certain but restricted. In the second, I sketch Galileo's appropriation of the *reductio* and its application to problems of natural philosophy, particularly to local motion. In the third, I claim that the validity of

⁸ Henry, 'Galileo and the Scientific Revolution', 6.

⁹ Henry, 20–30; Wallace, *Galileo and His Sources*; Clavelin, *The Natural Philosophy of Galileo*; Heilbron, *Galileo*.

Galileo's mathematics concerning the natural world relies on the assumption of the motion of the Earth.

2.1. From the Jesuits to God

Galileo's demonstrative terminology for mathematics (axioms, principles and definitions) follows the logical tradition of the Jesuits' version of Aristotelianism.¹⁰ There was a solid tradition of Jesuits' textbooks elaborating and interpreting the Aristotelian *corpus* in a particular way that was influential not only for Galileo but for Descartes as well. The Jesuit model of education, that operated a network of schools across Europe and the colonies, relied on the *Ratio Studiorum*—the order of studies—and associated manuals concerning particular topics under Aristotle's tutelage.¹¹ According to the definitions offered in Valla's *Logica*, the dominant textbook on Aristotelian logic, Aristotle had stated that axioms are propositions that must be known necessarily by anyone who would learn a science and require no demonstration, while positions (*positiones*) need not necessarily be foreknown.¹² Positions, at the same time, are twofold: suppositions (*suppositiones*)—wherein something is affirmed or denied of another—and principles or definitions (*principia sive definitiones*)—wherein no affirmation or denial is made. Galileo's early *De demonstratione* followed closely, yet not *verbatim* the terminology of Valla's *Logica*. Galileo, however, did not elaborate further these terms. In Galileo's version, axioms were neither suppositions nor positions, but propositions so easily known that no one refuses to acknowledge them. Suppositions were propositions assumed as true without demonstration, although they could be demonstrated. 'Terms or definitions' in Valla's *Logica* or 'principles or definitions' in Galileo were not suppositions because they were not propositions and did not affirm being or nonbeing; they expressed meanings but do not state their domain, that is, whether they are universal or not.¹³

¹⁰ Wallace, *Galileo and His Sources*.

¹¹ DesChene, *Physiologia*; DesChene, 'Forms of Art in Jesuit Aristotelianism (with a Coda on Descartes)'; Gabbey, 'Descartes's Physics'.

¹² Wallace, *Galileo and His Sources*, 112.

¹³ Wallace, 112–13.

At first sight, Galileo's use of this terminology appears consistent with the Jesuit teachings and with Valla's *Logica*. However, Galileo slightly transformed the use in order to advance on demonstrations about the natural world since his early studies on hydrostatics. The use of the strictly defined terms of mathematics to the 'proof of natural facts' immediately attracted the attention of natural philosophers. When Galileo's ideas began to appear in print, for example in the *Discourse on floating bodies*, his mathematical-mechanical approach to questions traditionally concerning natural philosophy did not pass unnoticed. The transgression of disciplinary boundaries, as well as the departure from the mechanical tendencies compatible with dominant trends of Aristotelianism, are summarised in the words of Vincenzo di Grazia, professor of natural philosophy at Pisa. Di Grazia considers that are far from the truth 'those who wish to prove natural facts by means of mathematical reasoning, among whom, if I am not mistaken, is Galileo'. In his view, all sciences and all arts 'have their own principles and their own causes by means of which they demonstrate the special properties of their own subject'. Di Grazia considered, following Aristotle, that 'we are not allowed to use the principles of one science to prove the properties of another'. Trying to prove natural properties with mathematical arguments is 'demented, for the two sciences are very different'. The correct procedure in enquiring on natural bodies is the study of 'motion as their natural and proper state', while the mathematician abstracts from all motion.¹⁴

This remark underlines one of the organising principles of Aristotle's division of knowledge, the so-called *metabasis*. Sciences were distinguished according to their subject; hence each one should be approached in a specific, unique way. The result is that the outcome of one science cannot be extrapolated to any other, that is, the principles or reasons achieved in one domain cannot be transplanted to any other.¹⁵ Di Grazia charged Galileo of investigating natural properties by 'mathematical arguments'. But Galileo was aware of this transgression and had anticipated the criticism:¹⁶

¹⁴ Quoted in Shea, *Galileo's Intellectual Revolution*, 34–35.

¹⁵ Funkenstein, *Theology and the Scientific Imagination*; Wallace, *Galileo and His Sources*; Shea, *Galileo's Intellectual Revolution*; Jardine, 'Problems of Knowledge and Action: Epistemology of the Sciences'.

¹⁶ Shea, *Galileo's Intellectual Revolution*, 35.

I expect a terrible rebuke from one of my adversaries, and I can almost hear him shouting in my ears that it is one thing to deal with matters physically and quite another to do so mathematically, and that geometers should stick to their fantasies, and not get involved in philosophical matters where the conclusions are different from those in mathematics. As if truth could be ever be more than one; as if geometry in our day was an obstacle to the acquisition of true philosophy; as if it were impossible to be a geometer as well as a philosopher, so that we must infer as a necessary consequence that anyone who knows geometry cannot know physics, and cannot reason about and deal with physical matters physically!

Consequences no less foolish than that of a certain physician who, moved by a fit of spleen, said that the great doctor Acquapendente, being a famous anatomist and surgeon, should content himself to remain among his scalpels and ointments without trying to effect cures by medicine, as if knowledge of surgery was opposed to medicine and destroyed it.¹⁷

In his defence, Galileo used an example from medicine, which had an unclear status in the Aristotelian divisions of knowledge.¹⁸ Medicine dealt with the motion of natural bodies and, in this sense, it was a causal, theoretical discipline. However, this knowledge was directed to heal, and in consequence, medicine also shared some characteristics with arts. Galileo appealed to the benefits that physicians—arguably belonging to the domain of natural philosophy—may obtain from surgery—associated to manual arts. With this example Galileo illustrates the idea that the truth is never in contradiction with the truth to support his claim that mathematical inferences cannot be in contradiction with natural philosophy. Avoiding the discussion on the approaches, Galileo shifted the emphasis from procedures to contents: the outcomes of disciplines cannot be in contradiction and in consequence their methods are not bounded to their initial subject.

From a different angle, Galileo's arguments against Di Grazia point to a salient problem of the application of mathematics to natural philosophy: how the mathematical principles may refer to the natural world ('that geometers should stick to their fantasies, and not get involved in philosophical matters where the conclusions are different from those in mathematics'). The point that the outcomes of

¹⁷ Shea, 35.

¹⁸ Lohr, 'The Sixteenth-Century Transformation of the Aristotelian Division of the Speculative Sciences'.

disciplines cannot be in contradiction because the truth is indivisible ultimately rests on God. Galileo's treatment of certainty in mathematics sheds light on how the terminology of mathematics became available for explaining the natural world in contrast with God's way to know the world. In the concluding remarks of the first day of the *Dialogo*, Galileo held that human understanding (*l'intendere*) can be conceived in two modes: extensive and intensive. Extensively, that is 'with regard to the multitude of intelligibles which are infinite', human understanding tends to be zero, because a thousand of certainties are zero when compared to infinity (*l'intender umano è come nullo*). However, intensively that is, the understanding of one proposition, human intellect knows it perfectly (*cioè perfettamente*), and 'thus in these it has as much absolute certainty as Nature itself has'.¹⁹ Only mathematical propositions are of this kind, and while human understanding knows less propositions, the mode of knowing intensively is comparable to the divine understanding. But how is it that a mathematical proposition has the same certainty that 'Nature itself has'?

One aspect of this problem has to do with the way of reaching at these mathematical certainties. In Galileo's views, the human way to grasp these propositions (*modo di conessere*) was also from God. Galileo explained this with a mathematical example: the knowledge of a circle. God, 'by a simple apprehension of the circle's essence, knows without time-consuming reasoning all the infinity of properties'. By contrast, 'our method' (*modo*) proceeds with reasoning step by step, from one conclusion to another'. In knowing a circle, the human understanding begins with one of the infinite properties and *assuming* it as the *definition*, proceeds by reasoning to another property, and from this to a third, and then to a fourth and so on. Then properties of things 'are included in the definitions of all things'. From the point of view of God, all these properties 'are perhaps but one in their essence'. In other words, our limited, step-by-step *modo* of reaching the mathematical truths presents to us a collection that, from God's point of view, is just one single truth. This piecemeal *modo* of knowing also implies that the human mind 'is clouded with deep and thick mists'

¹⁹ Galileo, *Dialogue*, 118–19; Galileo, *Opere*, VII, 128–29; Shea, *Galileo's Intellectual Revolution*, 121–29.

which disperses when we have mastered (*ci siamo fatti padroni*) conclusions firmly established and from them we proceed to make inferences, that is, to uncover further properties. Galileo illustrated this point: ‘what more is there to the square on the hypotenuse being equal to the squares on the other two sides, than the equality of two parallelograms on equal bases and between parallel lines? And is this not ultimately the same as the equality of two surfaces which when superimposed are not increased but are enclosed within the same boundaries?’ On the margin, Galileo writes that ‘infinite properties are perhaps but one’.

Our piecemeal way of reaching knowledge presents as multiple what might be one in Nature and from God’s ‘point of view’. A further problem appears from this multiplicity of properties: that we are able to *postulate* an infinite variety of definitions not necessarily corresponding to those in nature. Our ‘clouded’ understanding may take for natural just a way of conceiving things not dwelling in nature. The order of the mathematical reasoning does not correspond *per se* with the natural order. On the contrary, the finitude of the approach may constitute not only a limitation in terms of only acceding to pieces, but in following the wrong paths. In the *Discorsi*, introducing the question of motion naturally accelerated—a kind of motion he considered as accidental in his early works—, Galileo claimed that ‘it is appropriate to seek out and clarify the definitions that best agrees with the accelerated motion which nature employs (*utitur natura*)’. The problem is not that, for the sake of developing mathematics, it is possible to ‘invent at pleasure’ some kind of motion and explore its consequences in the way that some men have derived spiral and conchoidal lines from certain motions, though nature makes no use of these (*licet talibus non utatur natura*). Indeed, mathematics has widely benefited from assuming these motions from which man have laudably demonstrated their essentials from assumptions (*symptomata ex suppositione demonstrarunt*).²⁰ In the *Discorsi*, Galileo claimed to be confident that his definition of naturally accelerated motion ‘best agrees with the essence of naturally accelerated motion’ by looking into the properties of ‘acceleration for descending heavy bodies’. The definition is right because it has been found by ‘the very powerful reason that the essentials

²⁰ Galileo, *Two New Sciences*, 153; Galileo, *Opere*, VIII, 197.

successively demonstrated by us' are seen to be in agreement with that which 'physical experiments show forth to the senses' (*quae naturalia experimenta sensui repraesentant*).

Galileo solved this problem by adapting the procedures of the science of mechanics, particularly the *reductio*, to the study of the natural motion of bodies *pace* Guidobaldo. His studies on motion appeared in the *Discorsi* (1638), but these were developed in earlier. In fact, in the dramatic action of the Third Day, the participants in the dialogue discussed a Latin treatise that is said to be written by an Academician friend of Salviati. The order of presentation of the subject and the adoption of the *reductio* follow the early *De motu* (1589-1592).

2.2. 'Scienza nuova et ritrovata da me'

Since his early works, Galileo attempted to explain local motion by adapting procedures and concepts rooted in mechanics. The idea that the wondrous operation of complex machines could be elucidated if viewed as (that is, *reduced* to) an arrangement of balances, whose functioning was explained by mathematical principles and definitions, proved to be central to the ancient project of mechanics, particularly Pappus' and Hero's, as we have seen. By the mid-sixteenth century, this *reductio* was thought to cover also the central procedure of the *Mechanical Problems*, in which the entire operations of machines could be reduced (ἀνάγεται) to the lever and this to the 'miraculous' properties of the circle, as illustrated in Guidobaldo's *Mechanicorum Liber*. The efforts at systematising mechanics tailored this *reductio* to the axiomatic models of Euclid and Archimedes, putting further pressure on the mathematical principles on which mechanical explanation rested. However, the axiomatisation of mechanics in the hands of Guidobaldo, Benedetti or Moletti was not a simple adjustment of a ready-made science into a well-known model. Guidobaldo replaced the Aristotelian idea of the circle and its properties for Pappus' centres of gravity as the foundational principle of mechanics. However, the axiomatic (self-evidential) characteristics of the centres of gravity ended up being problematic in light of the observations drawn from experiments with the inclined balance. Guidobaldo reformulated his principle to the *gravitas secundum situm*,

already present in the tradition of the *scientia de ponderibus* that Guidobaldo fiercely despised. Nonetheless, the generation of mathematicians after Guidobaldo, for example, Galileo, Torricelli or Roberval, did not direct their efforts at replacing the *a priori* principles of the *scientia de ponderibus* or those of the *Mechanical Problems* for other *a priori* principles. They became less concerned with axiomatising mechanics on *a priori principles*. They were more interested in extending the applications and techniques of mechanics than in systematising its foundations, although both ideals were not mutually exclusive. Concerning foundations, their practice takes for granted that mechanics was a mathematical science and thus, their approach postulated principles in order to solve specific problems, following the practice attributed to Archimedes.²¹

In his early *Mechanics* (1599), Galileo assumed the centres of gravity as foundation of this science, but postulated as well the idea of *momentum*, meant as a ‘tendency to move caused not so much by the heaviness of the moveable body as by the arrangement in which heavy bodies have among themselves’.²² That is, through such *moment* a less heavy body can counterbalance some other heavier bodies, ‘as in the steelyard, a little counterweight is seen to raise a very heavy weight not by excess of heaviness but rather by its distance’. The determination of this *momentum* involved weight, distance and velocity. A mechanics built on centres of gravity was mainly concerned with the generation of equilibrium. However, the introduction of *momentum*, as different from the natural tendency to move downwards (called here *heaviness*), involved motion in the analysis of mechanical arrangements. In so doing, Galileo went out of the straight jacket of equilibrium to which Guidobaldo and Commandino had attempted to reduce mechanics in their quest for certainty. These early formulations proved to be particularly successful, for example, in reducing the screw to the lever by means of his analysis of the inclined plane. In the *Discorsi*, Galileo still praised himself in the form of the Academician for this achievement, coming from ‘an old treatise on mechanics written at Padua for the use of his pupils, demonstrated at length and conclusively in connection with his treatment of the

²¹ Palmieri, ‘Breaking the Circle’; Bertoloni Meli, *Thinking with Objects*; Bennett, ‘The Mechanical Arts’.

²² Galileo, *On Motion and On Mechanics*, 151.

origin and character of the marvellous instrument, the screw'.²³ An important consequence of the introduction of motion into the mathematical science of mechanics is Galileo's development of an alternative to deal with a natural philosophical issue in a way different from the qualitative approach of the Scholastics and from the natural philosophies based on principles such as sympathy and antipathy emerging as alternatives to this latter.²⁴

Galileo's mathematical approach to the motion of natural bodies has two important backgrounds that were ultimately rooted in the interpretation of Aristotelian texts that evolved as critical stances of them.²⁵ The first derives from the criticisms to Aristotle's ideas on projectile motion that was challenged as early as John Philoponus' (c.490-570) commentary on *Physics*. The most accepted interpretation of this phenomenon in *Physics*, labelled as the theory of *antiperistasis*, held that Aristotle assigned to the air the function of keeping the projectile in motion, for it was displaced backwards by the initial motion of the body but when it reached the back of the body pushed it forwards. Philoponus pointed out that air played inconsistent roles and, instead attributed the motion of projectiles to an incorporeal force. Similar criticisms and alternatives were put forward by Avicenna (c. 980-1037) and Jean Buridan (c. 1295-1363). These explained the sustained motion of projectiles by some kind of power or force transmitted by the mover to the projectile at the beginning of motion. The motion of projectiles belonged in principle to the domain of forced or violent motions, but debates on *antiperistasis* over the centuries brought up views challenging fundamental tenets of Aristotelian natural philosophy, for example, that projectile motion was a combination of natural and forced motion and, in consequence, that a body may have opposite tendencies at the same time; or that the mover and the moved shall be in contact action. Disciplinary and conceptual distinctions between natural and forced motions became blurry because some of these alternatives called upon mathematical approaches such as those of the fourteenth-century *calculatores* implicitly challenging fundamental claims of

²³ The demonstration referred is in *La Meccaniche* Galileo, 169–77.

²⁴ Henry, 'Galileo and the Scientific Revolution', 20–30; Feingold, 'The Occult Tradition in the English Universities of the Renaissance: A Reassessment', 89.

²⁵ Cf. Orozco-Echeverri, 'Force in Renaissance Science'.

Aristotelian natural philosophy.²⁶ On the other hand, Jesuits at the Collegio Romano had long debated on Aristotelian topics and especially on natural philosophy. Their readings of Aristotle's *Physics* dealt with some conceptual tensions in the general view of motion; they also focused on the specific aspects related to the causes of motion and the possibility of motion in a void. Jesuits were concerned with three issues: the possibility of motion between the natural and the violent; the requirements for the continuity of motion and finally the existence of a point of rest at the moment of reflection.²⁷

In *De motu* (c. 1590), Galileo put forward a mechanical explanation of natural motion, combining elements coming from Aristotle's mechanics, Archimedean hydrostatics and the medieval *scientia de ponderibus*. However, he presented these developments in the context of natural philosophy, not as an exercise in the mixed-science of mechanics. *De motu* starts off with equating Aristotle's explanation that bodies are heavy (*gravius*) because they have a natural tendency to move in some direction, with that of 'the philosophers' not adducing any other reason than Providence to explain the 'existing arrangement' of the world. By contrast, he 'anxiously sought from time to time to think of some cause, if not necessary, at least reasonable and useful'.²⁸ Galileo established that the cause generating all natural motions downwards and upwards is the 'essential heaviness or lightness' of the moving body (*lotionem omnem naturalem, sive deorsum sive sursum... gravitates vel levitate fieri*). All elements (earth, water, air and fire) were heavy, but some move upward and others downward depending on a difference of *gravitas* between the element and the medium in which they move, not because they were looking for their natural place, as Aristotle held.²⁹ In the absence of a medium, all bodies would simply fall because of their intrinsic *gravitas*, including those traditionally

²⁶ Clagett, *The Science of Mechanics in the Middle Ages*, 419–626.

²⁷ Wallace, *Galileo and His Sources*, 162–65.

²⁸ Galileo, *On Motion and On Mechanics*, 14–15.

²⁹ The 'motus elementorum' dispute in Pisa between Girolamo Borro, Francesco Buonamici and Francesco di Vieri had already advanced some ideas in this direction and circulated important criticisms to and qualifications of Aristotle coming from Simplicius, Philoponus, Alexander of Aphrodisia and Themistius Salvia, 'From Archimedean Hydrostatics to Post-Aristotelian Mechanics'; Camerota and Helbing, 'Galileo and Pisan Aristotelianism'.

considered light (air and fire). Specific *gravitas* is defined in terms of proportions:³⁰ ‘we define as equally heavy (*gravius esse*) two substances which, when they are equal in size, are also equal in *gravitas*’.³¹ Following this definition, Galileo remarked that lightness, central to the Aristotelian explanation of air and fire, was not a quality but just the negative difference between specific *gravitas* of the body and the medium. Then, Galileo moved on to ‘demonstrate’ some consequences: that bodies of the same *gravitas* as the medium move neither upwards nor downwards; that bodies lighter than water cannot be completely submerged. Right after, Galileo advanced in a notion of *velocitas* that departed from Aristotle. In his view, *velocitas* is the *ratio* of motion in function of the specific *gravitas* of the body.³² The immediate consequence of this approach is that, if motion is generated by the interaction of the body with the medium, in the void, the *velocitas* of the mobile would not be infinite and instantaneous as claimed by Aristotle but it would be as the specific *gravitas* of the body, for the numerical value of the medium’s weight would be zero. In Galileo’s words: ‘the body will move in a void in the same way as in a plenum’.³³

The image emerging from this conception of natural motion, in which bodies sink or float on Earth according to specific *gravitas*, supported the analogy between bodies moving naturally and the weights of a balance. If motion was explained as a function of the arithmetic difference between the specific *gravitas* of the body and the medium, this amounted to redefine motion in term of equilibrium, rather than tendencies or qualities: ‘We shall first consider what happens in the case of the balance, so that we may then show that all these things also happen in the case of bodies moving naturally’. After explaining the principle of the balance, Galileo

³⁰ Shea, *Galileo’s Intellectual Revolution*, 119; Camerota and Helbing, ‘Galileo and Pisan Aristotelianism’; Palmieri, ‘Breaking the Circle’.

³¹ Drake has pointed out that this conception of weight is roughly equivalent to specific gravity or relative density. Galileo, *On Motion and On Mechanics*, 13.

³² Translating *velocitas* as speed or velocity may be misleading. It is important to have in mind that this *velocitas* is a direct *ratio* between two homogeneous quantities; a *ratio* between space and time was inconceivable. Thus, *velocitas* is not an average (as we tend to conceive velocity after the calculus); it may refer to the space traversed in a given time (or distance) or the time elapsed to traverse a given space (time interval). Salvia, ‘From Archimedean Hydrostatics to Post-Aristotelian Mechanics’. On divergent approaches to *ratios*, cf. Section 6.2

³³ Galileo, *On Motion and On Mechanics*, 41–50.

concluded that ‘the heavier cannot be raised by the less heavy’ and, in consequence and contrary to Aristotle, ‘what moves moves, as it were, by force and by the extruding action of the medium’; that is, natural motion upward and downward is caused by the interaction of the body with the medium and not as a realisation of its inner tendency to move.³⁴ But beyond this, the redefinition of motion in mechanical terms turned the Aristotelian distinction between natural and forced motion upside-down. In fact, if motion was caused by the difference of *gravitas*, this would amount to claim that it is forced rather than natural in the Aristotelian view, since it is not the realisation of the internal tendency of the body looking for its natural place but the interaction with the surrounding medium which produces motion. The *analogy* between the natural motion of bodies and the weights of the balance was used by Galileo for the *reductio* of natural motion to the weights of a balance and for the subsequent application of the mathematical principles of mechanics to the understanding of natural bodies in motion:

the motion of bodies moving naturally can be suitably reduced (*congrue reducatur*) to the motion of weights (*ponderum*) in a balance. That is, that the body moving naturally plays the role of one weight in the balance, and a volume of the medium equal to the volume of the moving body represents the other weight (*pondus*) in the balance.³⁵

The *reductio* applied to the motion of natural bodies became, in Galileo’s hand, the crucial way to overcome the reservations that Guidobaldo and Benedetti had expressed concerning the impossibility of a mathematical science of motion.³⁶ Since *De motu*, Galileo’s application of *reductio* to natural motion began to appear as a redefinition of bodies in which their fundamental properties or ‘passions’ (*passiones*) were carefully isolated from those ‘accidental’ in mathematical definitions. Accidental properties were postulated as responsible for unexpected and unpredictable effects and hence, accounted for the disagreement between the mathematical achievements and the observations. Once defined, the essential properties could be handled mathematically and included into the principles from

³⁴ Galileo, 22–23.

³⁵ Galileo, *Opere*, I, 259; Galileo, *On Motion and On Mechanics*, 23, 39–41.

³⁶ Bertoloni Meli, *Thinking with Objects*, 68–79; Bertoloni Meli, ‘Guidobaldo Dal Monte and the Archimedean Revival’; M. Henninger-Voss, ‘Working Machines and Noble Mechanics: Guidobaldo Del Monte and the Translation of Knowledge’; Renn and Damerow, *The Equilibrium Controversy*.

which demonstrations and conclusions should be drawn. Galileo identified this procedure with the analytic method employed by mathematicians ‘to track down what we believe to be the true cause of this effect’.³⁷ For example, in *De motu*, Galileo claimed that *gravitas* is the essential cause of natural motions, as we have mentioned, and remarked that acceleration observed in free fall was only a perturbation generated by accidental causes. However, these perturbations do not run against the truth of the mathematical principles and their capability to grasp aspects of nature from which other conclusions may be demonstrated.

In *De motu*, Galileo formulated a *reductio* from which he later developed his ideas on motion naturally accelerated. This *reductio* also made blurry the distinction between natural and forced motion. Explaining the motion of a body descending uniformly on an inclined plane, Galileo claimed that this motion was equivalent to the motion of the same body uniformly falling through a media of greater and greater density, making use of the approach he established in the first chapters.³⁸ In other words, Galileo compared the motion on an inclined plane with the motion of a body going through mediums offering different resistance. Based on this comparison, Galileo concluded that the body will descend vertically with greater force than on an inclined plane in proportion as the length of the descent on the incline is greater than the vertical fall. However, the validity of this conclusion required a further clarification. Galileo remarked that ‘this proof must be understood on the assumption that there is no accidental resistance (occasioned by toughness of the moving body or of the inclined plane, or by the shape of the body)’. The accidents disturbing his proof and potentially showing a disagreement between his mathematical conclusion and observations are material. In consequence, mathematical reasoning requires that ‘we assume that the plane is, so to speak incorporeal or, at least, that it is very carefully smoothed and perfectly hard, so that, as the body exerts its pressure on the plane, it may not cause a bending of the plane and somehow come to rest on it, as in

³⁷ Galileo, *On Motion and On Mechanics*, 88. On the analytic method of demonstrative sciences to investigate nature, see especially Galilei, *Dialogue Concerning the Two Chief World Systems, Ptolemaic and Copernican*, 57–59, where Galileo contrasts Aristotle's use of experience with that advanced by the 'mathematical philosopher'.

³⁸ Roux and Festa, ‘The Engima of the Inclined Plane’; Salvia, ‘From Archimedean Hydrostatics to Post-Aristotelian Mechanics’.

a trap'. But not only the inclined plane should be assumed as 'perfectly hard'; the moving body 'must be assumed to be perfectly smooth, of a shape that does not resist motion, e.g., a perfectly spherical shape, and of the hardest material or else a fluid like water'.³⁹ Galileo followed Guidobaldo's perfect fit between the geometrical design and the material realisation of the machine. However, instead of seeking the material perfection adjusted to the geometrical design, Galileo developed a mathematical approach to control material conditions and, in a sense, to make them irrelevant in the demonstration. Indeed, the elimination of disturbances validated Galileo's mathematical conclusion on the motion of bodies on inclined planes, but he pushed this *reductio* forward by postulating that the motion of a perfectly shaped body on a perfectly smooth surface is parallel to the horizon: 'If everything is arranged in this way, then any body on a plane parallel to the horizon will be moved by the very smallest force, indeed, by a force less than any given force'.⁴⁰ The parallelism between the horizon and the path of the body is a condition that no body on Earth can fulfil but that appears here as basic for the demonstration. In so doing, this arrangement represented a body subject to no external resistance, that is, without any tendency to move: being parallel to the horizon, the angle of the plane does not exert any influence and being perfectly circular, it is perfectly balanced on its centre of gravity aligned with the only point of contact with the plane. The reduction of the motion to the 'essential' conditions allows Galileo to apply the principles of mechanics. Because this body is in perfect equilibrium, it is *like* an equal-armed balance with two equal ends, in which its weights are fully balanced by the resistance of the plane. Therefore, 'the least force' would set this body in motion, because the smallest force (or 'a force less than any given force') is sufficient to make it move along the same direction of the impulse it received, as in the case of the balance in perfect equilibrium.⁴¹

The situation described above faces two major challenges concerning Galileo's mathematical conception of motion when considered against the background of previous natural philosophy and mechanics. On the one hand, that the motion of the

³⁹ Galileo, *On Motion and On Mechanics*, 65.

⁴⁰ Galileo, 65–66.

⁴¹ Salvia, 'From Archimedean Hydrostatics to Post-Aristotelian Mechanics', 125.

body on the horizontal plane is ‘neither natural nor forced’. In fact, it is not natural because natural tendencies are counteracted and ‘if its motion is not forced motion, then it can be made to move by the smallest of all possible forces’. On the other hand, the idea of a balance whose weights suspended form a right angle had been highly controversial by the late sixteenth century, as it has been pointed in the previous chapter. In fact, Guidobaldo and Benedetti argued against Tartaglia that the lines of descent never made a right angle, because the weights directed towards the centre of the Earth were convergent.⁴² On this point Galileo replied that he assumed the angles to be right under ‘the protecting wings of the superhuman Archimedes’, who made a similar assumptions in his *Quadrature of the Parabola*. In other words, Galileo’s treatment of motion required the abstraction of disturbing material conditions (such as the lines of descent of a balance on the Earth). However, Galileo recognised that the *reductio* of the body moving on a horizontal plane to the (controversial) equal-armed balance presented a further difficulty: ‘that a plane cannot actually be parallel to the horizon, since the surface of the earth is spherical, and a plane cannot be parallel to such a surface’. Galileo then compared the motion of a sphere rolling around the ‘horizontal’ plane with that of a sphere spinning around its vertical axis, with its centre coinciding with the centre of the Earth. In this case, the principle of the ‘smallest force’ also applies and the motion of the sphere spinning around its centre would continue clockwise or anti-clockwise, depending on the direction of the initial impulse. From the solution to the problem of the parallelism of the plane Galileo drew some conclusions of his mechanical principles motion of celestial bodies (*Pari ratione de caelo est iudicandum*). If the sphere spinning around its centre is subject neither to an external force nor to a natural tendency ‘a star will be able to retard the motion only when it is being moved away from the place toward which it naturally tend. But this never happens in a rotation that takes place about the centre of the universe, for there never is upward and never downward motion. Therefore, the motion will not be retarded’.⁴³ The outcome of Galileo’s *reductio* of natural bodies to the essential properties of motion is a set of definitions, axioms and demonstrations whose validity ultimately rests on the

⁴² Wallace, *Galileo and His Sources*, 206; Bertoloni Meli, *Thinking with Objects*, 30–32.

⁴³ Galileo, *On Motion and On Mechanics*, 73–74; Salvia, ‘From Archimedean Hydrostatics to Post-Aristotelian Mechanics’, 125–27.

certainty of mathematics. However, Galileo was confident that his mathematical principles grasp the essence of the properties (*passiones*) which ‘nature does employ’.⁴⁴

This way of conceiving bodies has been read as the primacy of reasoning and mathematics over sense experience and experiments. This is a long-debated topic in Galileo’s scholarship. Whether this approach is Platonist, Neo-Platonist or Archimedean, Galilean reduction of bodies and their accidents to properties has a function in connection with mechanics. Bodies are defined in a mathematical way so that mathematical demonstrations may be certain of them; however, they may be different from what appears to the senses. It is important to keep in mind that accidents, as mentioned above, played a central role generating a discrepancy between the mathematical conclusions and experiments. In explaining the motion of projectiles as a form of parabolic motion, Galileo claimed that ‘considerable disturbance arises from the impediment of the medium; by reason of its multiple varieties, this disturbance is incapable of being subjected to firm rules, understood and made into science’.⁴⁵ The medium alters the heaviness, speed and shape of which ‘no firm science can be given’. Guidobaldo had already noticed this. However, the recognition of the existence of these elements, yet not ‘subjected to rules’, makes possible to operate in practise ‘under those limitations of experience’.⁴⁶ Mathematics, in this case, sheds light on the confusing elements of sense experience. A stronger claim on the primacy of certainty over experiments appears in the *Dialogo*, when discussing about the motion of a stone falling from the ship’s mast. The point discussed was that, according to Salviati, a stone falling from the ship’s mast strikes in the same place, no matter if the ship moves or stands still. In reaction to Simplicio’s denial of this point, Salviati asked whether Simplicio had ‘ever made this experiment of the ship’. Simplicio replied that he had never done it, but the ‘authorities who adduced it had carefully observed’. However, Simplicio asked Salviati if he had performed the experiments to ‘declare it to be certain’. Salviati replies that ‘without experiment, I am sure that the effect will happen as I tell you,

⁴⁴ Galileo, *Two New Sciences*, 153; Galileo, *Opere*, VIII, 197.

⁴⁵ Galileo, *Two New Sciences*, 224.

⁴⁶ Galileo, 225.

because it must happen that way', that is, because it is certain from a mathematical point of view. The point here is that the certainty of the conclusion depends on the validity of the mathematical procedure, not on experience.⁴⁷ The certainty of principles comes from reasoning, yet they have to explain our world. It is possible to postulate empirical principles in order to analyse natural phenomena. If their validation does not come from experience, how can Galileo be sure that 'nature does employ' them?

2.3.Systema mundi

From the previous sections, three elements clarify Galileo's place in the history of laws of nature. First, the adaptation of the mechanical *reductio* to the natural motion of bodies made possible to operate with definitions stating general properties from which new properties could be further discovered by applying mathematical procedures. In this way, Galileo shaped a mathematical science of motion. Second, the conception of certainty, contrasted with God's way to understand, conceived human reasoning as a piecemeal way to proceed, whereby a multiplicity of properties stands for what in reality might be one and the same thing. This provided a validation of the use of mathematics in understanding the natural world but at the same time, set limits to the human capacity to understand; a moderated form of scepticism regulates the 'optimism' deriving from the concept of certainty. Finally, this piecemeal approach rendered it possible to consider questions of natural philosophy without fulfilling the causal requirements of the dominant trends of Aristotelianism. In other words, Galileo provided a set of mathematical definitions and principles that may adumbrate human experience, but they did not ultimately establish why motion is that way and not otherwise, that is, why some specific kinds of motion exist in nature rather than others.

Since the first reflections on motion in *De motu*, Galileo built mathematical definitions that, inspired by the analysis of machines, involved natural motion, and attempted to include the motion of celestial bodies. Although these early demonstrations appeared problematic to Galileo and his immediate circle and

⁴⁷ Galileo, *Dialogue*, 168.

underwent important transformations through his intellectual development, the application of mechanical notions and procedures to motion tore down conceptual distinctions of Aristotelian natural philosophies, specially the distinction between motion of celestial and terrestrial bodies. From the analogy between the weights in a balance and a body in perfect equilibrium on a plane parallel to the horizon, Galileo made inferences reaching celestial bodies.

In this context, it seems reasonable to me why Galileo came to embrace Copernicanism with such enthusiasm: it provided a general foundation explaining why some kinds of motion were real and subject to mathematical analysis yet not immediately perceivable by sense experience. This view seems to emerge as soon as Galileo embraces Copernican astronomy. However, the circumstances of Galileo's discovery and enthusiasm for Copernicus' approach are not completely clear.⁴⁸ In the earliest surviving records of his Copernicanism, in letters to Mazzoni, May 30, 1597 and to Kepler August 4, 1597, Galileo claimed that he had been able to explain physical phenomena that could not be explained in the old astronomy. From '*Copernici sententiam*', Galileo claimed, he had been able to find out causes of multiple natural effects (*Ex tali positione multorum etiam naturalium effectum causae sint a me adinventae*).⁴⁹ Although the context of this claim is the theory of tides, it is hard to believe that Galileo did not have in mind as well the mechanical analysis of natural motion of bodies, including the consequences it had for celestial bodies. Galileo's telescopic discoveries of the first decade of the 1600s, the rule of free fall, the isochronism of pendulums and the analysis of parabolic motion led him to the idea of writing a *systema mundi*.⁵⁰ In *Sidereus Nuncius* (1610), Galileo presented his observations of the moon and attempted to explain the variations of its light depending on its position regarding the sun and the earth assuming the Copernican arrangement of bodies as the causes of these variations. He commented that 'these few remarks suffice us here concerning this matter, which will be more fully treated in our *System of the world*'. However, he sketched the plan of this book

⁴⁸ Koyré, *Galileo Studies*, 129; Heilbron, *Galileo*, 215; Henry, 'Galileo and the Scientific Revolution'; Drake, *Galileo: Pioneer Scientist*, 70.

⁴⁹ Galileo, *Opere*, X, 68.

⁵⁰ Henry, 'Galileo and the Scientific Revolution'; Drake, *Galileo: Pioneer Scientist*, 79–80.

in which ‘the solar reflection from the earth will be shown to be quite real—against those who argue that the earth must be excluded from the dancing whirl of stars for the specific reason that it is devoid of motion and light’. The main point shall be that the earth is a ‘wandering body surpassing the moon in splendour, and not the sink of all dull refuse of the universe’.⁵¹ This same enthusiasm is mentioned in a letter to the Tuscan Secretary of State, in exchanges concerning Galileo’s appointment as mathematician to the Duke in May 1610. In describing his future projects, Galileo mentioned ‘two books on *The system or the constitution of the universe*, an immense concept and full of philosophy, astronomy and geometry’. Interestingly, the letter also mentioned three books on local motion, ‘an entirely new science in which no one else, ancient or modern, has discovered any of the most remarkable properties which I demonstrate to exist in both natural and violent movement, whence I may call this a new science and one discovered by me [*scienza nuova et ritrovata da me*] from its very foundations’.⁵² Galileo’s discovery of Copernicus as a cause explaining motion balanced the asymmetrical attitude attributed to his early work: ‘the different attitude Galileo entertained toward *suppositiones* in astronomy and those in mechanics had to do with the possibility of establishing, by direct or indirect recourse to experience, the truth of the latter, coupled with a somewhat disinterested resignation over the impossibility of doing the same with the former’.⁵³ The truth of the Copernican system that he thought he could demonstrate implied that the properties postulated in mathematics could be dealt with as causes and effects of motions in the natural world.

As we have seen, central to Galileo’s approach was the idea that mathematical definitions may capture essential aspects of nature. This view of mathematical definitions was not intended to fill the requirements of Aristotelian four causes. Since his early logical works, Galileo came to replace the four Aristotelian causes with what he called *causa per se*, as opposed to the *causa per accidens* which ended up being operative categories of *reductio*. Galileo distinguished between the properties required to define and explain one thing and the accidents interfering with that

⁵¹ Galileo, *Opere*, III, 75.

⁵² Galileo, X, 351–52.

⁵³ Wallace, *Galileo and His Sources*, 261.

knowledge. In his mature works, Galileo put forward that mathematical definitions and principles refer to essences. In the Second Day of the *Dialogo*, Simplicio claimed that the cause of the motion downwards was ‘gravity’, in the Aristotelian sense of a tendency to move towards the centre of the Earth. Salviati replied that he was wrong: ‘what you ought to say is that everyone knows that it is called ‘gravity’, but its essence, of which essence you know not a bit more than you know about the essence of whatever moves the stars around’. Although we can postulate properties and make demonstrations, ‘we do not really understand what principles or what force it is that moves stones downwards’.⁵⁴ Defining properties of bodies, which naturally occur, does not amount to uncovering the ultimate essence of bodies. The knowledge of some aspects of the natural world cannot be mistaken for the whole. Humans’ knowledge is not God’s knowledge. In fact, Galileo claimed that ‘there is not a single effect in nature, even the least that exists, such that the most ingenious theorists can arrive at a *complete* understanding of it’.⁵⁵ However, Galileo is not claiming that mathematical knowledge of nature is a previous stage of an essentialist, more advanced form of knowledge. On the contrary, his contrast between the mathematical approach based on definitions and the Aristotelian presumed knowledge of the substance, points to an alternative conception of knowledge.

This may be clarified by bringing back the form of scepticism that I read as a regulative principle of the idea that we can attain knowledge of limited aspects of nature. The piecemeal approach is regulated by abandoning the idea that knowledge, to be such, has to be exhaustive; it can be, on the contrary, intensive and deductive and thus it can be revised and improved. In the *Discorsi*, Galileo presented the science of motion as ‘a gateway and a road to a large and excellent science (...) a science into which minds more piercing than mine shall penetrate to recesses still deeper’.⁵⁶ This scepticism is the counterpart of the anthropocentric, limited view of knowledge that Galileo attributed to Aristotelianism. Speaking through Salviati in the *Dialogo*, Galileo reminded Simplicio that thinking that God did not create anything in vain does not mean that we are able to understand the purpose of everything in the

⁵⁴ Galileo, *Dialogue*, 272.

⁵⁵ Galileo, 116.

⁵⁶ Galileo, *Two New Sciences*, 190.

universe: ‘it is brash for our feebleness to attempt to judge the reason for God’s actions, and to call everything in the universe vain and superfluous which does not serve use’. In Salviati’s words, there is a great ineptitude on those who ‘would have it that God made the universe more in proportion to the small capacity of their reason than to His immense, His infinite, power’.⁵⁷ The reduction of all possible knowledge—God’s view—to the human understanding is the root of Aristotelianism. Simplicio’s views are charged with implying ‘the vain presumption of understanding everything, that can have no other basis than never understanding anything’. In Salviati’s views, by contrast, ‘I cannot bring myself to believe that there may not be other things in the universe dependent upon the infinity of the universe of its wisdom, at least, so far as my reason informs me’.⁵⁸

* * *

To conclude, it seems that the best judgment on this moderated view of knowledge, with its emphasis upon certainty and its dependency on the Copernican premise as a condition of the mathematical demonstrations—together with the achievements on the mathematical motion of bodies showing the gap between Galileo and the ‘laws of nature’—is reported by Descartes when, writing to Mersenne, he claimed that ‘[Galileo] has not investigated matters in an orderly way, and has merely sought explanations for some particular effects’.⁵⁹ Certainly, Galileo would disagree with the first claim, but it seems to me that he would not be in disagreement with the idea that he was looking for the (mathematical) explanation of the particular effects following from an Earth in motion. However, this apparently fragmentary project was, in fact, his way of building the *systema mundi*, relying on a piecemeal approach of knowledge, to the discovery of only part of the secrets of nature.

⁵⁷ Galileo, *Dialogue*, 427, 429.

⁵⁸ Galileo, 116.

⁵⁹ Descartes to Mersenne, 11 October 1638 in Henry, ‘Galileo and the Scientific Revolution’, 4.

3. Kepler's rearrangement of (the science of) the stars

3.1. Kepler's laws revisited

The early success of Kepler's 'laws of planetary motion', boosted to a large extent by their central place in Newton's *Principia*, has led us to project a disproportionate image of these accomplishments in Kepler's works. Accounts of the scientific revolution depict Kepler's achievements in terms of laws. Indeed, Kepler's career is usually presented as an explicit quest for the laws of planetary motion.¹ Other studies, aware of Kepler's absence of a sound concept of law, rely on the use of quotation marks to call the attention to this. Rather than recognising this as a problem, they just go around it: 'Kepler himself was still not sure of the validity of the area theorem (to avoid the notion of 'law')'.²

Other historians have gone beyond the customary practise of prefixing 'law' to Kepler's three most famous findings. A wider use of the term supports stronger claims concerning the emergence of modern science or the revolution in astronomy. Koyré weighed up Kepler's contribution to modern science as an innovative conception of the universe governed by universal mathematical laws explaining its order: 'What is radically new in Kepler's conception of the world is the idea that the Universe is governed by the same laws in all its parts and that these laws are strictly mathematical'.³ A variant of this appears in Caspar's monumental biography: 'Kepler's greatest service [is] that he substituted a dynamic system for the formal schemes of the earlier astronomers, the law of nature for the mathematical rule, and causal explanation for the mathematical description of motion. Thereby he truly became the founder of celestial mechanics'.⁴ By rooting astronomy in physics, Kepler transformed the mathematical rules into laws of nature, the mathematical description into causal explanation.

¹ Kozhamthadam, *The Discovery of Kepler's Laws*, 1. Dreyer, *A History of Astronomy from Thales to Kepler*, 373.

² Graßhoff, 'Natural Law and Celestial Regularities from Copernicus to Kepler', 157.

³ Koyré, *Études d'histoire de La Pensée Scientifique*, 56.

⁴ Caspar, *Kepler*, 136.

In contrast with these readings, terminological studies of laws have pointed out that Kepler's use of 'law' is restricted to the description of specific phenomena, such as the law of reflection in optics or to the principle that governs the motion of a planet making possible to trace its trajectory in astronomy. A mathematical sense of law related to proportion or to geometrical relationships seems to be present as well.⁵ However, none of the occurrences of the term law refers to the sense that became fashionable after Descartes. A 'metaphysical use'—that is, laws in reference to God as lawgiver—also appears in a few places of Kepler's correspondence and works but it does not seem associated with any of the meanings previously mentioned.⁶

Nevertheless, these studies leave out important elements of Kepler's peculiar uses of the term law. 'Law' and 'laws of nature' expose important aspects of Kepler's lifetime vision of setting up astronomical enquiries on physical considerations; at the same time, these terms point to the traditions on which this project relies. Firstly, Kepler's mentions of 'law' in a theological sense—metaphysical, in Roux's classification—are connected to astronomy in a vague, general way. The conception of God as lawgiver emphasising his ruling of nature is prominent in debates on providence and the role of nature in the knowledge of God derived, such as in the works of the reformer Philip Melanchthon (1497-1560), widely influential during Kepler's formative years.⁷ Secondly, there is a mathematical sense of law referring to numerical, geometrical or harmonic proportions or to observable regularities ultimately representable in geometry. Because quantity is the essence of what exists—including matter—, most historians have misleadingly assumed that this sense of the term is equivalent to 'laws of nature'. However, not all mathematical relationships are termed as laws; in order to be so, they need to be compatible with a physical explanation and to reveal an archetype. In other words, these mathematical laws—which Kepler usually labelled as geometrical—do not contain, or are not by themselves, physical explanations, yet they have to be compatible with them.

⁵ Roux, 'Les lois de la nature à l'âge classique la question terminologique', 549–51.

⁶ Roux, 555.

⁷ Caspar, *Kepler*, 44–50; Methuen, *Kepler's Tübingen*; Barker and Goldstein, 'Theological Foundations of Kepler's Astronomy'; Kusukawa, *The Transformation of Natural Philosophy*, 124–73; Westman, 'The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory'; Gerdes, 'Johannes Kepler as Theologian'.

Thirdly, law can make reference to a rule of calculation; it may be a method of simplifying ways of constructing astronomical tables by applying geometrical procedures. In its most general sense, this law is a procedure or device and the relationships to which it points does not have to correspond to the physical world; this is what Kepler means when observing that in astronomy, geometrical and astronomical hypotheses are not the same, for geometrical hypotheses are used to calculate and the circles and figures there postulated do not dwell in nature.⁸ In the *Commentaries on the motion of Mars*, the ‘area law’ was initially formulated as an easier way to calculate the numerical values required by the ‘distance law’, which Kepler formulated on physical grounds (that the rate at which a planet moves in its orbit is inversely proportional to its distance from the Sun, given a force originating from the Sun).⁹ This interpretation of law as a rule of calculation became central in the reception of Kepler’s ‘area law’ and ‘distance law’.¹⁰ Fourthly, there is a sense of law encompassing mathematical and theological meanings, ultimately rooted in the archetypal cosmology. Because God made man and nature ‘embracing a certain pattern of the creation of their functions’, Kepler reasoned that man and nature ‘also observe the same laws along with the Creator in their operations’.¹¹ This idea supports the conclusions concerning harmonic proportions, for it makes possible to legitimise the connection between the sensible harmonies of human music to the intellectual harmonies of God’s universe.¹²

These variants of ‘law’—in line with terminological approaches—would be enough to revisit Kepler for the history of laws of nature. However, my claim is that Kepler’s relevance lies in the underpinnings allowing him to encompass mathematics and physics, that is, in the theologically inspired cosmology filling the gap between the demonstrative mathematics and the explicative (causal) requirements of natural

⁸ Jardine, *The Birth of History and Philosophy of Science*, 98.

⁹ In the *Epitome*, however, Kepler realised that the distance law and the area law are not equivalent. On this see Aiton, ‘Kepler’s Second Law of Planetary Motion’; Thoren, ‘Kepler’s Second Law in England’; Caspar, *Kepler*, 131–32.

¹⁰ Russell, ‘Kepler’s Laws of Planetary Motion: 1609–1666’; Aiton, ‘Kepler’s Second Law of Planetary Motion’; Thoren, ‘Kepler’s Second Law in England’; Harper, *Isaac Newton’s Scientific Method*.

¹¹ Kepler, *The Harmony of the World*, 146; Kepler, *KGW*, 1940, 6:104.

¹² Kepler, *KGW*, 1949, 14:21–41; Stephenson, *The Music of the Heavens*, 90–100; Martens, *Kepler’s Philosophy and the New Astronomy*, 136–41.

philosophy. Put otherwise, if historians seem to have agreed that in Kepler astronomy in the classical sense culminated and the foundation of a new celestial mechanics began, all this is just a consequence of his most important achievement: the discovery of the blueprints of the Creation. The complex Neo-Platonic reading of the Christian cosmogony underpinning the archetypal cosmology provides the link among different traditions, practices and concepts stemming from mathematics and natural philosophy. The archetypes operate at all levels of Kepler's endeavours, from the most abstract reflections on the nature of reality to the most specific calculations.¹³ Indeed, archetypes—and not laws—bind together God, man and nature, making of the universe a knowable entity. At this point, it becomes clearer why Koyré, Caspar and others attributed to laws of nature what in fact belong to the archetypes. The '*lois de la nature*' play in Descartes' philosophy a role comparable to the archetypes in Kepler's thought, for both laws and archetypes are eventually justified by metaphysics to make mathematical results relevant to natural philosophy. Kepler and Descartes followed similar paths: all material things are essentially defined by quantity (Kepler) or by extension (Descartes); knowledge is ultimately dependent upon eternal ideas of mathematics that God imprinted in man's souls, together with the idea of a deity; these ideas are sufficient to provide a full account of nature in causal terms. The next chapter will suggest that this similarity is not a mere coincidence. However, the most important difference is that Kepler's archetypes have no agency, they do not exert any kind of activity in the world; they are formal causes. The universe is ordered because it reflects or instantiates God's perfection; and this order can be grasped by identifying geometrical or harmonic patterns underlying the apparent chaotic, disordered material gathered from sense-experience. But neither the laws nor the archetypes actually govern the Keplerian universe; they do not operate as 'secondary causes'. Kepler made clear that archetypes have no causal efficacy (*Archetypos, et quia horum per se nulla efficacia est*).¹⁴

Therefore, Kepler should be included in the history of laws of nature because of his redefinition of the disciplines concerned with the heavens. Kepler is widely praised

¹³ Martens, *Kepler's Philosophy and the New Astronomy*. See, also Field, *Kepler's Geometrical Cosmology*; Kozhamthadam, *The Discovery of Kepler's Laws*.

¹⁴ Kepler, *KGW*, 1963, 8:63.

for being ‘the first to envision astronomy as a part of physics’.¹⁵ In Kepler’s account, astronomy had reached a dead-end in the works of Copernicus and Brahe, for the cosmological implications of their mathematical hypotheses were contradictory, yet both were observationally equivalent. Thus, it was not possible to decide which system corresponded to the actual constitution of the universe on purely mathematical grounds. His *Commentaries on the motion of Mars*, whose subtitle reads *New astronomy based upon causes or celestial physics*,¹⁶ was an explicit redrawing of the boundaries of mathematical astronomy in order to include natural philosophical considerations and thus to move astronomy away from its standstill. Kepler believed that astronomy was progressive, and the necessary move to set it in motion again was the introduction of elements from physics.¹⁷ Mathematical equivalence did not entail physical equivalence. Thus, the decision on which world-system or astronomical hypothesis provided a better account of the heavens should be settled by involving natural philosophy. This redefinition, which implied the consideration of physical problems into astronomy, provided the model for Descartes’ *a priori* physics, as I will argue in the next chapter, and stimulated the tradition of ‘elliptical astronomy’ in England by refreshing Gilbert’s accomplishments, a topic that I will explore in chapters 5 and 7.

In order to develop these claims, this chapter is divided in three more sections. The next contextualises Kepler’s works, especially the *Mysterium Cosmographicum* (or *The Sacred Mystery of the Cosmos*),¹⁸ in the Lutheran debates on the role of nature in the conception of the divine providence. Next, I will examine the archetypal cosmology on the redefinitions of God as an architect, matter as quantity and human knowledge as recollection/observation. It will appear how the archetypes operate as formal causes because they are co-eternal with God, find their place in matter and constitute the foundation on which all justification shall ultimately be founded. Finally, it will analyse Kepler’s reform of the sciences of the heavens and their

¹⁵ Stephenson, *Kepler’s Physical Astronomy*, 2.

¹⁶ Kepler, *Astronomia Nova*, 3.

¹⁷ Jardine, *The Birth of History and Philosophy of Science*, 222; Martens, *Kepler’s Philosophy and the New Astronomy*, 103.

¹⁸ Barker and Goldstein, ‘Theological Foundations of Kepler’s Astronomy’, 99.

mutual interaction derived from his cosmology. This section emphasises that Kepler's reform of knowledge aims at an astronomy founded upon physics, not to found physics *a priori*. This is an important case to contrast with the Cartesian use of metaphysics to validate the causal function of his laws of nature, as ultimate principles of his *a priori* physics in the next chapter.

3.2. The Keplerian problem

The redefinition of astronomy as a consequence of Kepler's cosmology is connected to the debates on providence promoted by Melanchthon and his followers. The distinction and interaction between disciplines, as well as their specific ways to deal with their subjects, are corollaries of the archetypal universe, in which not only geometric and harmonic *ratios* are principles of demonstration, but also Man as the centre and purpose of God's creation. Relevant to this context is that Melanchthon's main reason to turn down the cosmology of the *De revolutionibus*—not its mathematical astronomy—was the rejection of the theological implications of removing man from the centre of the cosmos.¹⁹ Kepler's *Mysterium*

Cosmographicum directly answers to this theological problem in two ways. First, the Earth—man's abode—occupies not the centre but a very special place, in the middle of the progression of celestial bodies from the Sun to Saturn; the centre is reserved to the Sun, representing God the Father around which all planets move. At the same time, man mirrors God in his capability to know the archetypes of the creation and in acting according to them. Kepler remarked that in their 'works men ordain in accordance with harmonic laws' (*quae hominis ab harmonicis leges ordinant*), laws consonant with God's archetypes. In so doing, Kepler put forward a conception of human knowledge encompassing archetypal, mathematical and physical levels. Kepler explicitly subscribed to a revised version of Neoplatonic *anamnesis* as the foundation of knowledge, yet the entire recollection is underpinned by the central role of observations.²⁰ Second, the archetypal character of the universe entails a redefinition of matter as quantity that Kepler nicely put in a shell: '*ubi materia, ibi*

¹⁹ Cf. Kusakawa, *The Transformation of Natural Philosophy*, 172–73; Westman, 'The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory'.

²⁰ Kepler, *The Harmony of the World*, 289–306.

geometria'.²¹ The view of nature in terms of mathematics allowed Kepler to claim that sound knowledge of God can be reached through the contemplation of his perfect works, not only his written word, the Scripture, but nature. Thus, the possibility of knowledge is a central, theological aspect of Kepler's defence of Copernicanism.

Melanchthon's early interests in astrology were related to his way of explaining signs from God that the present world was near the end. The observation of the comet of 1531 was widely read as an indication that something drastic was about to happen, yet neither Luther nor Melanchthon seem to have interpreted it in an apocalyptic way.²² However, during the same month of the observation of the comet, Melanchthon wrote a eulogy on the study of astrology and astronomy, in the form of a letter to his friend Simon Gyranaeus, that came to be prefixed to the widely spread edition of Sacrobosco's *De sphaera* (1531).²³ Melanchthon claimed that astronomy was valuable for Christians because it delivered teachings about Providence. Indeed, providence was defined as 'a knowledge by which God foresees everything and a government by which He protects His whole creation'.²⁴ Astrology was a legitimate way of reaching knowledge of God and, thus, an efficient way to increase faith. But not all kind of practices labelled as astrology were convenient to achieve this goal. An authentic astrology should be based in physics, in order to understand 'what effects the light of the stars has on simple and mixed bodies, and what kind of temperaments, what changes and what inclinations it induces'. Consequently, this astrology was not unfounded speculation but based on the 'observation of physical causes which are ordinances of God'. Just as physics looked for causes and effects in general, astrology uncovered causes and effects implicit in God's ordinances for the world.²⁵

²¹ This famous sentence occurs in *De fundamentis astrologiae certioribus*, Kepler, *KGW*, 1941, 4:17.

²² Kusukawa, *The Transformation of Natural Philosophy*, 124–25.

²³ Kusukawa, 126; Methuen, *Kepler's Tübingen*, 70–77.

²⁴ This definition is taken from the *Initia doctrinae physicae*, 203 as quoted in Kusukawa, *The Transformation of Natural Philosophy*, 154–55.

²⁵ Methuen, *Kepler's Tübingen*, 77.

This vindication of astrology as a way to reveal God's intentions was part of a wider conception of the function of natural philosophy in revealing Providence.

Melanchthon believed that the observation of celestial bodies could reveal that God is the creator and also that he cares for the world. From this knowledge it was also possible to draw rules for ethical behaviour and the appreciation of the scope of divine free will.²⁶ In other words, knowledge of nature revealed that there is a God, architect of the world. Also, this knowledge can offer guides to social and moral behaviour, although it cannot reveal the mysteries of Christ. Understanding the natural world cannot increase the faith in Jesus Christ, for nature did not reveal elements to recognise the incarnation and sacrifice of God the Son; revealed truths on salvation remained reserved for the gospel. This constraint was consequence of the fall, for human beings in their original state would have been able to derive the nature and will of God from the heavens. Philosophy then helps restoring the knowledge of natural law (in the sense of moral, ethical behaviour) lost by the fall.²⁷ Astronomy reveals God's governance of the heavens. Melanchthon saw traces of the original knowledge in Plato who, 'should be judged to have said not only eruditely but also in conformity with religion that eyes were given to us for the sake of astronomy. For eyes were certainly given especially for this reason: that they may be guides to seeking some knowledge of God'. Seeking for knowledge is aligned with religion, while those who despised astronomy 'were deliberate atheists'. Indeed, by not studying astronomy, they 'removed providence', because if these men 'had dealt with this doctrine they would have discovered the manifest footprints of God in nature' and, in so doing, they 'would have been forced to admit that this totality of things was both made and is governed by a certain Mind'. This was ultimately supported by the 'the authority from Scriptures where it is written, 'let them [the lights] be for signs and for seasons, and for days and years [Genesis 1.14]'.²⁸ Astronomy is presented here as a natural activity: for its sake, eyes were given to humans. In stating this, Melanchthon sketched the idea that removing providence

²⁶ Methuen, 78; Barker and Goldstein, 'Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal', 95; Kusukawa, *The Transformation of Natural Philosophy*, 129.

²⁷ Methuen, *Kepler's Tübingen*, 78; Harrison, *The Fall of Man and the Foundations of Science*, 96–103.

²⁸ Letter to Gyranæus (1531) quoted and translated in Kusukawa, *The Transformation of Natural Philosophy*.

from our understanding of nature is detrimental both for theology and astronomy. In his view, the connection between the study of nature and providence was not accidental or optional. ‘Epicureans’ were labelled as atheist because they did not recognise God as creator and of course they were not astronomers.

Furthermore, failing to recognise God’s governance of the world amounted to denying the law-like pattern exhibited by the regularity of celestial bodies. Drawing from this theological point, Melanchthon formulated a version of the argument from design, available to him in ancient sources such as Cicero’s *De Natura Deorum*. In his *Initia doctrinae physicae* (1549), the order and regularity of celestial motions demonstrated that ‘there is an architectonic mind of this world, because it is impossible for this most beautiful order of things (*corporum*) and of celestial motions, position of elements and conservation of species, mind and knowledge ruling life in man, to have come into being by chance or to subsist by chance’.²⁹ God is referred to as a ‘Mind’, stressing his architectonic role. This specific formulation of the argument from design is connected with some other ideas: God formed the human mind in such a way that its natural function is to look for the ‘consideration of nature’ like ‘swimming to a fish or singing to a nightingale’.³⁰ In this study of nature in which the human mind uncovers God’s providence, geometry and arithmetic function as ‘wings’ necessary for knowing God.

In Melanchthon’s cosmology, human beings occupied the central place of creation. This position is geographical and eschatological, and both meanings are inextricably connected. Men’s location in the centre of the cosmos is a sign of divine love. Certainly, God created ‘this great and wondrous work (...) so that it might be the dwelling place of human nature’. The universe and specially the celestial bodies and heaven were designed in such a way that human beings could uncover the divine providence, for ‘God wished to be known and beheld’. In other words, the entire universe was created by ‘the goodness of God and His immense love towards

²⁹ Kusukawa, 157.

³⁰ Melanchthon, *De astronomia et geographia*, 297 as quoted in Methuen, *Kepler’s Tübingen*, 85.

mankind'.³¹ But God's love towards men was not restricted to seeds 'vestiges of divinity' in the creation; he also endowed human beings with a soul 'inflamed with love and enthusiasm for the truth and rousing', providing them with a natural tendency to search for knowledge, and making men capable of reaching certainty. In Melanchthon's words,

The whole nature of things is like a theatre for human mind, which God wished to be watched. For this reason He placed in the minds of men the desire of considering things and the pleasure which accompanies this knowledge. These reasons invite healthy minds to the consideration of nature, even if no use followed. Just as vision delights, even if no use followed, so the mind also, by its own nature, is led to beholding things. Therefore, these are the reasons of this study, especially because to consider nature is to follow one's own nature, and consideration *per se* leads to the most pleasant joy, even if other uses did not follow.³²

The universe is an intentional creation of God for men's benefit. A clear sign of this divine love is the natural tendency to seek knowledge, even if that knowledge is not useful ('even if no use followed').

Furthermore, only heavens reveal God's providence in a direct way, yet all nature is God's creation, including animals and 'lower beings'. Melanchthon offered theological arguments for the distinction between the heavens—in which the perfect motion of celestial bodies can be described mathematically—, and the sublunary region of generation and corruption—where change and imperfection are outside the domain of quantity. Because mathematics provides the means to explain and understand the perfection of the heavens, it is the best way to reach the knowledge of God through nature. Although the sublunary region was also created following a divine plan, Melanchthon observes that its order points to the observer in the first instance to the influence of the stars and then only indirectly to God. Thus, if the human mind is expected to reach the knowledge of God, it has to transcend its immediate surroundings and make observations of the celestial bodies, by whose regularity and perfection it is possible to attain the knowledge of God.³³

³¹ Melanchthon, *Initia doctrinae physicae*, 213f quoted and translated in Kusukawa, *The Transformation of Natural Philosophy*, 154.

³² Melanchthon, *Initia doctrinae physicae*, 189 as quoted and translated in Kusukawa, 150.

³³ Methuen, *Kepler's Tübingen*, 84.

Mathematics lay at the centre of Melanchthon's reinterpretation of natural knowledge.³⁴ Although the reformer was using a Platonic idea (that geometry and arithmetic are 'wings'), the interpretation he made was done under his religious interests: mathematics were 'wings' necessary for knowing God. In his writings about education, Melanchthon put forward the idea that arithmetic and geometry were propaedeutic for the study of syllogisms and thus preparatory for philosophy; but beyond this pedagogical function, geometry accounted for God's governance as clearly as a picture appears to the eyes. Knowledge, in consequence, is natural to human beings:

I hope you (...) blaze with the desire of knowing that most beautiful teaching of celestial motions and effects. This step should be made through geometry. You will believe that you should thank God when you devote yourself to these studies. For it was said by Plato most gravely: 'God always geometrizes', that is, I understand it, He governs everything and with the surest law regulates the heavenly courses and the whole of nature (*gubernare omnia et certissima lege cursus caelestes et totam naturam regere*). Hence it is hardly doubtful that he approves the study of those who, as if observing the lines of the course of the heavens, know and worship the Ruler Himself.³⁵

Geometry delivered knowledge of the divine design and governance of nature from the regularity of celestial motions. But geometry was not like the ladder that should be thrown away after climbing up. Melanchthon interprets the Christian teaching stating that God created man in his own image as meaning that the human mind was created as capable of understanding nature through mathematics and, in this sense, geometry reveals the correspondence between God the geometer, the ordered nature and the fallen man that can understand the principles of natural law through philosophy.³⁶

³⁴ Methuen, *Kepler's Tübingen*; Westman, 'The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory', 75; Westman, *The Copernican Question : Prognostication, Skepticism, and Celestial Order*; Kusakawa, *The Transformation of Natural Philosophy*.

³⁵ Melanchthon, Praefatio in geometricam, 110 as quoted in Kusakawa, *The Transformation of Natural Philosophy*, 139–40.

³⁶ Methuen, *Kepler's Tübingen*, 83.

The governance of God is described in terms of ‘laws’, in a theological sense, with implications for mathematics. Sometimes Melanchthon switches from ‘laws of nature’ to ‘laws of motion’: ‘When the mind considers this wondrous order of nature, namely those laws of motion (*videlicet ipsas motuum leges*), the fixed species of planets and animated beings and their modes of generation and the periods of their duration, it is necessary to infer a prior and knowing cause, namely God the creator, by whose design this whole order is both established and governed as well as conserved, just as the teaching about God in the Church clearly teaches’.³⁷ This use of ‘laws of motion’ is close to the unspecific view that Jardine also identified in Reinhold, Peurbach and others.³⁸ However, given Melanchthon’s conception of astronomy, in which the order of nature is *specifically* understood in geometrical terms as related to the regularity of celestial motions, this use of the term constitutes a slight departure from the vague use of the term as synonym of the order of nature.

The reasons to reject the Copernican cosmology are related to Melanchthon’s connection between his moral philosophy as rooted in a natural philosophy in which man constituted the ultimate goal. Thus, man’s abode, the earth, had to be located in the middle of the physical universe, the centre of creation.³⁹ In consequence, Melanchthon and his followers accepted with enthusiasm the mathematical calculations of Copernicus because they led to better accuracy in predicting planetary positions; however, they rejected Copernican cosmological claims, on the ground of the patent contradiction with their basic premise of the human location in the centre of the universe.⁴⁰

These ideas widely circulated in Kepler’s Tübingen.⁴¹ Maestlin and Kepler studied theology under the tutelage of Jacob Heerbrand (1521-1600), a pupil of Melanchthon

³⁷ Melanchthon, *Initia doctrinae physicae*, 410, as quoted and translated in Kusakawa, *The Transformation of Natural Philosophy*, 158.

³⁸ Jardine, *The Birth of History and Philosophy of Science*, 242.

³⁹ Kusakawa, *The Transformation of Natural Philosophy*, 172–73; Methuen, *Kepler’s Tübingen*, 100.

⁴⁰ Westman, ‘The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory’.

⁴¹ Methuen, *Kepler’s Tübingen. Stimulus to a Theological Mathematics*; See also Barker and Goldstein, ‘Theological Foundations of Kepler’s Astronomy’; Kusakawa, *The Transformation of Natural Philosophy: The Case of Philip Melanchthon*; Caspar, *Kepler*; Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order*; Westman, ‘The Melanchthon Circle,

who wrote a famous *Compendium theologiae*. From the 1580s onwards, this became the textbook for teaching theology. It is worth noting that the major difference with Melanchthon is that, instead of emphasising the ethical and theological consequences of the creation, Heerbrand developed an interest in creation itself commenting on both Moses account in Genesis and Plato's *Timaeus*, giving priority to Moses' account. Heerbrand's discussions of the creation of the heavens exhibit an up-to-date knowledge of astronomical issues.⁴²

It appears that the context of the Lutheran Reform in which theology was redefined, establishing new connections with astronomy and natural philosophy set the context for Kepler's works. It is in this context that Kepler answers to pressing questions. These questions had assumptions and consequences different from those raised in the astronomical tradition of Ptolemy. By giving to astronomy a central place in this reformed theology, astronomy faced challenges without counterpart in the medieval tradition of mixed-mathematical sciences coming through the Arabs. Therefore, questions asking for the number of the planets, their true order, their mutual interaction or their effects on the life on earth were embedded in a specific theological context that sets the problems and provides concepts and methods not necessarily present in these terms in the tradition of mixed-mathematical sciences. In this light, Kepler's astronomy should be read, to some extent, as embedded in the Lutheran response to Copernicus. The opening sentence of the 'Praefatio Antiqua' of the *Mysterium Cosmographicum* (MC) locates the project within the context of a theological re-formulation of the geometrical cosmology:

I propose, reader, to demonstrate in this little book that most Good and Great Creator, in the Creation of this moving world, and the arrangement of the heavens, referred to those five regular solids, well known from Pythagoras and Plato to our own time, and that to their nature he fitted the number of the heavens, their proportions, and the plan (*ratio*) of their motions.⁴³

The 'secret' revealed in the work is the reason for the number and proportion of celestial motions rather than the determination of the position of the planets at any

Rheticus, and the Wittenberg Interpretation of the Copernican Theory'; Gerdes, 'Johannes Kepler as Theologian'.

⁴² Methuen, *Kepler's Tübingen*, 112–13.

⁴³ Kepler, *KGW*, 1963, 8:23.

given time.⁴⁴ The mention of God locates this ‘little book’ in the context of the providence as it relates to the Copernican astronomy/cosmology. Indeed, Kepler presented astronomy as the field in which his theological activity took place, as he remarked in a letter to Herwart von Hohenburg: ‘Since we astronomers are priests of the highest God in regard to the book of nature, it benefits us to be thoughtful, not of the glory of our minds, but rather, above all else, of the glory of the Creator’.⁴⁵ And, in a letter to the Faculty of Theology he is even more forceful: ‘I wished to be a theologian; for a long time I was troubled, but now see how God is also praised through my work in astronomy’.⁴⁶

3.3. Reading the blueprints

Kepler’s universe was modelled by God following a strict mathematical plan that human beings are created to decipher. From the study of astronomy, it followed that ‘there is a God, founder of all nature, and that in the very mechanics of it he had care for the humans that were to come’. Kepler praised himself for being the first to have uncovered the blueprints of the creation in six thousand years—the presumed age of the Earth. However, Kepler acknowledges that some predecessors, such as Plato and Pythagoras, had partial adumbrations.

The idea that the world was created by God following a mathematical plan that emanated from His own nature constitutes Kepler’s point of departure and arrival. The first direction can be appreciated in his first published work, the *Mysterium Cosmographicum* (1596). Kepler attempted to demonstrate that the Copernican arrangement of celestial bodies could be derived *a priori* from the geometrical structure of the universe upon which God drew when creating the world: the five ‘Platonic’ solids. Some years later, the harmonic proportions of the *Harmonice Mundi* (1619) played a similar role, for they were presented as patterns constituting *a priori* the motion of the planets and the eccentricity of their orbits, alongside with other phenomena such as the action of celestial bodies on earth and human music. In

⁴⁴ Barker and Goldstein, ‘Theological Foundations of Kepler’s Astronomy’, 100.

⁴⁵ Letter to Herwart von Hohenburg, March 26 1598, Kepler, *KGW*, 1945, 13:193.

⁴⁶ Kepler to the Theological Faculty, 28 February 1594 as quoted in Aiton, ‘Johannes Kepler and the “Mysterium Cosmographicum”’, 175.

the reverse direction, Kepler postulated that the authentic knowledge of nature disclosed that behind the apparent diversity of phenomena lay order, proportion and beauty and, therefore, an intelligent God that framed the world. Kepler deduced that atheism was not only dangerous for religion but incorrect in philosophy, for it implied a substantive limitation in the knowledge of nature; in fact, astronomers not recognising God as creator restricted astronomical demonstrations to the calculation of planetary positions ruling out the true knowledge of the heavens. Kepler refers explicitly to these two orders when putting together the arguments in the *Harmonice Mundi*: the ontological, deductive order would be God-nature-man and the epistemological man-nature-God.⁴⁷

The God of Kepler was adorned with the attributes of the Christian God (omnipotent, omnipresent, omniscient, merciful). However, Kepler emphasises his perfection and his power as creator, virtues that he considered as indissolubly linked. The connection between divine perfection and his role as maker of the universe underpins the idea that God followed a plan for creating the world. Kepler interpreted the perfection of God as implying that everything was created with reason or purpose according to the divine design. Because of this, Kepler usually portrayed God as an architect. Meanwhile, the universe somewhat shares the attributes of the creator. In the *Mysterium Cosmographicum*, Kepler claimed that ‘the most perfect builder must needs produce a work of greatest beauty, *for it is not now, nor ever was, possible* (as Cicero, following Plato’s *Timaeus* shows in his book on the Cosmos) *that the best should ever be anything but the most beautiful*’.⁴⁸ God is the most perfect creator and thus his works mirrored this perfection: ‘the nature of all things imitate God the founder (*conditorem*), to the extent possible in accord with the foundation of each thing’s own essence’.⁴⁹ From this characterisation of God, Kepler drew two cosmological consequences: that geometry is (in) the essence of God and, in consequence, (in) the essence of his works; then, God, nature and man mirror each other.

⁴⁷ Kepler, *The Harmony of the World*, 283.

⁴⁸ Kepler, *KGW*, 1963, 8:24. Cf. Martens, *Kepler’s Philosophy and the New Astronomy*, 173; Caspar, *Kepler*, 366; Bertoloni Meli, *Equivalence and Priority*, 424.

⁴⁹ Kepler, *KGW*, 1939, 2:19; Kepler, *Optics*, 19.

The characterisation of God as an architect placed geometry in the centre of cosmology. In the second chapter of the *Mysterium Cosmographicum*, which summarises the cosmology on which the theory of the solids rests, Kepler explained that for creating the world God preconceived an Idea. The origin of this idea of the world is but God himself, for he could only have derived it from his own essence; because only God existed before the creation, God himself had to be its source.⁵⁰ In explaining why God relied on specific patterns when creating the world, Kepler moves one step back and claims that the ultimate origin of the archetypes is geometry which is nothing but God himself:

Geometry, which before the origin of things was coeternal with the divine mind and is God himself (for what could there be in God which would not be God himself?), supplied God with patterns for the creation of the world, and passed over to Man along with the image of God: and was not in fact taken in through the eyes.⁵¹

The idea that geometry was co-eternal with God also appears in Kepler's annotations to Proclus's commentary on Euclid, quoted *in extenso* in the Book IV of the *Harmonice Mundi*. In this chapter, Kepler discussed the nature of mathematical things (*mathematicas species*) in order to justify the need for the harmonic ratios in explaining planetary motions. Kepler's argumentative strategy was to throw in with the Neoplatonic tradition of Proclus against Aristotle. Kepler claimed that mathematical ideas were prior to sensible things and thus could not be 'taken in through the eyes', while Aristotle in *Metaphysics* argued against Plato that universals were but abstracted from sensible things and do not exist independently from them.⁵² This conception runs against Kepler's project of using mathematics to demonstrate *a priori* some characteristics of the world as it was created by God. In fact, Kepler blamed Aristotle's atheistic view of nature for this philosophical mistake.⁵³ Reflecting on the nature of numbers in his early *De quantitibus libelli*, Kepler remarked that the higher questions concerning the origin of counting and numbers

⁵⁰ Kepler, *KGW*, 1963, 8:44–45.

⁵¹ Kepler, *The Harmony of the World*, 304.

⁵² See Lohr, 'The Sixteenth-Century Transformation of the Aristotelian Division of the Speculative Sciences'; Jardine, *The Birth of History and Philosophy of Science*, 225–27; Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*.

⁵³ Kepler, *The Harmony of the World*, 297–98; Kepler, *KGW*, 1940, 6:219–20.

remained ‘inaccessible to Aristotle, inasmuch as he was ignorant of the true God’.⁵⁴ The reply to Aristotle in the *Harmonice Mundi* took the form of a long quotation from Proclus’ Book on Euclid to which Kepler interpolated his Christian reading of the Neoplatonic conception of mathematics.⁵⁵ Maybe no other piece better displays Kepler’s interpretation, in his own Christian way, of the Neoplatonism as expressed by Proclus [Italics are Proclus’ quotation and round brackets are Kepler’s comments]:

*The soul itself is the generator of mathematical species and concepts. But if, containing them in itself as first patterns or paradigms, it makes them take their essential character, in such a way that their generation (the Christian understands, the creation of sensible things) is nothing but the propagation of species which were previously in it (that the mathematical reasons for the creation of bodies were coeternal with God, and that God is pre-eminently soul and mind, whereas human souls are images of God the Creator, even in essentials in their own way, is known to Christians) then we shall agree with Plato, in saying this, and the true essence of mathematics will have been discovered by us.*⁵⁶

The Neoplatonic claim stating that mathematical ideas were imprinted in human souls and that experience is but an awakening of them becomes, in Kepler’s hand, the Christian doctrine of the creation of man in God’s image. Moreover, the ‘propagation of species’ is also interpreted as the creation of bodies which, for Kepler, is the instauration of quantity in which geometry became three-dimensional matter or body.

If geometry belongs to the essence of God, body (*corpus*) was created to instantiate the divine ideas in God’s mind before the creation and, in consequence, its nature was geometrical. God created body to express the archetypes, geometric in nature.⁵⁷ Body was central to God’s plan because it allowed him to make a finite, perfect universe—infinity is a negative characteristic for Kepler.⁵⁸ Since God was perfect and thus had to create the most beautiful universe, he turned to the three-dimensional

⁵⁴ Cifoletti, ‘Kepler’s De Quantitatibus’, 235.

⁵⁵ Kepler, *The Harmony of the World*, 299–302; Kepler, *KGW*, 1940, 6:218–21.

⁵⁶ Kepler, *The Harmony of the World*, 299.

⁵⁷ Martens, *Kepler’s Philosophy and the New Astronomy*, 48–49.

⁵⁸ In fact, the indeterminacy implied in infinity is contrary to the idea of number and proportion that Kepler attributes to the universe as the work of a perfect God. Cf. Kepler, *KGW*, 1945, 13:35.

body in order to avail himself of rational constraints upon which to shape the created cosmos and determine a finite number of planets.⁵⁹ This cosmogonic idea is sketched in the *Mysterium Cosmographicum*:

In the beginning, God created body; and if we know the definition of body, I think it will be fairly clear why God created body and not any other thing in the beginning. I say that what God intended was quantity. To achieve it he needed everything which pertains to the essence of body; and quantity is a form of body, in virtue of its being body, and the source of its definition.⁶⁰

The geometric nature of body, its definition in terms of quantity rather than quality, also implied that ‘space without matter is negation’.⁶¹ In fact, the basic principles of creation (the curved and the straight) and space were derived from the nature of quantity as the essence of body. The creation of physical powers in matter in order to generate motion in the universe instantiate quantity as well.⁶² In the first edition of the *Mysterium Cosmographicum* Kepler claimed that ‘quantity in fact was created in the beginning together with matter; the heavens were created the second day’. For the 1621 edition, Kepler added: ‘Rather, ideas of quantity are and were co-eternal to God Himself. These ideas are also exemplars in souls made in the image of God (being their essence as well)’.⁶³

The idea that geometric ideas were innate, as defining the contemplative faculties of our soul, is but one small part of a Kepler’s theory of soul.⁶⁴ In line with Neoplatonism, Kepler characterised knowledge as *anamnesis*. Against abstraction, Kepler argued that:

Since quantities possess constructability not by virtue of the figures’ passing before the eyes, but in virtue of being clear to the eyes of the mind, in virtue not so much of having been abstracted from sensible things but of never having been associated with them, therefore, we have rightly established abstract quantity as the

⁵⁹ Davenport, ‘Did Johannes Kepler Have a Positive Theory of the Real Presence?’, 326.

⁶⁰ Kepler, *KGW*, 1963, 8:44.

⁶¹ Kepler, 8:65.

⁶² Cf. Martens, *Kepler’s Philosophy and the New Astronomy*, 109.

⁶³ Kepler, *KGW*, 1963, 8:26.

⁶⁴ See more in Boner, ‘Kepler’s Living Cosmology: Bridging the Celestial and Terrestrial Realms’; Escobar, ‘Kepler’s Theory of the Soul: A Study on Epistemology’.

terms for archetypal proportions, that is those which are constructible from the divisions of the circle.⁶⁵

This characterisation of knowledge occurs as part of Kepler's attempt to find the causes of the harmonic proportions in the division of a circle into equal parts in the *Harmonice Mundi*, which 'are made geometrically and knowably, that is, from the constructible regular plain figures'.⁶⁶ Kepler founded harmonic proportions in geometry as different from other theories of proportions based on the meaning of integers of the Pythagorean. Kepler's solution was that geometry, co-eternal with God, supplied patterns 'for the furnishing of the world, so that it should become best and most beautiful above all most like to the Creator'. But geometry not only supplied patterns to God when creating the world but also, insofar as man's soul (and other souls) are images of God, patterns provided the way to 'command each of their own bodies, to govern, move, increase, preserve and also particularly propagate them'. In the case of human mind, Kepler stressed that 'they use the very same [proportions] as laws for performing their functions and for expressing the same proportions in the motions of their bodies'.⁶⁷ Because of this, geometry connects God with humans, as Kepler writes in a letter to von Hohenburg: 'Those [geometric] laws are within the grasp of the human mind; God wanted us to recognise them by creating us after his own image so that we could share in his own thoughts'.⁶⁸

The view of God as an architect also legitimised the causal reasoning. In Kepler's view, any precise enquiry is but a variant of the more general form: why did God create things this way and not otherwise? Instead of describing the observable world and then formulating possible causes, his cosmology postulated *a priori* reasoning as the first step in the investigation of the world; causation thus is always necessary causation.⁶⁹ As we have seen, the purpose of creation was to exhibit the divinity of the Creator (*ad adumbrandum in mundo diuinitatem Conditoris*).⁷⁰ This divinity was

⁶⁵ Kepler, *KGW*, 1940, 6:303; Kepler, *The Harmony of the World*, 304.

⁶⁶ Kepler, *The Harmony of the World*, 9.

⁶⁷ Kepler, *KGW*, 1940, 6:104–5; Kepler, *The Harmony of the World*, 145–47.

⁶⁸ Kepler, *KGW*, 1945, 13:308–9.

⁶⁹ Barker and Goldstein, 'Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal'; Jardine, *The Birth of History and Philosophy of Science*; Jardine, 'Problems of Knowledge and Action: Epistemology of the Sciences'; Martens, 'Kepler's Solution to the Problem of a Realist Celestial Mechanics'.

⁷⁰ Kepler, *KGW*, 1963, 8:45.

expressed under the form of geometrical archetypes. Because of that, Kepler claimed that God distinguished between the curved and the straight when creating quantity in the world, in order to represent the divine and the creatures respectively. The distinction between the curved and the straight as the fundamental division appeared prominently in the Neoplatonic literature of the Renaissance typically referred to Nicholas of Cusa. Kepler pushed this claim forward and stated that the perfect curved figure, the sphere, is the image of the One God in the Holy Trinity: the Father is the central point, the Son the surface, and the Holy Spirit the regularity of the relation between the point and the circumference.⁷¹ The archetype of the Trinity, the image of God in geometrical terms, accounted for the most general and basic features of the visible world: ‘The Sun in the centre, which was the image of the Father, the Sphere of the Fixed Stars, or the Mosaic waters, at the circumference, which was the image of the Son, and the heavenly air which fills all parts, or the space and firmament, which was the image of the Spirit’.⁷² The necessary properties of the sphere, as the archetype of the Trinity, causally accounted for the characteristics of the visible world.

This interplay between the divine, the geometrical and the physical dominates Kepler’s works.⁷³ Kepler’s most important works, the *Mysterium Cosmographicum* and the *Harmonice Mundi* were attempts to uncover the geometric and harmonic archetypes ordering and explaining the actual constitution of the world. These works were the most important for Kepler because they dealt with his most cherished subject: the harmony—as synonym of order and beauty—of the world. Other works such as the *Astronomia Nova* were subordinated to cosmological interests. In a letter to von Hohenburg, Kepler confessed that his interest in having access to Tycho’s thorough observations was cosmological rather than astronomical: ‘One of the most important reasons for my visit to Tycho was the desire, as you know, to learn from him more figures for the eccentricities in order to examine my *Mysterium* and the

⁷¹ Kepler, 8:44.

⁷² Kepler, 8:24.

⁷³ Martens, *Kepler’s Philosophy and the New Astronomy*, 49.

just mentioned Harmony for comparison. For these speculations *a priori* must not conflict with experimental evidence; moreover, they must be in accordance with it'.⁷⁴

The *Mysterium Cosmographicum* explained (*cur ita, non aliter essent*) the number, quantity and motion of the orbs.⁷⁵ Kepler rejected numerological accounts, because in explaining the foundation of the world we should not argue based on the meaning that numbers have acquired from things following the creation.⁷⁶ Rheticus had claimed in the *Narratio Prima* that God created only six planets based on the perfection of the number. Instead, Kepler argued that the five regular geometric solids (the cube, tetrahedron, dodecahedron, octahedron and icosahedron), nested inside each other in a particular order, gave a precise account of the size of the planetary orbits and of the exact number of planets. The spheres circumscribing the solids would be the spaces of planetary orbits. Because there are only five solids, only six spheres could be circumscribed. But not only the exact number of planets was derived from the properties of this unique class of geometric solids; the position of planets was also connected to the properties of these figures, according to their resemblance to the sphere which determines their nobility. The sphere is the most perfect geometric figure because all its sides are equidistant from its centre; the centres of the faces of the perfect solids are equidistant from the centre of the solids and, in this sense, they participate of the perfection of the sphere. But this argument is ultimately archetypal. In the *Epitome* (1671-1621) Kepler claimed that the perfect solids 'imitate the sphere—which is an image of God (*Dei imaginem*)—as much as a rectilinear figure possibly can, arranging all their angles in the same sphere. And they can all be inscribed in a sphere'.⁷⁷ Kepler divided these solids into primary (the cube, the tetrahedron and the dodecahedron) and secondary (the octahedron and the icosahedron), according to their geometric properties.⁷⁸ The purpose of these elaborations was to define the order of the solids in the nested model. Bearing in mind God's love for humanity, the earth ('supreme and epitome of the entire world

⁷⁴ Kepler, *KGW*, 1949, 14:130; Baumgardt, *Johannes Kepler. Life and Letters*, 61.

⁷⁵ Kepler, *KGW*, 1963, 8:23.

⁷⁶ Kepler, 8:25.

⁷⁷ Kepler, *KGW*, 1953, 7:272; Kepler, *Epitome*, 28.

⁷⁸ See Field, *Kepler's Geometrical Cosmology*, 53–60; Caspar, *Kepler*, 60–71; Martens, *Kepler's Philosophy and the New Astronomy*, 40–56.

and the noblest among all stars') is the starting point of the measurement of all orbits and its position divides the primary from the secondary solids.⁷⁹ The position of earth is not arbitrary and, according to the archetype of the perfect solids, it occupies a special, unique place in the progression of celestial bodies from the sun to Saturn. In this disposition, Kepler praised the work of God the architect: 'From motion we have orbits, and bodies [the perfect geometric solids] from the number and magnitude. It only remains that we said with Plato that 'God always geometrises' and in this fabric of mobiles inscribed solid bodies within the spheres and these spheres within solids, so that no solid body remained unclothed in the inside and in the outside by mobile orbits'.⁸⁰ From this geometric model, Kepler derived *a priori* the sizes of the orbits and compared them with the *a posteriori* calculations resulting from Copernicus' *De revolutionibus*, a work performed by his former teacher Michael Maestlin at Kepler's request. However, Kepler asked Maestlin to accomplish the calculation having the true Sun as centre, instead of the mean sun—a point in space just off the true Sun that Copernicus had used for his calculations following astronomical practices. According to the *Mysterium Cosmographicum*, the Sun was *anima motrix* and then it was the starting point of the calculations of planetary positions. However, in this work Kepler did not offer more evidence that the Trinitarian archetype and the idea of action-at-a-distance to account for the central position of the Sun. A more detailed argumentation of this change was central to the *Astronomia nova*, where Kepler elaborated that geometrical points could not cause motion and then, geometric calculations in astronomy should be referred to the Sun.⁸¹ The coincidence between the polyhedral distances compared with the Copernican calculations was fit enough to count as an empirical support.⁸² See, for example, a comparison between the polyhedral (distances without the Moon) and the Copernican distances:

⁷⁹ Kepler, *KGW*, 1963, 8:52. In the *Epitome* Kepler expands this argument by claiming that 'the nature of man, the observer creature and future dweller on the Earth, was taken up among the archetypal causes (*inter causas Archetypicas fuit*)—as being one who was going to reckon the magnitude of the solar body and contemplate the differences of day and night'. Kepler, *KGW*, 1953, 7:317; Kepler, *Epitome*, 77.

⁸⁰ Kepler, *KGW*, 1963, 8:47.

⁸¹ Martens, 'Kepler's Solution to the Problem of a Realist Celestial Mechanics'; Martens, *Kepler's Philosophy and the New Astronomy*, 47; Stephenson, *Kepler's Physical Astronomy*; Field, *Kepler's Geometrical Cosmology*.

⁸² Martens, *Kepler's Philosophy and the New Astronomy*, 45–46; Field, *Kepler's Geometrical Cosmology*, 63–70.

				<i>Polyhedral distances</i>	<i>Copernican distances</i>	<i>Errors</i>
<i>If lowest point of</i>	Saturn		Jupiter	577	635	-9%
	Jupiter	<i>is 1000</i>	Mars	333	333	0%
	Mars	<i>highest</i>	Earth	795	757	+5%
	Earth	<i>point should</i>	Venus	795	794	0%
	Venus	<i>be</i>	Mercury or	577 707	732	-20% -2%

Figure 3. Polyhedral distances without the moon. Kepler, KGW, 8:82

Nevertheless, the geometric archetype of the polyhedra appears as insufficient to account for the *variation* of planetary motion once Kepler demonstrated that planets move in non-circular orbits. The *Mysterium Cosmographicum* explained the structure of the universe in a static way. But in the *Astronomia Nova*, Kepler demonstrated that planets move in ellipses whose one focus is the Sun and that a line drawn between the Sun and the planet sweeps equal areas in equal times of its orbit. Putting these assertions together indicated that the motion of the planet changes depending on its distance from the Sun: a planet moves faster when it is near its perihelion and slower by its aphelion. In the language of the polyhydric archetypes, the ‘cube and octahedron which are spouses do penetrate their planetary spheres somewhat; the dodecahedron and icosahedron which are spouses do not altogether follow theirs, whereas the tetrahedron exactly touches both’. If planets move in ellipses, the five regular solids were not enough to account for the observed motions. Kepler was aware of this:

From that fact it is evident that the actual proportions of the planetary distances from the Sun have not been taken from the regular figures alone; for the Creator, the actual fount of geometry, who, as Plato wrote, practices eternal geometry, does not stray from his own archetype. And that could certainly be inferred from the very fact that all the planets change their intervals over definite periods of time (...) [I]t is fitting that the Creator, if He paid attention to the proportion of the orbits in general, also paid attention to the proportion between the varying distances of the individual orbits in particular, so that the attention should be the same in each case, and that one should be linked with another.⁸³

⁸³ Kepler, KGW, 1940, 6:299; Kepler, *The Harmony of the World*, 407.

In order to explain the planetary *distances* and the *motion* of planets, Kepler postulated that the pattern of celestial motion was harmonic rather than polyhedral. Early thoughts on this were formulated in letters to Edmund Bruce—an Englishman in contact with Galileo, von Hohenburg and Maestlin in 1599.⁸⁴ Over the next years, Kepler penned some thoughts concerning the role of musical harmony in the explanation of different phenomena. The developments of these ideas on harmony were finally put together in the *Harmonice Mundi Libri V*, published in 1619. In it, Kepler formulated the relation between the sizes and the periods of planetary orbits which came to be known as the ‘harmonic law’. The book revived the ancient tradition of the ‘music of the heavens’, as a critical answer to Ptolemy’s *Harmonics* founded upon his geocentric model and monophonic understanding of music.⁸⁵

The *Harmonice Mundi* developed a conception of harmony beyond music. Kepler’s universal harmony encompassed astronomical, astrological and physical enquiries all of them underpinned by the idea that all these forms of harmony had the same mathematical basis.⁸⁶ In connection with his wider cosmological and theological commitments, Kepler criticised the Pythagorean numerological approach that defined harmony in terms of numerological relations between integers and put in its place an harmonic theory based on geometrical relations between physical quantities.⁸⁷ In addition, for a relation to be harmonic, its beauty had to be perceived by a soul; in the case of human music, this soul was the human mind; in the case of the planetary system, its harmony was designed to be perceived by its centre, a soul in the Sun.⁸⁸ The harmonic theory provided, then, further archetypal evidence on the Copernican arrangement. Furthermore, astronomy became ‘the most precise and objective field for harmonic discovery’.⁸⁹

⁸⁴ Caspar, *Kepler*, 85–96; Stephenson, *The Music of the Heavens*, 90–97; Martens, ‘Kepler’s Solution to the Problem of a Realist Celestial Mechanics’, 112–14.

⁸⁵ Stephenson, *Kepler’s Physical Astronomy*, 16–53.

⁸⁶ Stephenson, *The Music of the Heavens*, 4; Martens, *Kepler’s Philosophy and the New Astronomy*, 115.

⁸⁷ Stephenson, *The Music of the Heavens*, 131–34; Walker, ‘Kepler’s Celestial Music’.

⁸⁸ On this see Boner, ‘Kepler’s Living Cosmology: Bridging the Celestial and Terrestrial Realms’.

⁸⁹ Stephenson, *The Music of the Heavens*, 5.

Kepler's harmonic consideration began before he had access to Ptolemy's *Harmonics*. By 1607 he received a copy of the original Greek and was struck by the similarity between his thought and Ptolemy's.⁹⁰ However, Kepler considered himself in a better position than Ptolemy, for he had knowledge of the true arrangement of the heavens—the Copernican system—and of the polyphonic harmony developed by Gioseffo Zarlino, Vincenzo Galilei and others during the sixteenth century. The first two books of the *Harmonice Mundi* dealt with mathematics, establishing the geometrical foundations upon which Kepler would later postulate the harmonic ratios of nature and man. In the third book Kepler focused on musical harmony, formulating concepts such as *durus* and *mollis*, roughly similar to ours of major and minor and other musical concepts that he later applied to celestial harmony. In the fourth book, Kepler turned to astrology, rejecting a large part of the traditional practices.⁹¹ Kepler postulated an astrology concerned with the influence of stars when their harmonies were perceived by the soul of the Earth, generating effects such as winds, rain and all kind of storms; on the other hand, astrology was concerned with the influence of stars when perceived by the human soul, changing human temperaments.⁹² The fifth book dealt with the astronomical harmonies of the world. Kepler came up again with the polyhedra. At this point, Kepler came to believe that geometrical harmonies—harmonic ratios that give rise to musical harmonies—provide the underlying theory, the *ratio*, to account for all the polyhedral proportions. In this sense, the second chapter of the fifth book presented a wide range of ratios among numbers of edges, faces, plane angles and solid angles, among others.⁹³ Next, Kepler classified the polyhedra under these criteria. The outcome was that the regular polyhedra embodied the geometrical principles of abstract harmony and the nesting model accounted for the beauty of the *structure* of the Copernican arrangement of celestial bodies. However, the nested model only provided an answer to the structure of the universe, not to the universe in motion; in other words, the nested model could not explain the eccentricities and the periodic

⁹⁰ Kepler, *The Harmony of the World*, 390–91; Kepler, *KGW*, 1940, 6:289.

⁹¹ Cf. Walker, 'Kepler's Celestial Music', 245.

⁹² Field, 'A Lutheran Astrologer: Johannes Kepler'.

⁹³ Stephenson, *The Music of the Heavens*, 130–32.

times. The harmonic archetypes filled the gap and provided further evidence of God's perfection in creating the world.

Thus, Kepler's God was not only an architect but also a composer. This means that for establishing the motion of planets, their variations and the eccentricity of planetary orbits God relied upon musical harmonies. In this case, we see again an interplay between the divine, the geometrical and the physical. The basic tenet of Kepler's harmony of the heavens held that the ratio of variation of the motion of planets was equivalent to proportions of length string producing sounds.⁹⁴ If variations in the length of a string on Earth generated sounds perceived as beautiful and harmonious by the human mind, celestial motions produced no sound but give raise to a real harmony tuned to be perceived by the soul of the Sun. This perfect harmony was founded on the fact that the arc of the motion of planets as calculated from the Sun—centre of the music of the heavens and mover of the planets—followed harmonic proportions both between an individual planet's aphelial and perihelial motions and between adjacent planet's diverging and converging motions. Kepler calculated the diurnal motions and interpreted them in terms of his harmonic proportions.

Apparent diurnal motions in column 4 correspond to observations—that is, the results of astronomical calculations. Except for Mercury, there is a perfect harmony in the motion of single planets and in the case of paired planets (columns 1 and 2) the disagreement is minimal. Kepler adjusted one of the extreme motions for each planet, the perihelial motion for the inferior planets, the aphelial motion for Earth and superior planets, so these adjusted motions (in column 5) show how nearly harmonic the actual ('observed') proportions are.⁹⁵ Hitherto, Kepler has explained that the motion of planets followed harmonic proportions; the next step was to prove that there was a common scale in which these planetary motions were tuned. Kepler assumed the motion in the perihelion and the aphelion of Saturn, the slowest and

⁹⁴ Kepler understands musical harmonies as generated by the length of string rather than by wave lengths—as we do. This difference explains Kepler's understanding of musical harmonies in terms of geometrical proportions.

⁹⁵ Stephenson, *The Music of the Heavens*, 149–51.

<i>Harmonies of paired planets</i>	<i>Apparent diurnal motions</i>		<i>Closest proper harmonies of single planets</i>			
<i>Diverging</i>	Converging	Planet, apse	Minutes, seconds	Minutes, seconds		
1/3 (twelfth)	1/2 (octave)	Saturn, A	1'46''	1'48''	4/5	Major third
		Saturn, P	2'15''	2'15''		
1/8 (triple octave)	5/24 (double octave + minor third)	Jupiter, A	4'30''	4'35''	5/6	Minor third
		Jupiter, P	5'30''	5'30''		
5/12 (octave + minor third)	2/3 (fifth)	Mars, A	26'14''	25'21''	2/3	Fifth
		Mars, P	38'1''	38'1''		
3/5 (major sixth)	5/8 (minor sixth)	Earth, A	57'3''	57'28''	15/16	Semitone
		Earth, P	61'18''	61'18''		
1/4 (double octave)	3/5 (major sixth)	Venus, A	94'50''	94'50''	24/25	Diesis
		Venus, P	97'37''	98'47''		
		Mercury, A	164'0''	164'0''	5/12	Octave + minor third
		Mercury, P	384'0''	394'0''		

Figure 4. *Apparent extreme planetary motions*. Kepler, *KGW*, 6:311-312

most external of the six planets. Following his musical theory, these motions of Saturn represented (the lowest) G in the musical scale. Then, he calculated the distribution of the motions of the remaining planets in successive musical octaves, according to the observations of the apses established in the previous table. Based on these results, Kepler attempted to demonstrate that the universal consonances of the six planets existed in a similar way to a counterpoint of four human voices; Kepler assigned to planets human voices—soprano, alto, tenor and bass—although he was aware that planetary motions do not produce sound as human music. In the intelligible music of the heavens, Saturn and Jupiter are bass, Earth and Venus alto, Mars is tenor and Mercury soprano. The existence of a planetary harmony and a polyphonic celestial music has been demonstrated, in Kepler's view. The final step in the long and difficult Book V was the show the agreement between the polyhedral theory and the harmonic proportions. The polyhedra served as a 'rough model', while the harmonic proportions among the apparent motions determined the final dimensions of the planetary spheres.⁹⁶

⁹⁶ Stephenson, 230.

The theological orientation of Kepler's understanding of nature not only shaped his intellectual itinerary but his arrangement of knowledge and particularly the subordination of disciplines.

3.4. The subordination of sciences

The previous section offered elements to understand Kepler's reform of astronomy as embedded in a cosmology theologically driven. This reform is most visible in the *New Astronomy based upon causes or celestial physics, treated by means of commentaries on the motion of Mars* published in 1609, yet hints were already outlined in the *Apologia pro Tychone contra Ursum*, composed between 1600 and 1601.⁹⁷ Traditionally known as the *Astronomia Nova*, Kepler usually referred to this work by the less ambitious part of the title, *Commentaries on the motion of Mars*. The *Mysterium Cosmographicum* did not exhibit the technical knowledge in astronomy nor the mathematical skills characteristic of the *Commentaries on the motion of Mars* whereby Kepler would break with the 'spell of circularity'.⁹⁸ However, these advancements in astronomy were not detrimental to the early cosmology; the discoveries on planetary motion based on the enquiries on Mars led Kepler to set forth the harmonic archetypes which he eventually reconciled with the polyhedral model. The motivation behind Kepler's first visit to Tycho in 1600, in which he was assigned to elaborate the theory of Mars, was to gain access to the Danish's accurate observations in order to advance in his harmonic examinations, including the corroboration of the calculations of the *Mysterium Cosmographicum*.⁹⁹ But the peculiarities of the motion of Mars and the wide range of observations available occupied Kepler's mind until the summer of 1605, when he put together his enquiries on the topic.¹⁰⁰ The achievements in astronomy were developed within the framework of the early cosmology and its theological assumptions, as Kepler acknowledged in the Preface to the second edition of the *Mysterium*

⁹⁷ Jardine, *The Birth of History and Philosophy of Science*; Martens, *Kepler's Philosophy and the New Astronomy*, 57–68. Cf. Serrano, 'Trying Ursus: A Reappraisal of the Tycho-Ursus Priority Dispute'.

⁹⁸ Koyré, *The Astronomical Revolution. Copernicus, Kepler, Borelli.*, 225–79; Caspar, *Kepler*, 136–37; Stephenson, *Kepler's Physical Astronomy*; Field, *Kepler's Geometrical Cosmology*.

⁹⁹ Caspar, *Kepler*, 99–108.

¹⁰⁰ Caspar, 139.

Cosmographicum. According to him, his astronomical achievements ‘illustrate or perfect’ aspects already covered by the *Mysterium*.¹⁰¹

The process leading to the findings that eventually appeared in the *Astronomia nova* made manifest that the development of astronomical ideas, following the archetypal cosmology, entailed a re-definition of the discipline as it was practiced by the end of the sixteenth century.¹⁰² When Kepler arrived in Prague for his first visit to Brahe, he was assigned to resume Longomontanus’ work on Mars. But as Kepler tells to von Hohenburg, ‘I would already have concluded my researches about world harmony, had not Tycho’s astronomy so shackled me that I nearly went out of my mind’. The complexity of the observations prevented Kepler from rushing into conclusions just to support his harmonic speculations. In the same letter, Kepler added that ‘Those speculations may not *a priori* run counter to obvious empirical knowledge, rather they must be brought into agreement with it’.¹⁰³ Indeed, as Kepler became familiar with the intricacies of the motion of Mars, he realised that there was more at stake than the explanation of the motion of a single planet. Kepler saw himself as the architect that may use all these materials ‘according to a plan’. It is not hard to guess to which plan Kepler was making reference. In a letter of the period, Kepler summarises his views on Brahe and the potential use of his observations:

Tycho possesses the best observations and consequently, as it were, the material for the erection of a new structure; he has also workers and everything else which one might desire. He lacks only the architect who uses all this according to a plan. For, even though he also possesses a rather happy talent and true architectural ability, still he was hindered by the diversity of the phenomena as well as by the fact that the truth lies hidden exceedingly deep within them. Now old age steals upon him, weakening his intellect and other faculties or, after a few years, will weaken them that it will be difficult for him to accomplish everything alone.¹⁰⁴

¹⁰¹ Kepler, *KGW*, 1963, 8:9.

¹⁰² See Westman, ‘The Astronomer’s Role’; Westman, ‘Proof, Poetics, and Patronage: Copernicus’s Preface to *De revolutionibus*’; Barker and Goldstein, ‘Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal’; Jardine, *The Birth of History and Philosophy of Science*, 211–57.

¹⁰³ Caspar, *Kepler*, 127.

¹⁰⁴ Kepler as quoted in Caspar, 102–3.

In the light of the *Mysterium Cosmographicum*, it is not surprising that Kepler found that all the astronomical work done in Brahe's factory required order and direction. The collection and uses of observations to calculate planetary positions was but part of the task of the astronomer, for 'the truth lies hidden exceedingly deep within them'. In the metaphysics of the *Mysterium*, Kepler had claimed that God added the mind to the senses for the sole purpose of seeking the causes from our observations.¹⁰⁵ This deeply informed Kepler's conception of astronomy as a practice primarily concerned with observations.

A glimpse of this reformed view of astronomy before its full realisation in the *Astronomia Nova* was sketched in the *Apologia*. The first important aspect is the role of observation in Kepler's Neoplatonic conception of knowledge and its implications for astronomy. Against the sceptics and those claiming that astronomy was not capable of proper demonstrations,¹⁰⁶ Kepler argued that 'the astronomer ought not to be excluded from the community of philosophers who inquire into the nature of things'.¹⁰⁷ Kepler accepted the traditional definition of astronomy as concerned with the calculation of planetary positions in order to predict what motions will appear in the future: 'One who predicts as accurately as possible the movements and positions of the stars performs the tasks of the astronomer well'. However, 'one who, in addition to this, also employs true opinions about the form of the universe performs it better and is held worthy of greater praise'. Kepler explained that 'the former draws conclusions that are true as far as what is observed is concerned; the latter not only does justice in his conclusions to what is seen, but also (...) in drawing conclusions embraces the inmost form of nature'.¹⁰⁸ The reason behind this is the Neoplatonic claim that 'in all acquisition of knowledge it happens that, starting out from these things which impinge on the senses, we are carried by the operation of the mind to higher things which cannot be grasped by any sharpness of the senses'. However, this does not mean that the mind completes what we gather from the senses, as it

¹⁰⁵ Kepler, *KGW*, 1963, 8:17.

¹⁰⁶ See Jardine, 'Problems of Knowledge and Action: Epistemology of the Sciences', 225–47; Westman, 'The Astronomer's Role'; Westman, 'Proof, Poetics, and Patronage: Copernicus's Preface to *De revolutionibus*'; Rose, *The Italian Renaissance of Mathematics*.

¹⁰⁷ Jardine, *The Birth of History and Philosophy of Science*, 144.

¹⁰⁸ Jardine, 92/145.

happens when hypotheses are contrived to observe the celestial motions—a thesis that Kepler attributes to Ursus. On the contrary, Kepler claims that

we first of all perceive with our eyes the various positions of the planets at different times, and reasoning then imposes itself on these observations and leads the mind to recognition of the form of the universe. And the portrayal of this form of the universe thus derived from observations is afterwards called astronomical hypotheses.¹⁰⁹

In Kepler's view of knowledge, senses are driven by reason in a process ultimately leading to uncover the archetypes of the creation (the 'form of the universe').

Observations, then, are not raw material for the mind to interpret, but the starting point of a process in which the mind is awoken to uncover the structure underlying the arrangement and motion of celestial bodies.¹¹⁰ Therefore, an astronomical hypothesis is not an imaginary contrivance to explain observations, but the outcome of an intellectual process in which the mind eventually unearths the causes underlying these observations. In short, Kepler draws from his Neoplatonic conception of knowledge a disciplinary consequence: that astronomy, insofar as it is concerned with observations, uncovers the causes or form of the universe.

A second important aspect from the *Apologia* is Kepler's claim that astronomical hypotheses can provide an account 'of the form of the universe'. In his view, astronomy *could not* only be concerned with computing planetary positions, for *any* mathematical representation of the heavens *involve* natural philosophical assumptions. Kepler is not claiming that mathematical representations of celestial motions (i.e., world-systems or astronomical hypotheses)¹¹¹ may be used for natural philosophy; he is rather making the strong statement that *any* mathematical representation of celestial motions entailed physical consequences. A contrast with Ursus may clarify this point. In his *Tractatus*, Ursus had claimed that 'a hypothesis or fictitious supposition is a portrayal contrived out of certain imaginary circles of an imaginary form of the world-system, designed to keep track of celestial motions'. Ursus explained that these contrived hypotheses 'are nothing but certain fabrications

¹⁰⁹ Jardine, 92/144.

¹¹⁰ Escobar, 'Kepler's Theory of the Soul: A Study on Epistemology'; Boner, 'Kepler's Living Cosmology: Bridging the Celestial and Terrestrial Realms'.

¹¹¹ See Lerner, 'The Origin and Meaning of "World System"'.

which we imagine and use to portray world-systems'. These hypotheses only needed to agree with and correspond to a 'method of calculation of celestial motion, even if not to the motions themselves'.¹¹² That is, hypotheses 'can be preserved and maintained' if they could predict the position of a celestial object, although they may be in open contradiction with natural philosophy, 'for otherwise they would not be hypotheses, or (which is the same thing) fictitious suppositions, but true (not contrived) images of a true (not imaginary) form of the world-system'. Astronomers, in consequence, were required (*aitema*) to 'fabricate hypotheses, whether true or false and feigned, of such a kind as may yield the phenomena and appearances of the celestial motions and correctly produce a method for calculating them'.¹¹³ In his support, Ursus quotes *in extenso* Osiander's Preface, emphasising that since the astronomer 'cannot by any means apprehend the true causes, he must conceive and devise causes or hypotheses of such a kind that when assumed they enable those motions to be calculated correctly from the principles of geometry (...) provided that they yield a reckoning consistent with the observations'.¹¹⁴ The impossibility of reaching the true causes explaining the observed positions of celestial objects necessarily led astronomers to fictitious 'hypotheses'.

Kepler's entire *Apologia* is arguably the reply to this point, but I will only highlight two aspects. Kepler redefined the idea of 'hypotheses' by presenting a history of the use of the term and by establishing its precise meaning in the study of the heavens.¹¹⁵ In astronomy, 'we demonstrate with the help of numbers and figures some fact about a star we have previously observed, from things we have seen when carefully and meticulously examining the heavens. Then in the demonstration we have set up, the above-mentioned observation constitutes a hypothesis upon which that demonstration chiefly rests'. However, Kepler remarked, 'when we speak in the plural of astronomical hypotheses, we do so in the manner of present-day learned discourses. We thereby designate a certain totality of the views of some notable practitioner, from which totality he demonstrates the entire basis of the heavenly

¹¹² Ursus, *Tractatus* in Jardine, *The Birth of History and Philosophy of Science*, 41.

¹¹³ Ursus, *Tractatus*, in Jardine, 42.

¹¹⁴ Jardine, 42–43.

¹¹⁵ See Martens, *Kepler's Philosophy and the New Astronomy*, 50–56.

motions'. These premises are both 'physical and geometrical'. In truth, Kepler's strategy was to argue that astronomical and geometrical hypotheses were different, although both were implied in the calculation of planetary positions. Kepler illustrated his point with instances from the history of astronomy: 'when Ptolemy said that the motions of the planets slow down at the apogee and are speeded up at the perigee, he set up an astronomical hypothesis; but when he introduced the equant, he did so as a geometer for the sake of calculation'.¹¹⁶ Kepler polishes off his argument by stating that

Altogether there are three things in astronomy: geometrical hypotheses; astronomical hypotheses; and the apparent motions of the stars themselves. Accordingly, there are two distinct tasks for an astronomer: one, which truly pertains to astronomy, is to set up astronomical hypotheses such that the apparent motions will follow from them; the other, which pertains to geometry, is to set up geometrical hypotheses of whatever kind (for there can often be various kinds of geometry) such that from them those prior astronomical hypotheses, that is, the true motions of the planets unadulterated by the distortion of the sense of sight, both follow and can be worked out.

The distinction between geometrical and astronomical hypotheses were the soil in which the seeds of the *Astronomia Nova* germinated. Kepler condemned Ursus for the word 'hypotheses',¹¹⁷ for in making geometrical and astronomical hypotheses equal they reduced astronomy to its 'mechanical part', that is to the calculation of observations. Kepler admitted that astronomical hypotheses may be geometrically equivalent, that is, that two astronomical hypotheses may explain the same celestial observations without any apparent superiority from the mathematical point of view. However, 'even if the conclusions of two hypotheses coincide in the geometrical realm, each hypothesis will have its own peculiar corollary in the physical realm'.¹¹⁸ This is the crux of Kepler's reform of astronomy developed in the form of commentaries on the motion of Mars. The first part of the *Astronomia Nova* displayed the mathematical demonstration of the observational (that is, the computational) equivalence of the Ptolemaic, Braheian and Copernican systems ('the

¹¹⁶ Jardine, *The Birth of History and Philosophy of Science*, 154.

¹¹⁷ Jardine, 153.

¹¹⁸ Jardine, 141–42.

equivalence of hypotheses’).¹¹⁹ However, in Kepler’s view, only the (slightly-reformed) Copernican system provided a true account of celestial motions, only the physical corollary of the Copernican hypothesis was true.

Kepler’s reform of astronomy was developed in the *Astronomia Nova*. The title suggests a reform of all astronomical theory, but the work is mainly concerned with the study of Mars. However, the consequences of the study of Mars outlined the implications for the entire solar system, from which the work actually put forward a new astronomy. Moreover, the generalisation of the new astronomical theory to the entire system of the world occurred some years later in the *Epitome Astronomiæ Copernicanicæ* (1617-1621). In the introduction to the *Astronomia Nova*, Kepler claimed that the purpose of the work was ‘to reform astronomical theory in all three forms of hypotheses [Ptolemaic, Braheian and Copernican], so that what we compute from the tables may correspond to the celestial phenomena’.¹²⁰ The first part of the work, which Kepler calls the ‘the ground plan’, demonstrated that the main astronomical hypotheses are equivalent in observational terms. The bottom line here is that, from the mathematical point of view it was not possible to claim which hypothesis was better. In so doing, Kepler was reinforcing the position concerning the progress of astronomy mentioned above: rather than claiming that the astronomy of the ancients was perfect and needed to be re-established—such as Ursus held—, Kepler considered that the practice of astronomy required appealing to something outside mathematics in order to progress. Two assumptions are at play here: first, Kepler’s view of the development of human history and knowledge, in which the Christian revelation helped restoring the knowledge of nature that had been lost as a consequence of the fall.¹²¹ Second, the new cosmology championed in the *Mysterium* was centred on the notion of quantity as the essence of matter. From the disciplinary point of view, one important corollary of equating matter with quantity was that all use of geometry *necessarily* involved physical consequences.

¹¹⁹ Kepler, *Astronomia Nova*, 38.

¹²⁰ Kepler, *KGW*, 1937, 3:20; Kepler, *Astronomia Nova*, 19.

¹²¹ Harrison, *The Fall of Man and the Foundations of Science*.

The reform of astronomical theory entailed, therefore, a reform of astronomy as practice. In the *Mysterium* and in the *Apologia* were at play the metaphysical and epistemological reasons to challenge astronomy as practiced by the end of the sixteenth century. However, in the *Astronomia*, Kepler now faced a different challenge: to prove in the arena of astronomy that this reform was not only desirable and necessary but also possible. The way to solve this conundrum was surprising for the reader of the time:

Meanwhile, although I place this goal first and pursue it cheerfully, I also make an excursion into Aristotle's *Metaphysics*, or rather, I inquire into celestial physics and the natural causes of the motions. The eventual result of this consideration is the formulation of very clear arguments showing that only Copernicus's opinion concerning the world (with a few small changes) is true, that the other two are proved false, and so on.

Defining the true astronomical hypothesis led to 'an excursion into Aristotle's *Metaphysics*'.¹²² The outcome of this was that the reformed Copernican hypothesis was true and that the other hypotheses were 'proved false'. The adjustments to the accepted hypothesis came from the introduction of physical considerations into astronomy, being the most important one the postulation of the true Sun as the centre of astronomical calculations, following the physical postulate that 'the power that moves the planets resides in the body of the sun'.¹²³ In fact, Kepler argued that a mathematical point, such as the mean Sun or the centre of the Ptolemaic model cannot have the physical properties that the centre of the world should have to account for the fact that a planet 'is moved less vigorously when it recedes from the point whence the eccentricity is computed'; in other words, 'that the weakening of power is in the ratio of the distances'.¹²⁴ Physical considerations became the touchstone of astronomical computations.

For the reader of the time, appealing to natural philosophy to solve astronomical (*qua* mathematical) problems was to mix disciplines inadequately. Even readers

¹²² Kepler follows the Reading according to which *Metaphysics* dealt with *corpus mobile* as *sub ratione entis creati*. See Lohr, 'The Sixteenth-Century Transformation of the Aristotelian Division of the Speculative Sciences', 51.

¹²³ Kepler, *Astronomia Nova*, 578.

¹²⁴ Kepler, 279.

sympathetic to Copernicus, such as Maestlin, rejected Kepler's tactic. Maestlin had been enthusiastic of Kepler's approach in the *Mysterium*, especially concerning the possibility of demonstrating *a priori* astronomical conclusions. Maestlin was willing to accept the polyhedral hypothesis as an excursion into final causes;¹²⁵ after all, the idea that mathematics may account for final causes had precedents in the sixteenth-century debates on the nature of mathematics.¹²⁶ However, Maestlin could not accept the introduction of efficient causes in astronomy, such as 'the natural causes of motion'. In a letter concerning the *Epitome*, Maestlin reminded Kepler that the astronomical computations were 'founded upon Geometry and Arithmetic' and that 'physical conjectures' confused the reader instead of enlightening him. However, in Kepler's view, the connection between mathematics and physics was necessary for archetypal reasons. In the *Astronomia Nova*, Kepler clarified that

all things are so interconnected, involved, and intertwined with one another that after trying many different ways by which I might attain to the reform of astronomical calculations, some well-trodden by the ancients and others constructed in the emulation of them by their example, none other could succeed than the one founded upon the motions' physical causes themselves, which I establish in this work.¹²⁷

This interconnection between the mathematical level of astronomy and the physical level of efficient causes was guaranteed by the archetypal cosmology, particularly by the redefinition of bodies as quantities.

The reformed conception of astronomy, encompassing mathematics, 'physics and metaphysics' had been presented as well in 1604 in the 'Prooemium' to the *Astronomia Pars Optica*—a work that Descartes read and commented on to Beeckman.¹²⁸ Kepler begins in a familiar way: 'Astronomy, which deals with the motions of the heavenly bodies, principally has two parts'. The first part is concerned 'with the investigation and comprehension of the forms of the motions (*forma*

¹²⁵ Aiton, 'Johannes Kepler and the "Mysterium Cosmographicum"', 180.

¹²⁶ Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*; Jardine, *The Birth of History and Philosophy of Science*, 225–57; Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*.

¹²⁷ Kepler, *Astronomia Nova*, 19; Kepler, *KGW*, 1937, 3:20.

¹²⁸ Kepler, *Optics*, 13–15. On Descartes reference to Kepler's *Optica* see Gaukroger, *Descartes: An Intellectual Biography*, 128–30.

motuum)’ and is claimed to be subservient to (natural) philosophy. The other part, ‘arising from it’, investigated ‘the positions of the heavenly bodies at any given moment’: this second part was described as having ‘a practical orientation’ and ‘laying the foundation for prognosis’. Arithmetic and Geometry were presented ‘as Plato used to say, as a pair of wings’ supporting astronomy, for Arithmetic serves more ‘the practical part’ and Geometry ‘the contemplative’. The conclusion of astronomy would be arithmetical, comprising the tables of motions ‘and the ephemerides derived from them’. The division between theoretical and practical astronomy constituted Kepler’s attempt to present astronomy as a practice capable of providing an account of the heavens from which the calculations derive. In other words, by claiming that astronomy had two different but connected parts Kepler was introducing (natural) philosophical considerations into astronomy and, at the same time, modifying the goal of the computational tasks of astronomy, now bounded by the principles of natural philosophy. The fruit of astronomy was the tables of motion, a project undertaken by Brahe and culminated by Kepler in the *Tabulæ Rudolphinæ* (1627).

After this dual division, Kepler claimed that astronomy was founded upon two kinds of principles: ‘the observations’ and the ‘physical or metaphysical axioms’. The observational principles were long-recognised components of astronomy by the sixteenth century and included ‘the mechanical part, dealing with instruments and the way of using them, which the phoenix of astronomers, the late *Tycho Brahe*, published five years ago’ and the ‘historical part’ comprising the observation themselves. Kepler told that ‘twenty-four books of the most meticulous observations of this sort were left by Tycho Brahe’. Then, he added a third part concerning optics, ‘for the observed things in heaven and their motions take place through the mediation of light and shadow’. This is the subject of the book and Kepler recognised important predecessors in this field.¹²⁹ But Kepler’s novelty appeared in the presentation of a fourth, physical part comprising ‘the physical and metaphysical principles’, of a different kind from the observational.¹³⁰ These principles dealt with ‘the efficient

¹²⁹ See Lindberg, *Theories of Vision from Al-Kindi to Kepler*.

¹³⁰ Kepler, *Optics*, 13–14.

causes of the motions, or the movers; the formal causes, or figures that movers strive for; the material causes or orbs; and the physical intension or remission of motions'. Kepler presented a *celestial* physics as a constitutive part of astronomy, that is, a part of *astronomy* concerned with efficient, formal, material and final causes of the celestial motions. Kepler remarked that this fourth part would appear under the form of '*Commentaries on the motions of Mars (...) which I think I can call the key to a deeper astronomy*'. Physics may deal with the heavens from a metaphysical point of view that is, in the words of Benito Pereira, 'considering the substance of the heavens and the stars, whether it is ingenerable and incorruptible, whether it is simple or composite, whether it is elementary or is rather a certain fifth essence', but 'even if the physicist and the astrologer deal with the same heaven, they deal with it in different ways'. In fact, the astrologer was not 'concerned to seek and posit causes that are true and agree with the nature of things, but only causes of such a kind that he can universally, conveniently and constantly give and account of all those things which appear in the heavens'—words similar to Ursus'.¹³¹ Claiming that there was a 'celestial physics' modelled after the causal procedures of physics as part of astronomy was worrying for Kepler's contemporaries, for it clearly violates the division of the sciences or, as Osiander claimed in the preface to the *De revolutionibus* this would 'throw into confusion the liberal arts'.

The *Epitome Astronomiae Copernicanae* put together all the elements related to his reform of astronomy. Two elements from the *Epitome* further clarify that Kepler's reform cannot be read as the foundation of an *a priori* physics *à la cartesienne*. Kepler argued in favour of founding astronomy in physics, that is, borrowing the principles from a higher discipline in the order of sciences to act as framework for a derived one; this would solve the problem of the ignorance of the causes as presented by Pereira, Osiander, Ursus and others. Kepler's strategy fits within the Aristotelian conception of natural philosophy of the *Posterior Analytics*.¹³² To understand this qualification it is necessary to bring up the already-mentioned view according to which Kepler does not entirely reject Aristotle's achievements, but reads them as

¹³¹ Pereira, Benito, 1576, *De communibus omnium rerum naturalium principiis & affectionibus*, 47D as quoted in Jardine, *The Birth of History and Philosophy of Science*, 237.

¹³² Martens, *Kepler's Philosophy and the New Astronomy*, 103.

partially mistaken by his ignorance of the philosophical consequences of omitting the role of God in the creation. For Kepler, this was not a trivial claim. It implied Aristotle's mistaken views on the nature of mathematical entities and the idea of abstraction. Kepler corrected this view embracing a Christian version of *anamnesis*.¹³³ It is not surprising, then, that the title page of the fourth book of the *Epitome*, dealing with the heavens, was presented 'as a supplement to Aristotle's *On the Heavens*'.¹³⁴ In the introduction to the book, Kepler explained the differences: that he replaced the multiplicity of movements in single planets by demonstrating that the motion of the planet is not uniform and that this was causally explained by the eccentricities which, in turn, derived from 'the Archetype of the harmonic cosmos'; in consequence, 'it is established that this cosmos cannot be better than it is and that it is impossible that the world should not have been created at a fixed beginning in time'.¹³⁵

In the *Epitome*, Kepler also resumes his thoughts on astronomical hypotheses and their causes. In the scheme of the *Epitome*, Kepler divides astronomy in five parts (the same stated in the *Optica*, but in different order): Historical concerning the observations; optical concerning hypotheses; physical concerning the causes of hypotheses; arithmetical concerning tables and calculations and finally mechanical concerning instruments.¹³⁶ The physical part of astronomy is presented as 'in the highest degree relevant to the purpose' of which the astronomer cannot be 'dispensed'. For the astronomer 'should not have absolute freedom to think up anything they please without a reason'; on the contrary, the astronomer should be able to give '*causas probabiles*' for their hypotheses 'on which you propose as the true causes of the appearances'. In so doing, the astronomer establishes 'in advance the principles of your astronomy in a higher science, namely physics or metaphysics'. Kepler explained that the astronomer was not prevented 'from using those geometrical, physical or metaphysical considerations pertaining to those higher disciplines that are supplied to you by the very exposition of the specific discipline,

¹³³ Kepler, *KGW*, 1940, 6:303.

¹³⁴ Kepler, *KGW*, 1953, 7:250; Kepler, *Epitome*, 5.

¹³⁵ Kepler, *KGW*, 1953, 7:252; Kepler, *Epitome*, 7.

¹³⁶ Kepler, *KGW*, 1953, 7:23.

provided that you do not introduce any begging question'. Once the astronomer has advanced into this path, he is now 'master of what he has set out to do, insofar as he had devised causes of motion' which are in accord 'with reason' and fit 'to give rise to everything that the history of observations contains'. From the multiple enquiries into singular phenomena, making calculations and postulating causes, the astronomer now 'draws together in a single form those things'. At this point, the astronomer was no longer concerned with the 'demonstration of the phenomena', useful for everyday life, but 'aspires with the greatest joy in philosophising, to a higher end'. This higher end, to which the astronomer 'by geometrical and by physical arguments' directs his efforts is no other than place 'before the eyes an authentic form and disposition or furnishing of the whole universe'. This form is the book of nature, in which 'God the creator manifested and represented in part and by a kind of writing without words his essence and his will towards making'.¹³⁷ Kepler is aware of the controversial nature of the introduction of physics into astronomy, but he insisted on its necessity. As he claimed elsewhere in the *Epitome*, rejecting physics in astronomy was striking off the head of astronomy.¹³⁸ However, physics constituted the framework for solving astronomical problems by providing true causes in the form of principles, that is, by offering foundations on which the astronomical reasoning should be based. Physics and metaphysics established the nature of the heavens and the causes of motions which must be assumed in astronomical calculations.¹³⁹ In other words, because metaphysics and physics constituted the starting points of astronomy by providing principles from which the astronomer may derive probable causes, astronomy was ultimately capable of accounting for the form of the world. Kepler clarified that the astronomer put together 'in a single form those things which he had previously determined one at a time', that is, he built an astronomy consistent with physics and metaphysics. In so doing, the ordered universe ultimately revealed the creator and his intentions.

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¹³⁷ Kepler, 7:23, 25; Jardine, *The Birth of History and Philosophy of Science*, 250.

¹³⁸ Kepler, *Epitome*, 10.

¹³⁹ Martens, *Kepler's Philosophy and the New Astronomy*, 105–6.

The division of the sciences—and especially those concerning the stars—was articulated to the new cosmology. But Kepler did not see his work as a rebuttal of his predecessors, not even of Aristotle; on the contrary, Kepler portrayed himself as correcting them by his discovery of the blueprints of the creation. In so doing, Kepler attributed the determination of formal and final causes to the archetypal explanations, although they were not enough to account for nature; the world could not entirely be unveiled *a priori*. The world is not only structure (quantity), but also motion caused by forces ultimately associated to minds or souls considered by natural philosophy.¹⁴⁰ In the case of planetary motion, Kepler drew once and again on Gilbert's 'philosophy of magnetism' as a necessary element to justify his achievements in reforming Copernicus system. In the closing paragraph of the fourth book of the *Epitome*, Kepler contended that the souls to which he had alluded in connection with the motion of the planets were not intelligences 'which draw forth the celestial movements out of themselves as out of a commentary which employ consent, will, love, self-understanding'. On the contrary, these souls 'are of a lower family and bring in only an *impetus*—as if a certain matter of movement—by a uniform contention of forces, without the work of mind'. These souls, however, ultimately performed their actions according to the archetypes, as formal and final causes, for '[these souls] find the laws, or figure, of their movements in their own bodies, which have been conformed to Mind—not their own but the Creator's—in the very beginning of the world and attuned to effecting such movements'.¹⁴¹ There is no tension here between an 'animistic trend' going back to the Renaissance and a 'new mechanical world-view' just emerging. Kepler was re-drawing the boundaries of disciplines according to his view of the world in which 'all things are so interconnected, involved and intertwined with one another'.¹⁴² The full understanding of the creation implied, therefore, an interaction between the disciplines, not a reduction of them.

¹⁴⁰ Boner, 'Kepler's Living Cosmology: Bridging the Celestial and Terrestrial Realms'.

¹⁴¹ Kepler, *Epitome*, 120.

¹⁴² Kepler, *KGW*, 1937, 3:20; Kepler, *Astronomia Nova*, 19.

4. ‘Laws in nature’: Descartes’ *a priori* physics

The body of the Sun is the source of the power that drives all the planets around. Moreover, I have specified the manner [in which this occurs] as follows: that the Sun, although it stays in one place, rotates as if on a lathe, and out of itself sends into the space of the world an immaterial species of its body, analogous to the immaterial species of light. The species itself, as a consequence of the rotation of the solar body, also rotates like a very rapid vortex through the whole breadth of the world, and carries the bodies of the planets along with itself in a gyre, its grasp stronger or weaker according to the greater density or rarity it acquires through the law governing its diffusion.

Kepler, ‘Introductio’, *Astronomia Nova*¹

The matter of the heaven, in which the Planets are situated, unceasingly revolves, like a vortex having the Sun as its centre, and that those of its parts which are close to the Sun move more quickly than those further away; and that all the Planets always remain suspended among the same parts of this heavenly matter. For by that alone, and without any other devices (*machinamentis*), all their phenomena are very easily understood. Thus, if some straws or other light bodies are floating in the eddy of a river, where the water doubles back on itself and forms a vortex as it swirls; we can see that it carries them along and makes them move in circles with it. Further, we can often see that some of these straws rotate about their own centres, and that those which are closer to the centre of the vortex which contains them complete their circle more rapidly than those which are further away from it. Finally, we see that, although these whirlpools always attempt a circular motion, they practically never describe perfect circles, but sometimes become too great in width or in length, so that all the parts of the circumference which they describe are not equidistant from the centre. Thus we can easily imagine that all the same things happen to the Planets; and this is all we need to explain all their remaining phenomena.

Descartes, *Principia*, III, 30.²

4.1. Family resemblance

Kepler’s archetypal cosmology provided the *a priori* foundation for his mathematical astronomy. At the same time, this cosmology endowed mathematical conclusions with causal (physical) significance and thus Kepler claimed to have inaugurated a ‘celestial physics’. The most important astronomical (mathematical) implication of this change of perspective was the restoration of the central place of the Sun for the computations of the position of the planets, based on the philosophical claim that the Sun was the source of motive power of the planets. Largely, the so-called ‘Kepler’s laws’ emerged as mathematical consequences of this approach encompassing mathematics and physics, although both disciplines remained separated. As summarised in the epigraph to this chapter, Kepler developed a physical account of the motion of the planets introducing some species, similar to light, emanated from

¹ Kepler, *KGW*, 1937, 3:34.

² Descartes, *AT*, 1904, IX:92.

the rotating Sun and thus moved the planets with them in vortices. God impressed in the human soul the blueprints of the world; that is, the soul contained the archetypes from which Kepler claimed to have derived *a priori* the principles of his astronomy and his celestial physics. Because these principles were imprinted in the soul, human beings could attain certain knowledge of them and derive consequences. In a similar way, Descartes' sketched a natural philosophy proceeding from causes to effects, starting from 'laws of nature'. Descartes' strategy resembles Kepler's: his solution to the problem of the certainty of natural philosophy consisted in appealing to mathematics for providing causal principles ultimately founded on divine immutability. The connection between mathematics and a new metaphysics explaining the creation of, and relationship with the world guaranteed the certainty of the foundations of the knowledge of the natural world. Based on the laws of nature, Descartes put forth a cosmology in which streams of rotating particles, denominated vortices, accounted for the motion of planets, the phenomena on earth and light. Arguably, Descartes' reform of natural philosophy has a strong family resemblance to Kepler's celestial physics: providing *a priori* foundations to make possible the demonstration of the effects by their causes. This is summarised in *The World*, where Descartes' declared that 'apart from the three laws that I have explained, I wish to suppose no others but those that most certainly follow from the eternal truths on which mathematicians have generally supported their most certain and most evident demonstrations'. These truths were the principles of God's creation, by means of which 'He disposed all things in number, weight and measure'.³

The family resemblance between Kepler and Descartes goes beyond a mere coincidence. In the *Tentamen* (1689), Leibniz praised Kepler's astronomical discoveries, claiming that he was the 'first mortal man to make public the laws of the heavens'. Although Kepler could not 'determinate the causes of so many and so unvarying truths', Leibniz claimed, he opened 'the way to investigation of causes', such as the vortex explaining that rotating bodies' endeavour to recede from the centre along the tangent. 'But later Descartes made brilliant use of these reasonings,

³ Descartes, *AT*, 1909, XI:47–48; Descartes, *The World*, 31–32.

though in his usual manner he concealed their author'.⁴ Leibniz did not indicate the source of this, but he points to a connection between Kepler's laws and Descartes' plenist physics. Furthermore, Leibniz was right in pointing out that Descartes concealed Kepler in his published works, for the only few references to the German astronomer are located in the correspondence and in some unpublished manuscripts. These references are mainly concerned with optics, particularly with the law of refraction *circa* 1620, with one notable exception which I will analyse below. Descartes read and criticised the *Astronomia Pars Optica* in his early correspondence with Beeckman, but there is no reference to astronomy or to the archetypal cosmology.⁵

Another possible connection between Kepler and Descartes is implied by the antecedents surrounding the composition of *Le Monde* and the important changes related to Descartes' intellectual agenda between 1629 and 1633. Schuster, and Gaukroger following him, have noticed that when Descartes visited Beeckman in Dordrecht in the autumn of 1628, Beeckman was 'ploughing through the astronomical works of Kepler'.⁶ Schuster and Gaukroger attributed different meanings to the influence that this meeting had on Descartes. Schuster's account emphasises Beeckman's effort to interpret Kepler's explanation of the celestial phenomena in terms of particles in motion and his refutation of the claim that light is immaterial. Meanwhile, Gaukroger accentuates that Beeckman's reading of Kepler shows 'that there was no longer any reason to separate terrestrial and celestial mechanics, in the traditional Aristotelian way, for both realms were amenable to the same kind of mathematical and physical treatment'.⁷ Both readings provide valuable insights into the transformation of Descartes' intellectual career and the emergence of mechanism at this stage. From my perspective, the meeting between Beeckman and Descartes in the late 1620s implies another crucial aspect.

⁴ Leibniz, *An essay on the causes of celestial motion*, translated in Bertoloni Meli, *Equivalence and Priority*, 127–28.

⁵ Schuster, 'Physico-Mathematics and the Search for Causes in Descartes' Optics—1619–1637', 478; Schuster, 'Descartes Opticien: The Construction of the Law of Refraction and the Manufacture of Its Physical Rationales, 1618–29', 258–95; Gaukroger, *Descartes: An Intellectual Biography*, 139–46.

⁶ Schuster, *Descartes-Agonistes*, 471.

⁷ Gaukroger, *Descartes: An Intellectual Biography*, 220.

The purpose of this chapter is to argue that Descartes' innovative use of laws of nature emerged as part of a project of setting the foundations of a natural philosophy as capable of demonstration *a priori*, that is, of the effects through the causes. Descartes' reform of natural philosophy implied the rejection of powers and qualities in matter. In so doing, Descartes brought explicative resources from mechanics into natural philosophy which he ultimately justified by appealing to God's immutability. This claim has two essential assumptions. First, that Descartes' use of laws of nature is innovative. The idea of laws of nature has a long and continuous history from the Ancients, but before the seventeenth century 'the concept of laws of nature has been used in a rather vague way, to explain "all that we say comes into being". The concept was entirely non-specific'.⁸ This changed in the works of Descartes, who introduced laws of nature as codifications of the basic principles of interactions of the particles generating natural phenomena. Descartes' transplanted the concept of mathematical law into natural philosophy and, in so doing, he presented metaphysical arguments to support the new domain of explanation of the 'laws of nature'.⁹ Second, that the visit to Beeckman in the autumn of 1628 triggered in Descartes the idea of a systematic explanation of natural phenomena based on 'mathematical truths' that evolved into the 'laws of nature'; this 'whole physics'¹⁰ would be a more comprehensive project than the *Regulæ*. In this meeting, Descartes went through Beeckman's *Journal* and realised that his Dutch mentor was providing an explanation of the motion of planets according to Kepler's astronomy based on corpuscles in motion. This meeting would also provide a missing link between Kepler and Descartes, whose similarity has been noticed since their own time. Beeckman has been seen traditionally as an initiator of Descartes' career. However, Beeckman was not only an initiator at an early stage of Descartes' career but also a decisive influence in the second meeting in 1628, by providing Descartes with a fundamental insight: an all-encompassing natural philosophy founded on 'mathematical truths'. This approach better explains the famous break between Descartes and Beeckman, for it seems reasonable to consider that Descartes felt

⁸ Henry, 'The Theological Origins of Laws of Nature and Its Subsequent Secularisation', 67.

⁹ Henry, 'Metaphysics and the Origins of Modern Science'.

¹⁰ Descartes, *AT*, 1897, I:70; Descartes, *CSM*, 1984, III:7.

exposed when he realised that the formerly solitary Beeckman came in touch with the *république des lettres*, particularly through Gassendi and Mersenne.¹¹

The basic elements of Descartes' new natural philosophy were put together in the unpublished *Le Monde* (1633), but it is also probable that Descartes' initial project incorporated materials that he subsequently reworked and published in 1637, prefaced with the *Discourse*. I will highlight that Descartes' solution to the foundation of natural philosophy based in the laws of nature as causes had important consequences for understanding the activity of the world as extended matter. In a way clearly resembling mathematical reasoning, Descartes' efforts are directed at demonstrating that a few general principles were enough to 'explain all the phenomena of nature',¹² making powers and qualities redundant, unnecessary and metaphysically problematic. Furthermore, this change of perspective was wedded to a redefinition of motion and to the dismissal of traditional ideas of force, impetus and attraction. However, these laws should not be considered as mathematical abstractions or idealisations. This reading of laws of nature as abstract idealisations misrepresents Descartes' project; in fact, this interpretation is the reason for aligning Descartes' physics with Galileo's, although Descartes explicitly stepped aside from it and rejected Galileo's reductionist approach. Once I clarify the emergence of Descartes' laws between 1628-1630, I will return to the project of an *a priori* physics, highlighting the disciplinary transformations and particularly the resulting conception of mechanics.

4.2. The Beeckman connection

This section offers evidence in favour of the claim that Descartes' laws of nature emerged within the process of reform of natural philosophy that began in Descartes' mind after visiting Beeckman in 1628. This process can be followed in the correspondence with Mersenne between 1629 and 1630. When Descartes visited his Dutch friend, Beeckman was so involved in Kepler's astronomy that he seriously considered for the first time the possibility to circulate in print his ideas concerning

¹¹ Berkel, 'Descartes' Debt to Beeckman'; Berkel, *Isaac Beeckman on Matter and Motion*, 60–65; Schuster, *Descartes-Agonistes*, 390–94.

¹² Descartes, *AT*, 1897, I:70; Descartes, *CSM*, 1984, III:7.

natural philosophy.¹³ Indeed, Beeckman was sketching a systematic explanation ‘of the motions of the stars and the Earth’ by ‘correcting’ Kepler’s celestial physics based on his understanding of the conservation of motion and on his explanation of the causes of celestial phenomena based on matter in motion. Beeckman rejected Kepler’s idea that light was immaterial and, in its place, developed a corpuscular conception of light integrated to his view of activity in the world—virtually the same way to articulate topics in *Le Monde, ou traité de la lumière*.¹⁴

The details of the 1628 meeting between Descartes and Beeckman have been reconstructed by different scholars.¹⁵ On 8 October he found his old Breda friend in the Latin School of Dordrecht, where Beeckman was rector. According to Beeckman’s *Journal*, Descartes informed him of his activities since 1618, mentioned his work on algebra—which he promised to send when he returned to Paris—and renewed the intention to collaborate. Descartes added that during his travels in France, Germany and Italy ‘he had not found anyone, with whom he could discuss his ideas as freely and from whom he could expect so much help in pursuing his studies’.¹⁶ By January 1629, Beeckman received from Paris a manuscript on algebra, together with some discussions on a mathematical problem that Descartes apparently mentioned the previous autumn. In March 1629 Descartes finally settled in Holland and once again visited Dordrecht on his way to Amsterdam. There is no further communication until the letter of September or October 1630 in which Descartes asked Beeckman to return the *Compendium Musicae*, as an angry reaction to Mersenne’s insinuation that Beeckman had claimed being Descartes’ teacher back in Breda.¹⁷

When Descartes visited Beeckman in late 1628 and in early 1629, the rector of Dordrecht was fascinated by the study of Kepler’s astronomy. He was particularly concerned with ‘correcting’ the causes of celestial motion that Kepler had adduced.

¹³ Berkel, ‘Descartes’ Debt to Beeckman’, 56; Berkel, *Isaac Beeckman on Matter and Motion*, 246n24.

¹⁴ Schuster, *Descartes-Agonistes*, 471–72.

¹⁵ Berkel, ‘Descartes’ Debt to Beeckman’, 50–52; Berkel, *Isaac Beeckman on Matter and Motion*, 46–75; Schuster, *Descartes-Agonistes*, 471; Gaukroger, *Descartes: An Intellectual Biography*, 220.

¹⁶ Beeckman, *Journal*, 1945, 3:94–95; Berkel, ‘Descartes’ Debt to Beeckman’, 50–51.

¹⁷ Beeckman, *Journal*, 1953, 4:194.

Beeckman openly rejected all explanation based on occult qualities, supernatural powers and ‘magnetical powers’, in line with Stevin’s ‘wonder to no wonder’ attitude in natural philosophy.¹⁸ For example, in reviewing *De nive sexangula* in February 1628—a work that Descartes read in a critical moment during the gestation of *Le Monde*, as I will examine later—, Beeckman judged Kepler’s causal explanations as ‘ridiculous and unworthy of a philosopher’ (*ridicula et philosopho indigna*), for ‘this is not adducing a cause (...) but obscuring it’.¹⁹ Thus, the correction consisted in replacing Kepler’s natural-philosophical causes for those satisfying Beeckman’s criteria for intelligible explanations. These criteria deeply relied on values guiding the artisan’s practice; the world is thought of as consisting of ‘tangible, concrete things that act upon each other in a way he could visualize. He [Beeckman] views the world like a craftsman inspecting a machine he is about to repair’.²⁰ No mechanics would appeal to occult qualities or immaterial causes to explain the functioning of a machine; on the contrary, mechanical devices show that only motion or pressure can produce ‘the rearrangement of parts and hence produce work and, for theoretical purposes, the causes of motion and pressures are other motions and pressures’. Indeed, Beeckman’s attitude was that ‘there is no point in talking about effects if you cannot imagine how they are produced’. From this perspective, Beeckman’s work was an attempt to develop a thorough transformation of the approach and understanding of the natural world.²¹ Indeed, the *Journal* shows Beeckman attempts to reform natural philosophy based on mechanical intelligibility.²² Gassendi, after meeting Beeckman and reading his *Journal* in 1629, wrote to Peiresc that Beeckman was ‘le meilleur philosophe que j’aye encore rencontré’.²³

¹⁸ Berkel, *Isaac Beeckman on Matter and Motion*, 130–36; Gaukroger and Schuster, ‘The Hydrostatic Paradox and the Origins of Cartesian Dynamics’; Arthur, ‘Beeckman, Descartes and the Force of Motion’.

¹⁹ Beeckman, *Journal*, 1945, 3:34; Cf. Berkel, *Isaac Beeckman on Matter and Motion*, 146–47.

²⁰ Berkel, *Isaac Beeckman on Matter and Motion*, 137.

²¹ Schuster, ‘Descartes and the Scientific Revolution, 1618–1634: An Interpretation’, i, 59–60.

²² Gaukroger and Schuster, ‘The Hydrostatic Paradox and the Origins of Cartesian Dynamics’, 553.

²³ Mersenne, *Correspondance*, 2:131; Beeckman, *Journal*, 1953, 4:153; Cf. Berkel, *Isaac Beeckman on Matter and Motion*, 58.

Under this light, Kepler's astronomy became a fertile field for trying out new natural-philosophical explanations.²⁴ As the friendship with Descartes was going through this fleeting renewal in 1628, Beeckman optimistically brushed some traces of his celestial mechanics that moved from the solution of dispersed problems—a defining trait of his physico-mathematics—to a more articulated set of explanations about the motion of celestial objects. Between July 1628 and June 1629, 21 out of 59 pages of Beeckman's *Journal* dealt with celestial mechanics,²⁵ so that when Descartes examined Beeckman's *Journal* during his visits of 1628 and 1629, a considerable number of pages were devoted to this topic.

Beeckman acquired the *Astronomia Nova* in August 1628. In the *Astronomia Nova* the motion of planets is generated by an immaterial *species* emanating from the Sun, analogous to the immaterial *species* of light. Because the Sun rotates on its axis, the emanation of this *species* accounted for the motion of the planets. The analogy between (immaterial) light and (immaterial) causes of motion runs through the book. The first thing that Beeckman noticed is that Kepler corrected the claim made in the *Optics* that the force of light weakened as the distance from the source increased. Beeckman contended that the force of light did not decrease; the right explanation was that, as the light moved away from its source, equal quantities of light must illuminate spheres of increasing surface area, because 'light is corporeal' (*lumen esse corpus*), a particular type of heat emitted by stars—something that Kepler did not admit and, in Beeckman's eyes, induced him to embrace erroneous conclusions.²⁶

Before this review of Kepler's cosmological works, Beeckman had explained the motion of planets around the Sun as the preservation of God's initial action of creation in a medium that did not offer any resistance; in other words, as a consequence of his principles of the conservation of motion.²⁷ The criticism of the nature of light led Beeckman to realise the potential of streams of light-particles for

²⁴ Schuster, *Descartes-Agonistes*, 471.

²⁵ Schuster, 471n.

²⁶ Beeckman, *Journal*, 1945, 3:73–74; Berkel, *Isaac Beeckman on Matter and Motion*, 101–3; Schuster, *Descartes-Agonistes*, 472.

²⁷ Berkel, *Isaac Beeckman on Matter and Motion*, 101; DesChene, *Physiologia*, 276ss.

explaining the solar system, in a world less empty than initially thought. Beeckman was fully committed to the idea that ‘what can be done with a few means is said to have been done badly with many’, so apparently one stream of light-corpuscles may account for what Kepler explained by multiple means.²⁸ Initially, he outlined some brief ideas about the balance of attractive and repulsive streams of particles holding the Moon in its orbit. This balance would be the outcome of the solar rays (repulsive) reflected by the Earth and the Earth’s own (magnetic) rays (attractive).²⁹ The initial formulation bore some difficulties, for example, that unreflected rays of the Sun would attract the Moon to it. A few pages after recording Descartes’ visit in the autumn of 1628, Beeckman extended his explanation of the Moon’s balance to the solar system, substituting the fixed stars for the Sun and the Sun for the Earth:

it seems that the same may be said of all planets (...) that the light or corporeal virtue of the eighth sphere [the sphere of the fixed stars] reflected by the Sun draws (*trahat*) all the planets to the Sun and the Sun repels them. And thus each planet will be affected by each of the virtues according to its magnitude or rarity (*magnitudo aut raritas*) and therefore they will be located at different distances from the Sun.³⁰

The ‘rarity’ or ‘magnitude’ of each planet and its motion can be explained in terms of their distance from the Sun; for the closer to the Sun a planet was, the stronger was the repelling action of the Sun and, conversely, the weaker the pushing strength of the ‘corporeal virtue’ of the sphere of the stars. From these draft-annotations emerges a general image of a solar system traversed by streams of particles—emanating from the stars and the Sun and bouncing on the planets—whose balance explained both the location of the planets, their ‘magnitude or rarity’ and their motion. Although these annotations did not constitute a fully elaborated explanation of the solar system, Beeckman saw with optimism the potential of this line of reasoning and extended the formulation to include controversial topics such as the motion of the comets and the three motions attributed to the Earth. On the physical explanation of the motion of the Earth—a cutting edge problem in 1628, for Galileo’s *Dialogo* was only published

²⁸ Berkel, *Isaac Beeckman on Matter and Motion*, 148; Cf. Beeckman, *Journal*, 1939, 1:10; Beeckman, *Journal*, 1945, 3:26.

²⁹ Beeckman, *Journal*, 1945, 3:74–75. A more detailed explanation of this mechanism can be found in Schuster, *Descartes-Agonistes*, 472–73.

³⁰ Beeckman, *Journal*, 1945, 3:100. I have modified the translation offered by Schuster, *Descartes-Agonistes*, 473n31.

in 1632—, Beeckman remarked that ‘I have shown that all three earthly movements are performed without any fictitious internal force (*insitâ vi ficticiâ*) and that they follow in a mathematical way from the movement of particles that are emitted by the sun’.³¹

Entries on Beeckman’s *Journal* have fuelled debates about his influence on Descartes since the seventeenth century. John Smith, after reading *Mathematico-physicarum Meditationum, Questionum, Solutionum Centuria*—a compilation of entries of the *Journal* published by Beeckman’s brother in 1644—wrote to Constantijn Huygens that ‘I recently have read (...) Descartes’ remarks on the magnet, but afterwards I read *Centuria* (already written in 1628 but published only recently), in which, under the numbers 36, 77 and 83 shows that these corpuscles were not first thought of by Descartes’.³² Although the influence is still matter of debate, my interest is to highlight the historical singularity of Beeckman’s exercise. Beeckman was revising some causal explanations of celestial motions—which he considered equivocal—based on mathematical (mechanical) principles: that is, Beeckman was offering an alternative to the natural-philosophical aspects of Kepler’s astronomy borrowing elements from physico-mathematics.³³ The central point here is that, rather than solving physico-mathematical problems *via* analogical reasoning between the universe and machines—something more common than most historians of science have been willing to accept,³⁴ the criticism of Kepler’s natural philosophy led to generalisations encompassing corpuscularian (visual) explanations with mathematical-mechanical principles. Beeckman’s reading of Kepler was specifically and explicitly regulated by two principles that he had derived from his

³¹ Beeckman, *Journal*, 1945, 3:108; Cf. Berkel, *Isaac Beeckman on Matter and Motion*, 101–2.

³² Berkel, ‘Descartes’ Debt to Beeckman’, 58. For further details on Beeckman’s mechanical explanation of the solar system and its connection with Descartes’ mechanical universe see Schuster, *Descartes-Agonistes*, 101; Berkel, *Isaac Beeckman on Matter and Motion*, 75–103; Gaukroger, *Descartes: An Intellectual Biography*; Arthur, ‘Beeckman, Descartes and the Force of Motion’; Aiton, *The Vortex Theory of Planetary Motions*; Koyré, *The Astronomical Revolution. Copernicus, Kepler, Borelli.*; Koyré, *Galileo Studies*; Westfall, *Force in Newton’s Physics*; Descartes, *AT*, 1908; Ariew, *Descartes and the Last Scholastics*.

³³ Cf. Beeckman, *Journal*, 1939, 1:244; Cf. Gaukroger, *Descartes: An Intellectual Biography*, 68–69; Berkel, *Isaac Beeckman on Matter and Motion*, 22–24.

³⁴ See Bennett, ‘The Mechanical Arts’; Bennett, ‘The Mechanics’ Philosophy and the Mechanical Philosophy’; Gabbey, ‘Between Ars and Philosophia Naturalis’; Hattab, *Descartes on Forms and Mechanisms*; Hattab, ‘From Mechanics to Mechanism’.

practice: the corpuscular nature of light and the idea that *change* of motion (rather than motion) required an explanation:

Earlier I have written things about the motions of the stars and the Earth that are slightly different from his—that is, Kepler’s—ideas and perhaps, when I get some free time and am set free from this very heavy load, unsuited for all thinking [his position in Dordrecht], I will treat these matters much more accurately than he does, on the one hand because I start from the fundamental insight, which he refuses to acknowledge, that light, as I said before, is corporeal, and on the other hand, because he does not know what is very true, that is: everything that is moved, will continue to move unless it is hindered (*omnia semel mota, semper moveri nisi impediuntur*).³⁵

Beeckman was not too interested in the specifics of the mathematical astronomy or in the details of astronomical observations. However, he was not interested in the *a priori* validation of these principles, as Kepler was.

These things that Kepler writes about the motion of Mars, in terms of physics, please me very much, especially because long before I read those things I had thought of the same things and had the intention of using them for a reconstitution of astronomy. This can be seen in many places in this book [in the *Journal*], especially where I discussed the motion of the Earth in a physical way. Now that Kepler has earned this glory before me, I hope that once I will be able to finish a work about this subject on the basis of my meditations, which he has not seen.³⁶

In summary, apart from stimulating hints on the action of diverse ‘corporeal virtues’—streams of particles—motion of planets, Descartes found in Beeckman a generalisation of physico-mathematical explanations based on a few principles, that is, an innovative way to solve problems. In his correspondence with Mersenne and in other public appearances during the years of 1628 and 1629, Descartes kept praising his discoveries in optics and the importance of method and he made it appear as coherent a set of tenets on method and mathematics that he had failed to articulate in the *Regulæ*. Descartes’ self-presentation, particularly in the *Discourse*, would give the impression, even to scholars in the twentieth century, that there is continuity (or unity) rather than a major reformulation in the developments between the *Regulæ*

³⁵ Beeckman, *Journal*, 1945, 3:74; Berkel, *Isaac Beeckman on Matter and Motion*, 56.

³⁶ Beeckman, *Journal*, 1945, 3:103; Berkel, ‘Descartes’ Debt to Beeckman’, 56–57.

and *Le Monde*.³⁷ Praising the successes of the method, rather than its failure and reformulation, was of course, in Descartes' best interest.³⁸ Nevertheless, the visits to Beeckman and the influence of his reading of Kepler on Descartes' formulation of the vortex theory underpinned by the laws of nature are complex, non-linear historical process. The family resemblance between Descartes' vortex theory and Beeckman's criticism of Kepler's celestial mechanics shall not lead us to believe that the cosmogony of *Le Monde* has as unique source the correction of Beeckman or that Descartes immediately realised the potential of Beeckman's elaborations when reading the *Journal*; other factors also converge in the genesis of *Le Monde* and the vortex theory.³⁹ However, my point is that in this process of reshaping his intellectual identity,⁴⁰ during the critical years of 1628 to 1630, one central thread in this process was Descartes' idea of an *a priori* physics anchored on the laws of nature, after Beeckman's style and values in re-writing Kepler's celestial mechanics guided by his physico-mathematical principles. At the same time, the emergence of this new physics progressively moved Descartes away from the Galilean way to explain natural phenomena by assuming general conditions open to mathematical treatment.⁴¹

4.3. 'Unexpected riches'

During his first months in Holland, Descartes was mainly concerned with the emerging dualism after the *Regulae* and with some optical problems. On 8 October 1629, Descartes wrote to Mersenne that he had received an account of the parhelia and decided to suspend all other work to 'make a systematic study of the whole of meteorology'. Reports on parhelia were circulating among mathematicians, particularly of the observations that Scheiner reported in March 1629 that subsequently were commented on by Gassendi.⁴² Descartes announced to Mersenne

³⁷ Gilson, *Études sur le rôle de la pensée médiévale dans la formation de système Cartésien*, 9–10.

³⁸ Schuster, *Descartes-Agonistes*.

³⁹ Schuster, 453–524; Gaukroger, *Descartes: An Intellectual Biography*, 226–89; Garber, *Descartes' Metaphysical Physics*; Gaukroger and Schuster, 'The Hydrostatic Paradox and the Origins of Cartesian Dynamics'.

⁴⁰ Schuster, 'The Young René Descartes—Lawyer, Military Engineer, Courtier, Diplomat ... and, We Might Add, Ambitious "Savant"', 7–8; Cf. Koyré, *Études galiléennes*, 127.

⁴¹ Garber, 'A Different Descartes: Descartes and the Programme for a Mathematical Physics in His Correspondence'.

⁴² Descartes, *AT*, 1897, I:29n12.

that he was planning to give some explanation of the phenomenon in a ‘small treatise’ and that this treatise would also provide ‘the explanation of the colours of the rainbow ... and for all sublunary phenomena in general’. Immediately after, Descartes requested Mersenne not to speak to anyone about this small treatise that he had decided to publish ‘as a specimen of my philosophy (*comme un échantillon de ma Philosophie*) and to hide behind the picture in order to hear what people will say about it’.⁴³ It does not seem a coincidence that just after mentioning the new project—which, hitherto seems to be moving from the particular (optical) explanation of parhelia to the explanation of ‘all the sublunary phenomena’—and the confidentiality request, Descartes thanked Mersenne for ‘informing him of the ingratitude of his friend [Beeckman]’. In a dismissive tone, Descartes suggested that Beeckman may have claimed that he was his master ‘il y a dix ans’ to cause a better impression on Mersenne. From the existing record, Beeckman did not claim such a thing but apparently Mersenne ‘loved to get people into disputes’.⁴⁴ For now, however, Descartes appeared not to perceive Beeckman as a major threat and he remained satisfied just with eroding Beeckman’s intentions in alluding to their former apprenticeship in Breda.

In the next letter to Mersenne, on 13 November 1629, Descartes showed his gratitude for sending some information on parhelia and for offering to print the ‘small treatise’, but he apologised because it is not going to be ready as soon as he expected, because ‘rather than explaining just one phenomenon’, he had ‘decided to explain all the phenomena of nature, that is to say, the whole of physics’. The optimism does not stop right there. Descartes admitted that he had found ‘a way of unfolding’ all his thoughts.⁴⁵ This did not sound exactly as the previous meteorological treatise covering ‘all the *sublunary* phenomena’; the meteorological treatise was evolving into a full-length physics. If the road to the natural philosophy of *Le Monde* started off with parhelia, some remarks in this letter gives us a few hints about the direction of the project. In the previous letter of October, Descartes had replied to a question that Mersenne had sent concerning the time it takes a weight to fall when it is

⁴³ Descartes, I:23; Descartes, *CSM*, 1984, III:6.

⁴⁴ Descartes, *AT*, 1897, I:24; Berkel, *Isaac Beeckman on Matter and Motion*, 61.

⁴⁵ Descartes, *AT*, 1897, I:69–70; Descartes, *CSM*, 1984, III:7–8.

attached to a cord 2, 4, 8 and 16 feet long.⁴⁶ Descartes had based his answer on his 1620s' findings on the fall of bodies. Between October and November, Mersenne enquired back for the calculations behind the answer. In November, Descartes replied:

As for your question concerning the basis of my calculation (...) I shall have to include this in my *Physics*. But you should not have to wait for that; so I shall try to explain it. Firstly, I make the assumption that the motion impressed on a body at one time remains in it for all time unless it is taken away by some other cause: in other words, in a vacuum that which has once begun to move keeps on moving at the same speed.⁴⁷

Descartes explained that as the weight falls, it retains its heaviness and this pushes it downwards, but at each moment it receives a new force (*novas vires*) that makes it fall. That is why, as the time increases a weight falling covers some distances more quickly.⁴⁸ The solution to the problem posed by Mersenne explicitly assumes the conservation of motion. Interestingly, Descartes enunciated this assumption in two ways that he considered equivalent: in the first, he emphasised the conservation of impressed motion over time in the absence of other causes; in the second he clarified that this means that when a body moved in a vacuum it kept moving 'semper & æquali celeritate'.⁴⁹ Interestingly, Descartes characterised the conservation of motion as remaining *in* the body, not as a state. Also, because this formulation of the conservation of motion assumes that motion is preserved *in* bodies, the only possible way to depict the conservation 'in it for all times' is by imagining a body moving *in vacuum*.

From the times of *Physico-mathematica*, both Beeckman and Descartes had been using some form of the conservation of motion, with different emphasis, in connection with the explanation of the fall of bodies.⁵⁰ The point in the letter of November 1629 is that the explanation of the fall of weights based on the conservation of motion was to be included in his *Physics*—without mentioning that

⁴⁶ Cf. Palmerino, 'Infinite Degrees of Speed: Marin Mersenne and the Debate over Galileo's Law of Free Fall'.

⁴⁷ Descartes, *AT*, 1897, I:71–72; Descartes, *CSM*, 1984, III:8.

⁴⁸ Koyré, *Études galiléennes*, 122–27; Schuster, *Descartes-Agonistes*, 144–53.

⁴⁹ Descartes, *AT*, 1897, I:72.

⁵⁰ Cf. Roux, 'Découvrir Le Principe d'inertie'.

Beeckman provided the connection between fall of bodies and conservation in 1618, of course.⁵¹ In answering to Mersenne, Descartes alluded again to the delay in delivering the treatise because its scope has been enlarged. In fact, it should strike as evident to Mersenne—as it may do to us—that a ‘small treatise’ that aimed to account for an *optical* phenomenon now included the mathematical calculation of the fall of *graves* relying on natural-philosophical assumptions about the causes of motion.

The next letter to Mersenne on 18 December 1629, suggests the beginning of an important variation in the scope of Descartes’ project. For the first time Descartes alluded to the foundations of physics. Descartes thanks (again) Mersenne for offering ‘to take care’ of the ‘little treatise’. However, Descartes admitted that he had decided not to put his name on it and not to publish it until receiving the ‘painstaking scrutiny’ of Mersenne. This ‘on account of theology, which has been so deeply in the thrall of Aristotle that it is almost impossible to expound another philosophy without its seeming to be directly contrary to the Faith’. Descartes assumed that a new natural philosophy may pose a threat to religion. He asked to the Father Mersenne ‘whether there is anything definite in religion concerning the extension of created things, that is, whether it is finite or infinite and whether in all these regions called imaginary spaces there are genuine created bodies’.⁵² The problem Descartes seemed to be facing here was the medieval question of the plurality of worlds.⁵³ The point here is not Descartes’ direct interest in metaphysics and theology but rather the derivative necessity he expressed in them given his way to formulate a new physics; Descartes was worried about the possibility of modifying some philosophical ideas that may be contrary to religion. A clue about the roots of this concern appears when Descartes was talking (again) about free fall and particularly by refuting Beeckman’s conception. Descartes respectfully corrected Mersenne’s interpretation of the increase of *gravitas* per time unit in free fall and, in so doing, mentions again the conservation of motion. There is no significant change in the content of it. But this time, Descartes dealt with it in a different way; rather than presenting it as an

⁵¹ Cf. Descartes, *AT*, 1908, X:219.

⁵² Descartes, *AT*, 1897, I:85–86; Descartes, *CSM*, 1984, III:14.

⁵³ Descartes, *The World*, 21n41.

assumption, he remarked that he will *demonstrate* it: ‘We need to bear in mind that we are assuming that what is once in motion will, in a vacuum, always remain in motion. I shall try to demonstrate this in my treatise’.⁵⁴ The change seems subtle, but it is further clarified in connection with other elements of the letter. The enunciation of the principle, with the comment that ‘*in meo tractatu demonstrare conabor*’, is a note that Descartes added on the margin of the letter when he was emphatically differentiating his understanding and use of this principle from Beeckman’s. Descartes strategy was a *reductio ad absurdum* of Beeckman’s conclusions concerning the speed of motion as he had reported them to Mersenne, particularly that once a falling weight reaches a certain point it always continues to fall at a constant speed, for the faster the body falls the more air it finds opposing to this downwards motion and, in consequence, the air creates a balance. Descartes remarked that, Beeckman, supposes that ‘what once begun to move continues to move of its own accord’. The conclusions Beeckman extracted from this principle were highly plausible ‘and those who are ignorant of arithmetic might be convinced by it; but one needs only to be able to count to see that it is unsound’. Next, Descartes offered his geometrical demonstration and ironically concluded ‘*Ac proinde Mathematicè demonstratur illud quod Becmmannus scripserat esse falsum*’ (Thus it is demonstrated mathematically that what Beeckman wrote is false).⁵⁵

In this letter of December 1629, Descartes was not only confident in his understanding of the mathematical consequences of the conservation of motion, but he also remarked that he will offer a demonstration of it in his treatise. From October to December as the idea of an *a priori* physics began to take shape, Descartes has moved from dealing with the conservation principle as an assumption to a demonstrated principle.

The first letter of 1630 contains three important indications about the development of Descartes’ project. First, Descartes began to consider an explanation of the human body as part of his physics. At the opening of the letter, Descartes asked Mersenne to

⁵⁴ Descartes, *AT*, 1897, I:90; Descartes, *CSM*, 1984, III:15.

⁵⁵ Descartes, *AT*, 1897, I:94; Descartes, *CSM*, 1984, III:16–17.

take care of himself when reporting that ‘M. Montais is ill’, until he found if it is possible ‘to discover a system of medicine which is founded on infallible demonstrations, which is what I am investigating at present’.⁵⁶ Although the reference is succinct, it is important to notice that Descartes’ approach is now presented in terms of finding a medicine founded on infallible demonstrations. In the *Discours* (1637), Descartes presented the subjects of the unpublished treatise that we now identify with *Le Monde* as including something ‘about man, because he observes these [celestial] bodies’.⁵⁷ The consideration of human body offered particular difficulties, according to the *Discours*, because he ‘did not have yet sufficient knowledge to speak of them in the same manner as I did of the other things—that is, by demonstrating effects from causes’.⁵⁸ However, by January 1630 Descartes was to integrate medicine into his larger, renewed natural philosophy. Second, for the first time in the correspondence with Mersenne, Descartes showed an interest in ‘qualities’. Descartes thanked Mersenne for sending him ‘the qualities that you have extracted from Aristotle’ and added that he already had a longer list ‘derived from Verulam’ and remarked that ‘this is one of the first things that I will try to explain. This should not be incredibly difficult, because once the foundations have been laid, they will follow from them’.⁵⁹ While this mention of qualities occurred in the middle of a list of short topics in which Descartes is replying to Mersenne’s requests, its central importance for the project is evident when Descartes claimed that this is one of the first things he will try to explain. In fact, the core of *Le Monde* will be an innovative explanation of qualities in terms of matter in motion, as an attempt to invalidate the naturalist philosophies of the Renaissance and, of course, the Scholastic explanations based in forms and qualities.⁶⁰ But the passage also suggested that qualities are not assumed as principles explaining phenomena. Descartes may need to explain why, however, qualities are so important in other philosophies. In consequence, he was interested in finding an exhaustive list of them,

⁵⁶ Descartes, *AT*, 1897, I:105–6; Descartes, *CSM*, 1984, III:17.

⁵⁷ Descartes, *AT*, 1902, VI:42; Descartes, *CSM*, 1985, I:132.

⁵⁸ Descartes, *AT*, 1902, VI:44; Descartes, *CSM*, 1985, I:134.

⁵⁹ Descartes, *AT*, 1902, VI:109.

⁶⁰ Hattab, *Descartes on Forms and Mechanisms*; DesChene, ‘Forms of Art in Jesuit Aristotelianism (with a Coda on Descartes)’; Joy, ‘Scientific Explanation from Formal Causes to Laws of Nature’; Ariew, *Descartes and the Last Scholastics*.

consulting on Aristotle, Verulam and his own memory. (Interestingly, the section of *Le Monde* explaining the three elements and their qualities opens alluding to what ‘The Philosophers’ maintain).⁶¹ Third, Descartes asked Mersenne if he had made experiments to see ‘whether a stone thrown with a sling, or a ball shot from a musket, or a bolt from a crossbow, travels faster and has greater force in the middle of its flight than it has at the start, and whether its power increases’. Descartes expounded that the common opinion was that this is the case, although his own view differed: ‘I find that things that do not move of their own accord but are impelled must have more force at the start than they have straight after’.⁶² In fact, from Aristotle to Aquinas, the common belief about the motion of projectiles claimed that their violent motion implied a greater speed in the middle of the flight than at the beginning or end.⁶³ Similarly Cardano, in *De Subtilitate*, had subscribed to the same opinion, explaining that this was caused by the disturbance of the air put in motion by the projectile.⁶⁴ It is important to notice that Descartes was asking about experiments performed by Mersenne, that is, he was enquiring for empirical evidence that may be different from the common conclusions on the matter.

The letter on 25 February 1630 suggests that Descartes was exploring the consequences of the conservation of motion in explaining diverse phenomena, such as the rebounding of balls that he had previously discussed with Mersenne. In the letter of January, Descartes had conceded that the rebounding occurred partially because the air inside the ball pushed the ball upwards in rebounding ‘like a spring’.⁶⁵ Descartes explained that the main cause of this was ‘the continuation of the motion which is present in all rebounding bodies’, that is, in his words: ‘From the fact that a thing has begun to move it follows that it continues to move for as long as it can: and if it cannot continue to move in a straight line, rather than coming to rest, it rebounds in the opposite direction’. This explanation was not restricted to balls rebounding, but also to ‘the matter of all other bodies, both of which rebound and

⁶¹ Descartes, *AT*, 1908, X:23; Descartes, *The World*, 16–17.

⁶² Descartes, *AT*, 1897, I:113–14; Descartes, *CSM*, 1984, III:18.

⁶³ Cf. Orozco-Echeverri, ‘Force in Renaissance Science’.

⁶⁴ Cardano, *The De subtilitate of Girolamo Cardano*, vol. I, pp. 139–40.

⁶⁵ Descartes, *AT*, 1897, I:105; Descartes, *CSM*, 1984, III:17.

those against which other rebound, such as the strings of a tennis racket, the wall of a tennis court, the hardness of the ball'.⁶⁶ The analogy with the bouncing tennis balls that Descartes had used in explaining the nature of light in refraction is now extended to explain the interaction of any part of matter.⁶⁷ the conservation of motion as presented here, states that the body will move 'as long as it can', that is, Descartes has deprived the formulation from concepts such as force and vacuum. Also, he included the straight direction at a fundamental level. However, this variant is not formulated as an independent principle, but it seems to be implied in the idea of the conservation of motion.

In the letter of 4 March 1630, Descartes made a direct reference to Kepler, in the context of his meteorological enquiries. After mentioning that it snowed 'a bit here, at the same time that you mentioned', Descartes complained that the winter was 'so warm in this country that we have not seen any ice or snow and I had already considered writing you to complain that I had not known how to make any remark about it in my *Meteors*'. After the anecdote, he asked 'if Mr. Gassendi has any other comments concerning snow, apart from those that I had seen in Kepler, and I had already remarked in winter, *De nive sexangula et grandine acuminata*, I would be very glad to know them'.⁶⁸ At first glance, Descartes' mention may be just part of his attempt to collect evidence in order to support his developments or to avoid controversial claims. However, *De nive* provided Descartes' with a concrete geometrical understanding of the distribution of particles in space that can be appreciated in Chapter 13 of *Le Monde*, when Descartes explains the corpuscular nature of light integrating it with his cosmology.

Kepler's *De nive* (1611) starts off with the question 'why snowflakes in their first falling, before they are entangled in larger plumes, always fall with six corners and with six rods, tufted like feathers?'. Kepler contended that there has to be a necessary cause, 'for if it happens by chance, why do they not fall just as well with five corners

⁶⁶ Descartes, *AT*, 1897, I:117; Descartes, *CSM*, 1984, III:18.

⁶⁷ Schuster, 'Descartes Opticien: The Construction of the Law of Refraction and the Manufacture of Its Physical Rationales, 1618–29'; Schuster, 'Physico-Mathematics and the Search for Causes in Descartes' Optics—1619–1637'.

⁶⁸ Descartes, *AT*, 1897, I:127.

or seven?’⁶⁹ In a move typical of his archetypal approach, explored similar cases in nature and found that a similar condition can be appreciated in the honeycombs, because they are built ‘on a six-cornered plan’. When considered in their three dimensions, these cells formed regular rhomboids. Kepler explored the geometrical properties of these solid rhomboid to conclude that ‘this is the geometric figure, as near as possible to a regular solid, which fills space, just as the hexagon, square, and triangle are the fillers of a plane surface’.⁷⁰

This aptness—that the ‘sixcorneredness’ is the most capable distribution of particles for filling space—, is also seen in the pomegranate: ‘if one opens up a rather large-sized pomegranate, one will see most of its loculi squeezed into the same shape’.⁷¹ Kepler explained how the round loculi are squeezed and concluded that ‘when collected in any vessel [equal pellets] come to a mutual arrangement in two modes, according to the two modes of arranging them in a plane’. In order to pack ‘solid

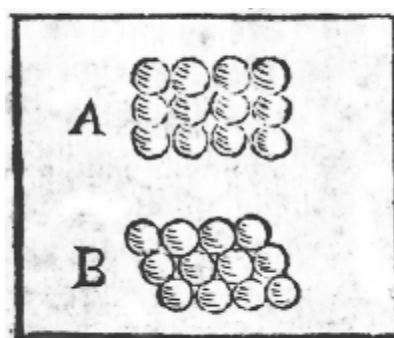


Figure 5. Kepler's arrangements of solid bodies to fill space, either squared (A) or in triangles (B). From *De Nive Sexangula* (1611).

bodies as tightly as possible’, the pellets will be either squared (A) or in triangles (B). In the case where pellets are squared, Kepler explained that any pellet is touched by four neighbours in the same plane and by one above and one below. However, if the pellets are packed in triangles, every pellet is touched by its four neighbours in the same plane but also by four in the plane above and by four below, and so throughout

⁶⁹ Kepler, *The Six-Cornered Snowflake*, 7.

⁷⁰ Kepler, 11.

⁷¹ Kepler, 13.

one will be touched by twelve and ‘under pressure spherical pellets will become rhomboid’.⁷²

The fact that Descartes read *De nive* and commented on it with Mersenne shows that his knowledge of Kepler was not restricted to optics and that in the crucial period of formulation of his new physics, he was reading Kepler. In chapter 13 of *Le Monde*, Descartes articulated his theory of light at a cosmological level. Descartes’ intention was to explain the generation of light in his universe full of corpuscles of matter. In the specific case of the solar system, Descartes focuses on describing the tendencies of the second element according to the laws of nature by providing a micro-corpuscular analysis. Schuster has suggested that most of the mathematical elements of the analysis ultimately derive from hydrostatical investigations that Descartes conducted in 1618-9.⁷³ Descartes represented the motion of fluids—water—in a way inverted to how light was presented in *Le Monde*. However, while early fluids were represented as continuous (Figure 6), the second element was represented as particles (*boules*) (Figure 7). Indeed, Descartes depicted in *Le Monde* the second element in the triangular distribution in which Kepler had argued that the *loculi* should be

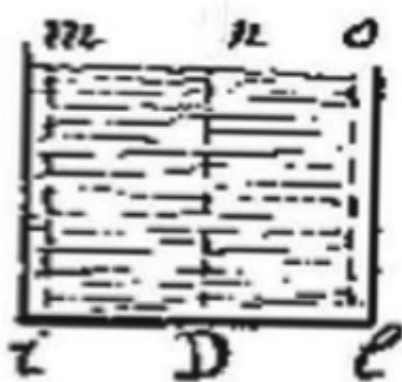


Figure 6. Representation of a fluid with a tendency to fall in a hydrostatical manuscript 1618. Schuster, 2013: 510.

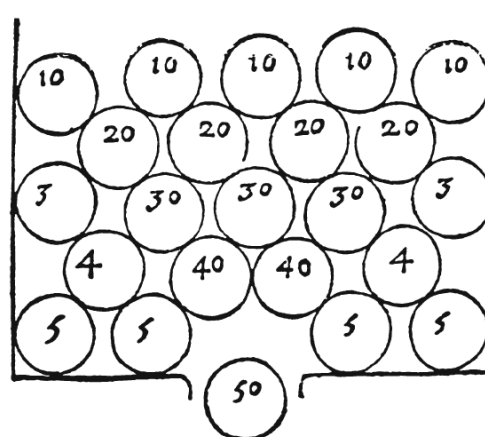


Figure 7. Second element filling the whole space. Descartes, AT XI: 93

placed to leave the minimum space between them. In explaining how light from the Sun was seen on a specific point, Descartes needed to explain how the particles follow their tendencies without interference. In order to prove his claim, he first

⁷² Kepler, 14–15.

⁷³ Schuster, *Descartes-Agonistes*, 501–7.

supposes that the particles of the second element move in an empty space and then he adds that this space is, in fact, ‘filled with the matter of the first element’. However, they do not interfere with the motion of the particles of the second element because ‘they are as ready to leave the places they occupy as other bodies are to enter them’.⁷⁴ The figure 7 is part of explanation of how the parts of the second element that must advance towards a point—the human eye—will stop after having filled the whole space, full with particles of the first element. Because these *boules* are ‘arranged in a space which is narrower than that which they leave’, they completely filled the space in the triangular distribution mathematically demonstrated by Kepler.

So far, the correspondence with Mersenne has shown that Descartes extended the initial project of explaining a particular phenomenon into a full-scale reform of natural philosophy requiring a new foundation. Reading the letter of 18 December 1629, we find that Descartes had already anticipated that an *a priori* physics accounting for all natural phenomena would be in conflict with well-established principles derived from Aristotle and, in consequence, may lead to controversies in theology and religion. The letter of 15 April 1630 shows Descartes’ introduction of laws of nature within the framework of his project and the subsequent creation of a new metaphysics based on reason, justifying his new *a priori* enterprise. For these reasons, this letter can be considered as the final stage of Descartes’ formulation of the project of a new physics, displaying the full scope of this project. However, the awareness of the extent of the project indicates how far Descartes was from completing this project, for as he recognised three years later, when he abandoned it, that it still required further elaboration.⁷⁵ But the point here is that in the path leading from the first interest in parhelia to the emergence of laws of nature, Descartes sketched a reform of natural philosophy in tension with accepted views of Aristotelian philosophy. His solution was the recourse to a new metaphysics, that is, the assertion of metaphysics as validating the foundations of knowledge.⁷⁶ This new

⁷⁴ Descartes, *AT*, 1909, XI:96; Descartes, *The World*, 61.

⁷⁵ Descartes, *AT*, 1897, I:270; Descartes, *CSM*, 1984, III:40–41.

⁷⁶ On Descartes’ metaphysics as a transformation of Scholastic and Aristotelian metaphysics see DesChene, ‘Forms of Art in Jesuit Aristotelianism (with a Coda on Descartes)’; DesChene, *Physiologia*; Ariew, *Descartes and the Last Scholastics*; Henry, ‘Metaphysics and the Origins of Modern Science’; Garber, *Descartes’ Metaphysical Physics*; Hatfield, ‘Force (God) in Descartes’

natural philosophy based on metaphysics ultimately removed powers and qualities from matter and accounted for the activity of matter in a way that was subject to mathematical analysis. After April 1630 Descartes would not extend further his project but would dedicate himself to the articulation of it by developing different branches, from the human being to the stars.

Three aspects indicate the edges of Descartes' project in the letter of 15 April 1630. The first is a manifestation of the awareness of its full scope when updating Mersenne about it. From the previous letters it is possible to infer that Mersenne was urging Descartes to send the tract and in return Descartes solved some problems, usually observing that the ultimate solution would be contained in his physics. These comments gave the impression that the treatise was going to be materialised soon. However, in this letter Descartes sets himself a deadline of three years to deliver the full text. Rather than adding new phenomena explaining this new date, Descartes confessed to Mersenne, that when he moved to Holland he was forced to abandon the tracts on which he was working in Paris for he acquired 'unexpected riches'. This alludes to the attempt to solve specific problems that finally directed him towards *Le Monde*: 'It is as if a man began building a house and then acquired unexpected riches and so changed his status that the building he had begun was now too small for him'. These unexpected riches were found in his 1628 encounter with Beeckman. Descartes acquired the *idea* of his new natural philosophy, a general framework on which he set *a priori* the foundations of a new explanation of the world. The limit of the project is visible here because at this point, Descartes had managed to transform the mathematical ideas into principles of demonstration which, in his view, will remain as such: 'I am sure that I will not change my mind again; because what I now possess will stand me in good stead no matter what else I may learn; and even if I learn nothing more, I shall still carry out my plan'.⁷⁷

The second and central aspect of this letter is the allusion to a new way of validating the 'laws of nature' as mathematical truths explaining the world. Descartes

Physics'; Hattab, *Descartes on Forms and Mechanisms*; Gilson, *Études sur le rôle de la pensée médiévale dans la formation de système cartésien*; Gilson, *The Unity of Philosophical Experience*.

⁷⁷ Descartes, *AT*, 1897, I:137–38; Descartes, *CSM*, 1984, III:20–21.

introduced the topic when making some remarks on a book—not identified—dealing with theology that Mersenne had sent. Descartes, who did not do anything for nothing, claimed that although the question was theological and, in consequence, ‘beyond my mental capacity’, it does not seem outside his province because it does not concern revelation ‘which is what I call theology in the strict sense’. Descartes then commented that he had been concerned with metaphysics for long time and that during the first months in Holland he was planning to write a treatise on it. In fact, he claimed that he was still interested in developing it but ‘I do not think it opportune to do so before I have seen how my treatise on physics is received’. In the following years, Descartes will consolidate the idea that metaphysics is only interesting to the extent that it is connected with natural philosophy; beyond this, metaphysics was unnecessary and even adverse, for ‘your mind will be drawn too far away from the physical and observable things, and become unfit to study them’.⁷⁸ Back to the letter, Descartes moved from the comments on the book to introduce his own ideas, as he planned to present them in his ‘treatise on physics’. Descartes was interested in checking with Mersenne and others possible objections and controversies concerning his views,⁷⁹ but the intention seems to be circulating a new way ‘to speaking of God’ which he now considered necessary to back up his natural philosophy. The concept of laws of nature makes its first appearance precisely when Descartes is articulating physics, mathematics and metaphysics:

In my treatise on physics I shall discuss a number of metaphysical topics and especially the following. The mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures. Indeed, to say that these truths are independent of God is to talk of him as if he were Jupiter or Saturn and to subject him to the Styx and the Fates. Please do not hesitate to assert and proclaim everywhere that it is God who has laid down these laws in nature [*lois en la nature*] just as a king lays down laws in his kingdom.⁸⁰

⁷⁸ This is said in the *Entretien avec Burman* Descartes, *AT*, 1903, V:165.

⁷⁹ As in other letters, Descartes indicates that he is expecting ‘to put this [that mathematical truths are laws of nature] in writing, within the next fortnight, in my treatise on physics; but I do not want you to keep it secret. On the contrary, I beg you to tell people as often as the occasion demands, provided you do not mention my name. I should be glad to know the objections which can be made against this view’. Descartes, *AT*, 1897, I:146; Descartes, *CSM*, 1984, III:23.

⁸⁰ Descartes, *AT*, 1897, I:145; Descartes, *CSM*, 1984, III:21–22.

For a seventeenth-century reader, it was not immediately evident why a treatise on physics shall include mathematical truths that is, why mathematical principles may have place in the explanation of natural phenomena. Of course, the Galilean way had associated mathematics with natural philosophy, but Descartes was aware that Mersenne also worked under this banner and had opted for the probabilistic approach of an operational natural philosophy based on appearances.⁸¹ Descartes himself had been solving problems within the Galilean approach, although the development of his own natural philosophy based on laws as causes would make it impossible for him to remain working in this way.⁸² Therefore, it was not that evident why he should deal with metaphysical issues in a treatise on physics. This awkwardness will become even more evident in Descartes' replies to Mersenne in May 1630. Descartes strategy consisted in providing a metaphysical validation of these mathematical truths (a 'demonstration') to guarantee their applicability in natural philosophy, as he had cautiously suggested in the letter of 18 December 1630. In a succinct but clear way, Descartes explained that he will deal with the metaphysical issue of the mathematical truths, that is, to conceive them as creatures of God or 'laws in nature'. The metaphysical claim here is that the mathematical truths that operate as principles are introduced directly by God in nature and, in consequence they 'depend on him entirely no less than the rest of his creatures'. In the equivalent passage of *Le Monde*, Descartes utterly supports his idea of mathematical laws as creatures in nature with the famous reference to the Bible saying that 'God ... disposed all things in number, weight and measure'.⁸³

Since the early days of the scholarship on laws of nature, the idea that the concept ultimately derived from the Judeo-Christian tradition has made a long career.⁸⁴ However, it is interesting to appreciate at this level of detail in the correspondence,

⁸¹ Dear, *Mersenne and the Learning of the Schools*, 42ff.

⁸² Garber, 'A Different Descartes: Descartes and the Programme for a Mathematical Physics in His Correspondence'.

⁸³ Descartes, *AT*, 1909, XI:47–48; Descartes, *The World*, 31.

⁸⁴ Zilsel, 'The Genesis of the Concept of Physical Law'; Needham, 'Human Laws and Laws of Nature in China and the West (I)'; Needham, 'Human Laws and Laws of Nature in China and the West (II): Chinese Civilization and the Laws of Nature'; Milton, 'The Origin and Development of the Concept of the "Laws of Nature"'; Milton, 'Laws of Nature'; Henry, 'Metaphysics and the Origins of Modern Science'; Lehoux, 'Laws of Nature and Natural Laws'.

that Descartes was not deriving the concept of laws of nature from the Christian tradition but rather relying on some theological and metaphysical claims to justify his use of an idea of law coming from mathematics as a principle of his physics. The metaphysical underpinning of mathematical truths transforming them in laws is, in fact, a solution to the problem of the legitimation and applicability of the mathematical (mechanical) principles to the domain of ‘tous les phænomenes de la nature’. The main difference between Descartes’ laws of nature and the previous occurrences of the term concerns their specificity.⁸⁵ Indeed, Descartes did not formulate laws as a metaphor of order; the metaphor he mentions in the letter of April 1630 is between God and a king imposing laws on nature and on his kingdom respectively, but Descartes is clear enough in pointing out that laws are creatures *in* nature. The metaphysical validation of mathematical truths as laws of nature asserts their existence in nature. Because of this, Descartes emphasised their direct dependency on God, although the point is not entirely clear so Descartes has to go through it once and again in May clarifying ‘*in quo genere causæ Deus disposuit æternas veritates*’ (by what kind of causality God established the eternal truths).⁸⁶

The metaphysical foundation of mathematical truths was not restricted to their existence in nature but Descartes also extended it to account for their cognoscibility. This was a consequence of the fact that this new metaphysics validates principles of knowledge. In a way that clearly resembles Kepler’s account of the human knowledge of the archetypes, Descartes claimed that ‘there is no single one [mathematical truth] that we cannot grasp if our mind turns to consider it. They are all *mentibus nostris ingenitæ* (inborn in our minds) just as a king would imprint his laws on the hearts of all his subjects if he had enough power to do so’. The full metaphysical justification for the inborn ideas will constitute the core of the third of the *Meditationes*. However, it is important to notice that since the introduction of laws in natural philosophy, Descartes was aware that the metaphysics validating this move could not be restricted to the relationship of God with nature but it also required to account for the way in which human beings may have access to the

⁸⁵ Henry, ‘Metaphysics and the Origins of Modern Science’; Henry, ‘The Theological Origins of Laws of Nature and Its Subsequent Secularisation’.

⁸⁶ Descartes, *AT*, 1897, I:151; Descartes, *CSM*, 1984, III:25.

knowledge of that relationship. In fact, the very possibility of a natural philosophy *a priori* depends on this cognoscibility, as it is stated in *Le Monde*:

The knowledge of these truths is so natural to our souls that we cannot but judge them infallible when we conceive them distinctly, nor doubt that if God had created many worlds, they would be as true in each of them as in this one. Thus those who know how to examine the consequences of these truths and of our rules sufficiently will be able to recognise effects by their causes. To express myself in scholastic terms, they will be able to have *a priori* demonstrations of everything that can be produced in this new world.⁸⁷

The previous quotation displays both directions of Cartesian metaphysics emerging from the introduction of laws of nature: the necessity of laws because of their dependence on God and their *a priori* cognoscibility in virtue of their innate character. The connection between these two directions is clarified in the Fifth part of the *Discourse*, when Descartes said that he had ‘noticed certain laws which God has so established in nature, and of which he has implanted such notions in our minds, that after adequate reflexion we cannot doubt that they are exactly observed in everything which exists or occurs in the world’.⁸⁸ In a way similar to Kepler’s the connections among God, nature and men, explained in metaphysics, constitutes the foundation of the order in the world and, in consequence, the possibility to reach a certain knowledge of such order. Descartes was aware that his metaphysics required further developments and, in consequence, anticipated some objections.

The details of Descartes’ doctrine of the creation of eternal truths have attracted the attention of notable scholars for decades.⁸⁹ However, my interest here is to point to the fact that when introducing laws of nature in his treatise of physics Descartes was aware that he was transgressing the boundaries of natural philosophy and metaphysics by dealing with God in this way. As I have mentioned before, Descartes asked Mersenne to circulate the idea that God established laws in nature. Immediately after this request, Descartes claims: ‘I hope to put this in writing, within

⁸⁷ Descartes, *AT*, 1909, XI:47–48; Descartes, *The World*, 31–32.

⁸⁸ Descartes, *AT*, 1902, VI:41; Descartes, *CSM*, 1985, I:131.

⁸⁹ Osler, ‘Eternal Truths and the Laws of Nature: The Theological Foundations of Descartes’ Philosophy of Nature’; Osler, *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in the Created World*; Henry, ‘Metaphysics and the Origins of Modern Science’; Garber, *Descartes’ Metaphysical Physics*; Hatfield, ‘Force (God) in Descartes’ Physics’.

the next fortnight, in my treatise on physics ... I want people to get used to speaking of God in a manner worthier, I think, than the common and almost universal way of imagining him as a finite being'.⁹⁰

A third important aspect of the 15 April 1630 letter is Descartes' approach to corpuscles which appears connected with the rejection of vacuum. In previous letters, Descartes had formulated the conservation of motion in terms of bodies moving in a vacuum. The metaphysical impossibility of vacuum, associated with the identification of bodies, space and extension, became a central assumption of the Cartesian world, connected as well with the circularity of motion and the homogeneity of matter. In a world full of matter, when a body leaves its place, it always enters into that of another and thus 'all motions that occur in the world are in some way circular'.⁹¹ This development occurred when Descartes was replying to a question posed by Mersenne on rarefaction and condensation:

The corpuscles (*ces petits cors*), which enter a thing during rarefaction and exit during condensation, and which can penetrate the hardest solids, are of the same substance as visible and tangible bodies; but you must not imagine that they are atoms, or that they are at all hard. Think of them as an extremely fluid and subtle substance filling the pores of other bodies. You must admit that even in gold and diamonds there are certain pores, however tiny they may be; and if you agree also that there is no such thing as a vacuum, as I think I can prove, you are forced to admit that these pores are now full of some matter which can penetrate everywhere with ease. Now heat and rarefaction are simply an admixture of this matter.

Rarefaction and condensation were traditionally explained in terms of interstitial voids within the particles of a body. In a previous letter to Mersenne of 25 February 1630 Descartes had explained rarefaction and condensation by comparing bodies with pores with a sponge full of water, claiming that 'it is certain that when something is condensed it loses some of its parts, and retains the bulkier parts, just as a sponge which is full of water loses water when you press it'.⁹² In putting forward his improved solution in the letter of April 1630, Descartes dropped some hints

⁹⁰ Descartes, *AT*, 1897, I:146; Descartes, *CSM*, 1984, III:23.

⁹¹ Descartes, *AT*, 1909, XI:19; Descartes, *The World*, 14.

⁹² Descartes, *AT*, 1897, I:119; Descartes, *The World*, 19.

indicating the rejection of vacuum. In contrast with the explanations based on void and the influential ether of Sebastian Basso,⁹³ Descartes postulated the homogeneity of matter, that is, both the solid and tangible bodies and the particles going through their corpuscles ‘are of the same substance’. Descartes had identified matter with extension since the *Regulæ*, but the consequences of this claim only become evident here when the introduction of laws allowed him to dismiss the concept of vacuum and, in consequence, he reduced motion to impact. In so doing, Descartes distanced himself from ancient atomism. In the succinct explanation of rarefaction, Descartes claimed that the *petits cors* entering the bodies through rarefaction and leaving in condensation accounted for the varying densities of bodies. Nevertheless, these corpuscles should not be considered as atoms but as extremely ‘fluid and subtle’. It is not accidental that these characterisations of corpuscles as ‘not at all hard’ interacting with subtler corpuscles filling the pores of the body appeared just a month after having mentioned Kepler’s *De Nive*, in which the same concepts accounted for the shape of snowflakes.

4.4. Men experienced with machinery

The examination of Descartes’ correspondence with Mersenne between 1628 and 1630 has revealed the development of basic tenets of Cartesian natural philosophy and metaphysics, triggered by the transformation of some principles and values of his practice coming from mechanics. The elaboration of these into ‘laws of nature’ called for a metaphysical foundation and, as a consequence, all this implied the re-working of the central tenets on which Aristotelian physics was based. We have seen that, as the principles of motion became ‘laws of nature’, Descartes also substantiated the early claim that matter is extension as a philosophical affirmation concerning its particulate, homogenous nature and consequently rejected any void. The ‘laws of nature’ offered an alternative natural philosophy to that based on powers and qualities of matter; a natural philosophy founded *a priori* could explain better the phenomena of nature and, in consequence, the previous concepts accounting for the activity of nature and its properties should be dismissed. Although

⁹³ Palmerino, ‘Rarefaction and Condensation’; Wang, ‘Rarefaction and Condensation’.

Cartesian views were challenged since their very formulation, not only in the Continent but also across the Channel, their influence during the seventeenth century remains beyond doubt. ‘If Cartesianism was seductive to seventeenth-century thinkers—Henry explains—it was almost certainly because it seemed to offer the only fully worked out system that was capable of replacing the still prevailing scholastic philosophy lock, stock, and barrel’.⁹⁴ The purpose of this final section is to argue that when extending mechanical principles to explain natural phenomena, Descartes also projected some core tenets of mechanics, a mixed-mathematical science, on his new natural philosophy. In particular, the new natural philosophy based on laws of nature and corpuscles adapted to the natural world the mathematical structure of explanation that late sixteenth-century and early seventeenth-century mechanics had developed to account geometrically for the operation of machines. A not well-noticed consequence of this is that Descartes then conceived mechanics as a branch of natural philosophy, rather than as a mathematical discipline.

The new natural philosophy entailed a redefinition of the terms in which natural explanations had been traditionally couched. If the world was full of homogeneous matter and all phenomena arose by collision according to motions codified in the laws, any void became metaphysically impossible, powers and qualities unnecessary, space was but the three-dimensionality of body, and motion, once introduced by God, was transmitted by collision. This redefinition has two different but interdependent layers: on one level, Descartes tried to carve new metaphysical definitions out of the vocabulary of Scholastics, particularly in the *Principia Philosophiæ*, to fully articulate (and to make intelligible to his readers) his natural philosophy; sometimes this was done in order to clarify the innovative use of old terms, although this approach usually generated confusions and controversies—for example, Malebranche’s occasionalism relies on Descartes’ ambiguous use of *concurrentia* and Henry More’s criticism of Descartes’ conception of motion stems from the classification of motion as a *modus*. On the other hand, Descartes incorporated terms coming from mixed-mathematics into his new natural philosophy; but again, he constantly contrasted the proper ‘philosophical’ (his) meaning with the

⁹⁴ Henry, ‘The Reception of Cartesianism’, 118.

‘vulgar’ or ‘common sense’ (others’) use. Among these, I will focus on the idea of *dispositio* as new *explanandum* in the reformed natural philosophy.⁹⁵

In a letter to (again) Mersenne of 26 April 1643, Descartes summarised his achievements on occasion of solving a concrete problem. This letter is important for several reasons: at this time, Descartes had already published his *Meditationes*, so he had fully elaborated the metaphysical underpinnings of his physics. Also, the contrast between laws of nature and powers and qualities is brought up to solve concrete questions formulated by an unidentified correspondent through Mersenne, so from here it is possible to appreciate an application of laws of nature in solving mechanical problems. Finally, because his physics was not yet published, Descartes’ answer provided a summary of the core tenets of the *Principia Philosophiæ*.

An unknown correspondent wrote to Mersenne asking to refer to his friend Descartes three issues on which he disagrees with someone else (also unidentified). The first is whether two missiles equal in every aspect, that is matter, size and shape, starting off with the same speed, in the same direction and in the same medium must necessarily travel the same distance or whether their behaviour may depend on the time during which they have been in contact with the moving force (*force*). The second asks whether it is necessary that a body that impressed motion on another travels, after the impact, with the same speed that the one on which motion was impressed. The third one is whether there is any other quality—such as the one pulling the missile to the earth or like the heat produced in the iron by the fire—, apart from the air resistance, that could cause the motion impressed on a missile to perish.⁹⁶

Descartes felt compelled to provide an *abrégé* of his physics. He could not answer without making clear that some of the problems involved in these questions concerned the concepts used in their formulation. Before offering his answers, Descartes postulated two ‘principles of physics’. The first is that ‘I do not suppose there are in nature any *real qualities*, which are attached to substances, like so many little souls to their bodies, and which are separable from them by divine power.

⁹⁵ Cf. Psillos, ‘Laws and Powers in the Frame of Nature’, 89–90.

⁹⁶ Descartes, *AT*, 1899, III:654–55.

Motion, and all the other modifications of substance which are called *qualities*, have no greater reality, in my view, than is commonly attributed by philosophers to shape, which they call only *a mode* and not a *real quantity*'.⁹⁷ This negative formulation reveals interesting aspects of the redefinition of motion. Descartes claimed that he did not assume real qualities attached to substances separable from bodies by divine power, that is, he openly rejected the doctrine of substantial forms and his Scholastic reappraisal involving God's action.⁹⁸ Put otherwise, Descartes rejected the idea that bodies had powers and qualities as real attributes, that is, that exist and may be (at least in thought) considered as separated from bodies by the unlimited power of God. Interestingly, Descartes illustrated his criticism with the misleading conception of motion as a *quality*. Motion was not a real quality but something comparable to shape, that in Scholastic terminology was called a *modus*—a terminological choice that would be at the heart of the exchange with Henry More.⁹⁹ In order to support his first 'principle of physics', Descartes offered two arguments: the first is that he does not see that 'the human mind has any notion, or particular idea, to conceive them [qualities], so that when we talk about them and assert their existence, we are asserting something we do not conceive and do not ourselves understand'. This is the negative version of the argument claiming that the mathematical truths, as principles of physics, were imprinted in our souls.¹⁰⁰ The second argument asserted that Scholastics introduced the idea of qualities because they thought they could not explain all phenomena without them; however, Descartes claimed that he will show that phenomena are better explained without them.

The second principle is that 'Whatever is or exists remains always in the state in which it is, unless some external cause changes it; so that I do not think there can be any *quality* or *mode* which perishes of itself'. Descartes explained this state in comparison with shape, in the sense that 'if a body has a certain shape, it does not

⁹⁷ Descartes, III:648–49; Descartes, *CSM*, 1984, III:216.

⁹⁸ DesChene, *Physiologia*; Ariew, *Descartes and the Last Scholastics*; Hattab, *Descartes on Forms and Mechanisms*.

⁹⁹ Hatfield, 'Force (God) in Descartes' Physics'; Webster, 'Henry More and Descartes: Some New Sources'.

¹⁰⁰ On this vexed philosophical issue, the best analysis is still Gueroult, *Descartes' Philosophy Interpreted According to the Order of Reasons*, 1:27–75.

lose it unless it is taken from it by collision with some other body; similarly if it has some motion, it should continue to keep it, unless prevented by some external cause'. Instead of offering arguments for this principle, he just rounded off the question claiming that he proved 'this by metaphysics'.¹⁰¹ Here, Descartes drew his first law from the rejection of power and qualities. In the correspondence with Mersenne between 1628 and 1630, the formulation of what this law stated changed considerably over time and was stripped of vacuum just when it became a law of nature, together with the development of the corpuscles as opposed to solid atoms; that is, when the principle, initially formulated as a mathematical abstraction, was inserted into natural philosophy as accounting for the observable changes in the real world acquired a different dimension. It is worth noting that Descartes formulated the first law as a statement on the activity of nature; that is, that the laws of nature accounted for motion in a world where matter did not play any causal role. As we know since April 1630, this law was ultimately demonstrated metaphysically. For this necessary connection between laws and metaphysics, Descartes sketched some metaphysical ideas underpinning the laws of nature in the 1643 letter namely, that God is 'entirely perfect and unchangeable' so that it seem absurd that 'any simple thing' which existed contained in itself the principle of change, particularly 'the principle of its destruction'. Descartes highlighted with this argument that the first law is but a consequence of the principle of the conservation of the quantity of motion in the world and of the divine immutability.

In this letter, the definition of motion was a consequence of these two principles:¹⁰² 'Since motion is not a *real quality* but only a *mode*, it can be conceived only as the change by which a body leaves the vicinity of some others: and there are only two kinds of change to consider, the one, change in its speed, and the other, change in its direction'. The idea that motion is to body *just* as shape, modified all the conceptual relations that Aristotle had established between substance and its accidents and made trivial the innumerable amount of medieval accounts based on the idea of motion as

¹⁰¹ Descartes, *AT*, 1899, III:649; Descartes, *CSM*, 1984, III:216.

¹⁰² In the *Principia*, Descartes shall present laws of nature as a logical consequence of his definition of motion (II, 36). This order is not accidental, see Gueroult, *Descartes' Philosophy Interpreted According to the Order of Reasons*, 1:7–8.

alteration. Descartes had criticised the Aristotelian idea of motion since the *Regulæ*, rejecting the ‘misleading’ idea that almost any (quantitative and qualitative) alteration was a case of motion and should be explained in terms of the rich Aristotelian lexicon of act and power, substance and accident, generation and corruption. In *Le Monde*, Descartes formulated as paradoxical the Scholastic idea that motion could occur ‘without any body’s changing place, such as those they call *motus ad formam, motus ad calorem, motus ad quantitatem*’. In his view, ‘I know of no motion other than that which is easier to conceive of than the lines of geometers, by which bodies pass from one place to another and successively occupy all the spaces in between’.¹⁰³ In other words, the only *real* change was that of place or local motion, but this could not be understood as a change arising from the qualities of the body. The rejection of qualities as causes implied that motion in the ‘ordinary sense’, as ‘*the action by which a body travels from one place to another*’, was wrong. Because there were no qualities or powers moving the bodies but only bodies pushing each other, motion was ‘*the transfer of one piece of matter, or one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies*’. Descartes was aware of the controversial character of his definition and explained why the rejection of qualities entailed this new meaning: ‘And I say ‘the transfer’ (*translationem*) as opposed to the force or action which brings about the transfer, to show that motion is always in the moving body as opposed to the body which brings about the movement’.¹⁰⁴

If matter was deprived of powers and qualities, to the point that the foundation of Cartesian natural philosophy was this lack of any capacity of self-movement in bodies, what is the source of activity in nature? How can the variety of phenomena emerge? If the motion of particles is the ultimate way of explaining, what is the ultimate source of this motion? How is it distributed in matter to generate phenomena as diverse as the circulation of blood, the rainbow, the snow and the motion of planets? The answer is God and the laws of nature. At the metaphysical level, God’s immutability guaranteed the certainty and universal applicability of these laws in

¹⁰³ Descartes, *AT*, 1909, XI:39–40; Descartes, *The World*, 26.

¹⁰⁴ Descartes, *AT*, 1904, VII:54; Descartes, *CSM*, 1985, I:233.

natural philosophy. But from the point of view of natural philosophy, God was also the (primary) cause of all phenomena, for ‘we must say that God alone is the author of all the motions in the world in so far as they exist and in so far as they are straight’,¹⁰⁵ that is, at the most fundamental level of laws of nature. Descartes specified that God ‘started to move the matter’ with a determined quantity of motion which remained constant because of the laws of nature.¹⁰⁶ The interesting question, then, is not what is the efficient cause (God) but, instead, how ‘it’ operates. In *Principia*, Descartes explained that

After this consideration of the nature of motion, we must look at its cause. This is in fact twofold: first, there is the universal and primary cause—the general cause of all the motions in the world; the second there is the particular cause which produces in an individual piece of matter some motion which it previously lacked. Now as far as the general cause is concerned, it seems clear to me that this is no other than God himself. In the beginning, he created matter along with its motion and rest; and now, merely by his regular concurrence, he preserves the same amount of motion and rest in the material universe as he put there in the beginning.¹⁰⁷

This article sets out the metaphysical foundation of laws of nature and, as we have seen, of the Cartesian explanation of the activity in the world. Descartes claimed that the universal cause of motion is God, while the particular causes are the laws of nature. Instead of relying on powers and qualities in nature, Descartes put in their place the idea of a God creating extended matter and setting it in motion according to his immutable nature. In fact, all the diversity of phenomena arose when the particles began to move: ‘let us think of the differences that He creates within this matter as consisting wholly in the diversity of the motions He gives to its parts’.¹⁰⁸ From this immutability derives the character of necessity of the laws. If God is immutable, his actions in a sense do not change. The combination of God and laws of nature as cause of motion has been subject of intense debates since the seventeenth century, for the formulation is ambiguous enough to allow occasionalist and concurrentist interpretations.¹⁰⁹ From the point of view of the transformation of disciplines,

¹⁰⁵ Descartes, *AT*, 1909, XI:46–47; Descartes, *The World*, 30.

¹⁰⁶ Descartes, *AT*, 1899, III:49; Descartes, *The World*, 32.

¹⁰⁷ Descartes, *AT*, 1905, VIII:61; Descartes, *CSM*, 1985, I:36.

¹⁰⁸ Descartes, *AT*, 1909, XI:34; Descartes, *The World*, 23.

¹⁰⁹ Ott, *Causation and Laws of Nature in Early Modern Philosophy*; Pessin, ‘Descartes’s Nomic Concurrentism’; Clatterbaugh, ‘Cartesian Causality, Explanation, and Divine Concurrence’; Platt,

Descartes was reframing natural philosophy in a way that included claims whose validity depended on a new metaphysics, concerning a conception of God and his connection with the world. The connection is such that, from the divine immutability Descartes inferred that ‘it is most reasonable to think’ that the quantity of motion in the world remained the same and, in consequence, the study of natural philosophy was the clarification of the specific changes of motion in the particles of matter or bodies according to the laws derived from God’s attributes. These changes occurred according to the three laws of nature and the rules of motion that, seen from this perspective, are but laws and rules governing the redistribution of motion in particular interactions.¹¹⁰ The causal role of laws of nature implies that these interactions occur in a determinate way *because* of the laws of nature, rather than *according* to them.¹¹¹ In other words, laws of nature determine—as powers and qualities did before—the outcome of the interaction between any particles or bodies in the world, although they are not directly observable. Descartes’ point seems to be that laws ‘in nature’ are enough to account for all the diversity of phenomena without any resource to powers and qualities.

Descartes’ laws of nature were explicitly formulated as the starting point of a new way to understand nature. All phenomena should be explained in a way that ultimately referred to the ‘laws of nature’. But the fact that ‘laws of nature’ provided the principles of demonstration did not entail that all demonstrations shall be reduced to them. Put otherwise, the formulation of laws did not entail that the totality of phenomena may be logically derived from them; if laws constituted the basic principles of explanation, it was the task of the natural philosopher to build up accounts based on them, not to show that every phenomena could be reduced to laws, as we saw in the first two chapters. In a letter to Mersenne, Descartes explained that the discovery of the order underlying natural phenomena ‘is the key and foundation of the highest and most perfect science of material things which men can ever attain.

‘Divine Activity and Motive Power in Descartes’s Physics’; Hattab, *Descartes on Forms and Mechanisms*; Osler, *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in the Created World*.

¹¹⁰ Henry, ‘Metaphysics and the Origins of Modern Science’; Roux, ‘A Deflationist Solution to the Problem of Force in Descartes’; DesChene, *Physiologia*.

¹¹¹ Psillos, ‘Laws and Powers in the Frame of Nature’.

For if we possessed it, we could discover *a priori* all the different forms and essences of terrestrial bodies, whereas without it we have to content ourselves with guessing them *a posteriori*'.¹¹² If the 'laws of nature' provided the foundations for explaining the natural world, their formulations did not paint the whole picture. In *Le Monde*, Descartes claimed that 'we must say that God alone is the author of all the motions in the world in so far as they exist and in so far as they are straight, but that it is the various dispositions of matter that render the motions irregular and curved'.¹¹³ A parallel claim occurs in *Principia*, just after Descartes formulated the first law of nature: 'what is in motion always, so far as it can, continues to move'. Descartes immediately added: 'But we live on the Earth, whose composition is such that all motions occurring near it are soon halted, often by causes undetectable to our senses'. In other words, because on Earth—that is, in the range of our experience—we do not immediately observe the first law, it does not mean that it does not *underly* the phenomena that we perceive through experience. The validity of laws of nature guaranteed their applicability to phenomena, but how can we arrive at the explanation of specific, singular phenomena which are the ultimate interest of natural philosophy? Descartes explained that in this he was greatly 'helped by considering artefacts'. In his view, there was no difference between 'artefacts and natural bodies except that the operations of artefacts are for the most part performed by mechanisms which are large enough to be easily perceivable by the senses—as indeed must be the case if they are to be capable of being manufacture by human beings'. However, natural effects depend on structures 'so minute that they completely elude our sense'. But, given that 'mechanics is a division or special case of physics, and all the explanations belonging to the former also belong to the latter' it is not less natural 'for a clock constructed with this or that set of wheels to tell the time than it is for a tree which grew from this or that seed to produce the appropriate fruit'. Put otherwise, because mechanics was but a part of natural philosophy, their way of explanation were also aligned. Interestingly, as Beeckman did before him, Descartes appealed to the form of knowledge in mechanics to explain knowledge of the natural world:

¹¹² Descartes, *AT*, 1897, I:251; Descartes, *CSM*, 1984, III:38.

¹¹³ Descartes, *AT*, 1909, XI:46; Descartes, *The World*, 30.

Men who are experienced in dealing with machinery can take a particular machine whose function they know and, by looking at some of its parts, easily form a conjecture about the design of the parts, which they cannot see. In the same way I have attempted to consider the observable effects, and parts of natural bodies, and track down the imperceptible causes and particles which produces them.¹¹⁴

This passage is traditionally quoted to support Descartes' reduction of nature to mechanical operations and, in consequence, natural philosophy to mechanics. Sometimes it is considered the birth certificate of the mechanical philosophy. However, in the light of the evidence I have presented, this is rather simplistic. Descartes claimed that in order to understand the production of natural effects he has been helped 'by the consideration of artefacts'. It is easy to conclude from here that he was formulating the famous idea of the universe as machine.¹¹⁵ But from the disciplinary point of view, Descartes only claimed that he relied on the *way* in which mechanics uncovered the design of the parts of a machine. However, the case of uncovering the secrets of nature is *not* as the case of the mechanician *building* a machine, for this would put men in the place of God. In Descartes' analogy, an experienced mechanician is presented with a machine that he had not created. Because he 'was experienced' in how machines function, he could 'easily form a conjecture about the design of the parts, which they cannot see'. In the case of natural philosophy, the 'experience' of the mechanician is replaced by the *knowledge* of the laws of nature from which he can 'track down the imperceptible causes and particles which produces them', that is, to explain the effects by causes. Instead of reducing the world to a machine, Descartes claimed that we know the world in the same way that a mechanician knows a machine that he did not create: 'conjecturing' about the *design* of the parts. The difference is that our 'conjectures' are laws of nature derived from divine immutability. And this is exactly what the Cartesian natural philosopher does: 'understand how all the things in nature could have arisen', that is, conjecturing about their design (*disposition*), which accounts for the observable effects.

¹¹⁴ Descartes, *AT*, 1905, VIII:326; Descartes, *CSM*, 1985, I:288–89.

¹¹⁵ Hattab, 'From Mechanics to Mechanism'.

Nevertheless, because human knowledge is not divine knowledge, ‘it should not therefore be inferred that they [natural phenomena] were in fact made this way’. Instead of uncovering the blueprints of the creation, Descartes’ philosophy was directed towards ‘the application of ordinary life’, that is, its main interest was to find a way to understand how things could have been done. To illustrate the contrast between human and divine points of view, Descartes borrowed an example from mechanics, claiming that ‘just as the same mechanician could make two clocks which tell the time equally well and look completely alike from the outside but have completely different assemblies of wheels inside, so the supreme mechanician of the real world could have produced all that we see in several different ways’.¹¹⁶ In so doing, Descartes incorporated the ideal that knowledge is not *certain* concerning particular things by uncovering their genesis; on the contrary, knowledge is certain because we can establish, based on the metaphysical certainty of mathematical laws, how things *are* in general, that is, how they necessarily may arise from a specific design. In summary, the world can be understood as (a mechanician understands) a machine: by forming ‘a conjecture’ on its design. This idea, as we have seen in the first chapter, was elaborated in the late sixteenth-century as mechanics became a geometrical explanation of physical (artificial) objects.

The previous argument rests on the idea that mechanics was a branch of natural philosophy, so ‘all explanations belonging to the former also belong to the later’. All *mechanical* explanation was a *natural-philosophical* explanation because mechanics was a branch of natural philosophy. Garber interprets this claim in exactly the opposite way: ‘mechanics subsumes physics: everything in physics now receives a mechanical explanation, that is to say, everything is explained as if it were a machine’.¹¹⁷ But Descartes claimed exactly the opposite. In concluding the *Principia*, the work traditionally considered as the first modern alternative to Aristotelianism, Descartes noticed that:

In attempting to explain the general nature of material things, I have not employed any principle which was not accepted by Aristotle and all other philosophers of every age. So this philosophy is not new, but the oldest and most common of all. I

¹¹⁶ Descartes, *AT*, 1905, VIII:327; Descartes, *CSM*, 1985, I:289.

¹¹⁷ Garber, ‘Remarks on the Pre-History of the Mechanical Philosophy’, 17.

have considered the shapes, motions and sizes of bodies and examined the necessary results of their mutual interaction in accordance with the laws of mechanics, which are confirmed by reliable everyday experience.¹¹⁸

This claim seems to run at odds with Descartes' attempt to establish a *new* natural philosophy. But Descartes' point was that his principles, that is, the laws of mechanics, were accepted 'by Aristotle and all other philosophers of every age'. In this Descartes followed Mersenne who, at the same time, followed Baldi, in believing that all the mechanical laws of Archimedes, Guidobaldo and Baldi stemmed from the *Mechanical Problems*.¹¹⁹ The difference is that, while Aristotle and others conceived that these principles apply to machines only, Descartes claimed that in the way of explaining of mechanics was hidden the right way of accounting for natural phenomena. In this way, mechanics was founded on natural philosophy.

* * *

The connection between natural philosophy and mechanics appears in a letter from Descartes to Fromondus in 1637. Fromondus criticised Descartes because his philosophy was similar to mechanics, in the sense that his philosophy 'seems too "crass" for him, because, like mechanics, it considers shapes and sizes and motions'. Descartes answer was historical. In his view, mechanics has always been a true part of natural philosophy but, when the latter was corrupted, mechanics 'took refuge with the mathematicians'. 'This part of philosophy—Descartes added—has in fact remained truer and less corrupt than the others, because it has useful and practical consequences, and so any mistakes in it results in financial loss. So if he despises my style of philosophy because it is like mechanics, it is the same to me as if he despised it for being true'.¹²⁰ Descartes 'reestablished' the status of mechanics not by providing an axiomatic organisation of its principles—like Guidobaldo and friends did—but by re-placing it in the domain of natural philosophy. When the tree of physics ripens, some of its fruits are mechanical.

¹¹⁸ Descartes, *AT*, 1905, VIII:323; Descartes, *CSM*, 1985, I:286.

¹¹⁹ Mersenne, *Les questions theologiques, physiques, morales, et mathematiques*, 91; Cf. Dear, *Mersenne and the Learning of the Schools*, 119–29; Hattab, *Descartes on Forms and Mechanisms*, 92.

¹²⁰ Descartes, *AT*, 1897, I:421; Descartes, *CSM*, 1984, III:64.

5. ‘The outer shell’: Cartesian laws in England

The French philosophy will inform us, that the Earth as well as other bodies is indifferent in itself to rest, or its contrary.

Glanvill, *The Vanity of Dogmatizing* (1661).¹

5.1. The interregnum

It is often claimed that laws of nature, such as Descartes’, became the landmark of the ‘new science’ by replacing explanations based on causes. In the 1980s, Drake argued that the search for ‘causes of events in nature that guided the Aristotelian science’ was superseded by a ‘quest for laws of nature based on experiment and measurement’.² This ‘process by which causes gave way to laws in science may be considered as having begun’ in Galileo’s early hydrostatical works.³ More recently, Lynn Joy refined this approach, claiming that different Aristotelian traditions held rival forms of explanation and therefore the rejection of the ‘scientific innovators’ is more complicated than the refutation of a monolithic doctrine of causation. Joy claims that these innovators ‘widely reject Aristotle’s account of the four kinds of causes as a source of acceptable theories in the specific sciences’.⁴ However, Joy characterises this transformation as the establishment of ‘laws in terms of lawlike regularities, according to which the observable features of any ordinary body are explained as effects of the organization and motions of the body’s constituent atoms’, a view which she attributes to Robert Boyle.⁵ Recently, Harrison rounded off this view claiming that ‘for most natural philosophers laws replaced explanations in terms of Aristotelian causes. As Newton put it, there are ‘general Laws of Nature by which the Things are form’d; their truth appearing to us by Phenomena, though their Causes, be not yet discover’d’.⁶

¹ Glanvill, *The Vanity of Dogmatizing*, 85–86.

² Drake, *Cause, Experiment and Science*. Cf. Schmaltz, ‘From Causes to Laws’; Ducheyne, ‘Galileo’s Interventionist Notion of “Cause”’.

³ Drake, *Cause, Experiment and Science*, xxv; Cf. Ducheyne, ‘Galileo’s Interventionist Notion of “Cause”’, 449. Cf. Chapter 2.

⁴ Joy, ‘Scientific Explanation from Formal Causes to Laws of Nature’, 70. Cf. Sections 3.4 and 4.4.

⁵ Joy, 92.

⁶ Harrison, ‘Laws of Nature in Seventeenth-Century England. From Cambridge Platonism to Newtonianism’, 148. Cf. Section 7.3.3.

Approaching laws of nature as replacing causes faces at least two difficulties. First, a glimpse into historical evidence from Descartes to Newton—such as the one presented in this chapter—reveals that the growing talk about ‘laws of nature’ did not replace causes but stimulated new forms of laws (such as the laws of motion) adapted to views on causation resultant from experimental forms of enquiry; causes did not simply give way to ‘experiments and measurement’. Second, the idea of replacing Aristotelian-causes for Cartesian-mechanistic-laws reinforces the claim that ‘laws of nature’ appeared in a process from Descartes to Newton in which natural philosophers were concerned with ‘identify[ing] the lawlike regularities exhibited in the organisation and motions of these fundamental elements or atoms’.⁷ This approach takes for granted that once Descartes introduced ‘laws of nature’, this form of explanation was widely embraced except for some corrections to their contents; the new way of explaining in terms of ‘laws of nature’ became dominant just after revisions of their stipulations. This attitude has its roots in a common view on the ‘mechanical philosophy’ claiming that Descartes provided the first systematic alternative to Scholastic philosophy by introducing a new way to understand nature in terms of matter and motion. However, Descartes’ mechanistic world ended up being too restrictive, so Newton introduced gravitational force into the mechanical philosophy. The architecture of the *Principia* would allow him to claim that gravity really exists and is explained by the inverse-square law although its cause remained occult. However, Newton privately aimed ‘to reduce the action of an ‘attractive’ force of universal gravity to an effect caused by matter and motion, so that his system of the world would ultimately conform to the received philosophy of nature’.⁸ Claiming that laws replaced causes aligns Newton with this problematic view of the development of mechanical philosophy. In this version, the period between Descartes and Newton is usually depicted as one of dissemination of the Cartesian idea of laws, a mere interregnum.⁹

⁷ Joy, ‘Scientific Explanation from Formal Causes to Laws of Nature’.

⁸ Cohen, ‘The Principia, Universal Gravitation, and the “Newtonian Style”, in Relation to the Newtonian Revolution in Science:’, 56–57.

⁹ This approach appears in different degrees in Roux, ‘Les lois de la nature à l’âge classique la question terminologique’; Steinle, ‘From Principles to Regularities: Tracing “Laws of Nature” in Early Modern England and France’; Steinle, ‘Negotiating Experiment, Reason and Theology: The

However, a reading sensitive to the disciplinary settings—particularly regarding mechanics—leads to different conclusions.¹⁰ It is often assumed that mechanics became a rather demarcated discipline in the second half of the seventeenth century, as the study of motion in terms of Descartes' laws; mechanics would have provided the fundamental analogy uncovering the workings of nature by postulating the principles of motion and the homogenous nature of matter, a view allegedly epitomised by Boyle. In this traditional view of mechanics—embedded in the story of the 'mechanical philosophy' just mentioned—Huygens, Wren and Wallis partially corrected the visible flaws of Descartes' rules until the definitive version appeared in Newton's *Principia*, given Newton's elucidation of force, momentum and the distinction between mass and weight. However, the characterisations and practices of mechanics and their connections with mathematics and natural philosophy were not monolithic during this 'interregnum'. Because Descartes formulated his laws as a redefinition of the foundations of *physics*, altering the function or amending the contents of 'laws of nature' entailed consequences for Descartes' view of physics as established *a priori*.

The talk about laws that became prominent in English natural philosophy after the 1650s, and the developments oriented by the experimental commitments promoted by, but not restricted to the establishment of the agenda of the Royal Society of London converged in the rejection of the foundations of Descartes' laws: the divine immutability and the doctrine of innate ideas. From Barrow to Locke, the Cartesian foundations of physics were represented as fictions, whose consequences were dangerous for social and religious orders.¹¹ The discomfort with the 'Cartesian way' reinforced a probabilistic attitude with a mitigated scepticism that was in vogue in the early Royal Society.

Concept of Laws of Nature in the Early Royal Society'; Harrison, 'The Development of the Concept of Laws of Nature'; Harrison, 'Laws of Nature in Seventeenth-Century England. From Cambridge Platonism to Newtonianism'.

¹⁰ See Gabbey, 'Between Ars and Philosophia Naturalis'; Gabbey, 'Descartes's Physics'; Gabbey, 'Newton's Mathematical Principles'; Garber, 'Remarks on the Pre-History of the Mechanical Philosophy'; Roux, 'Forms of Mathematization (14-17 Centuries)'.

¹¹ Shapin and Schaffer, *Leviathan and the Air-Pump*; Shapin, *A Social History of Truth*.

This chapter outlines the appropriation of Descartes' laws in England before Newton. During this period, Cartesian laws were integrated into English philosophies involving active principles, experimental practices and a physically-oriented mathematical astronomy. Instead of igniting the spark of the mechanical philosophy in England—as suggested by the interregnum approach—, English natural philosophers reworked the contents and function that Descartes had attributed to laws by rejecting their underpinnings and relocating them into the mathematical sciences. Descartes' laws were integrated into disciplinary settings different from that in which they were initially postulated. I will explain this in the following sections. In the next, I show that English natural philosophers, mathematicians and scholars appropriated 'laws of nature' as part of the critical reception of Cartesianism in general; in this process, a distinctive, new meaning of laws coming from Descartes made its way to England. This is visible in the growing talk of laws as 'indispensable' and 'catholique', in contrast with other more limited uses of the term (for example, the laws of optics or the law of the lever).¹² In the third section, I analyse how this new meaning interacted with local traditions and how the function, contents and foundations of Cartesian laws were modified by encompassing them with active principles, experimental commitments and debates on the role of motion in mathematics. English natural philosophers adopted the new terminology of laws; however, they modified the idea of 'laws of nature' according to the background against which they weighed up the 'French philosophy', particularly by making of them mathematical descriptions or measurments—instead of causes—of motion elucidated by experiments but whose ultimate causes remained unknown. In the final section, I explain the appropriation of 'laws of nature' in the local tradition of 'magnetical cosmology', from which it is possible to draw conclusions concerning the transformation of 'laws of nature' in principles that may be uncovered by experiments and *represented* by mathematics, dealing with forces and powers acting in matter. However, the ultimate causes of these forces and powers remained unknown.

¹² See, for example, Ruby, 'The Origins of Scientific Law'; Roux, 'Les lois de la nature à l'âge classique la question terminologique'; Steinle, 'From Principles to Regularities: Tracing "Laws of Nature" in Early Modern England and France'.

5.2. Descartes ‘Englished’

The reception of Cartesianism in England has been characterised as a mixture of enthusiasm and criticism. An English version of the *Discourse* (1649) compared the work of the translator with that of ‘those who cannot compose the Originals of *Titian* and *Van-Dyke*, [then] are glad to adorne their Cabinets with copies of them; so be pleased favourably to receive his [Descartes] Picture from my hand, copied after his own Designe’. The method of the *Discourse* was presented as the ultimate guide to ‘Enquire into Nature’, to ‘attain to the Knowledge of the Truth’; for ‘all lovers of Learning’, said the translator, ‘I have Englished this Essay’. This ‘Englished’ Descartes was not limited to the translation. In line with the trends of his home country, the anonymous translator highlighted that Descartes ‘invites all lettered men to his assistance in the prosecution of this search [the Knowledge of Truth]; that for the good of Mankinde, They would practise and communicate Experiments, for the use of all those who labour for the perfection of Arts and Science’.¹³ The passages of the *Discourse* promoting experiments provided the translator the occasion to align Descartes with the ‘House of Solomon’. There was no mention of metaphysics, of the doctrine of inborne ideas or of the proofs of the existence of God which appear all over the *Discourse*. On the contrary, Descartes was seen under the light of the experimentalism and the collective fact-gathering enterprise; a call to the communal endeavour of building natural knowledge for the ‘good of Mankinde’.

One of the first enthusiasts and, at the same time, critic of Descartes in Cambridge was Henry More.¹⁴ He saw in Descartes a powerful ally against atheism but subsequently moved towards a harsher position. Although More was critical since his first exchange with Descartes in the late 1640s, he widely discussed the implications and assumptions of some of the most influential Cartesian theses. In so doing, he

¹³ Anonymous, ‘To the Reader’ in Descartes, *A Discourse of a Method for the Wel-Guiding of Reason and the Discovery of Truth in the Sciences*.

¹⁴ Henry, ‘The Reception of Cartesianism’; Henry, ‘Occult Qualities and the Experimental Philosophy’; See also, Levitin, *Ancient Wisdom in the Age of the New Science*; Leech, *The Hammer of the Cartesians: Henry More’s Philosophy of Spirit and the Origins of Modern Atheism*; Webster, ‘Henry More and Descartes: Some New Sources’; Jesseph, ‘Mechanism, Skepticism, and Witchcraft’.

spread Cartesian ideas and vocabulary.¹⁵ However, the reception of Descartes in England, and consequently of his laws, was not restricted to More. As early as 1648, William Petty critically referred to Descartes by echoing Bacon's characterisation of Scholastics as spiders: '[Descartes] employed in spinning the cobwebs out of bare suppositions & out of Principles, which though may be true, yet are remote abstracted & generall'.¹⁶ The remoteness of these principles, that is, their disconnection from experience, was perceived as problematic both for their validity and for their utility in explaining natural phenomena.

Writing almost at the same time as More, another Cambridge scholar put forward a sharp criticism of the Cartesian world governed by laws. In his view, the 'Cartesian hypothesis' could not account for the diversity of phenomena and, at the same time, it was not a good way 'to think about God'. The young and ascending Isaac Barrow considered in 1652 'that the Cartesian hypothesis concerning matter and motion by no means satisfies the principle phenomena of nature'.¹⁷ This *oratio* summarises Descartes' second part of the *Principia*, including the most relevant premises for the 'laws of nature'. The innumerable external objects we perceive should be conceived, according to Descartes, as mere extension which is inseparable from the idea of corporeal matter. 'Which thing being supposed, while a great many different things can easily be deduced, such as what space is, to wit, that there is no vacuum or empty space, [...] most importantly it is possible to deduce the following: that the world is one, cohering together everywhere, and occupying all imaginable space, with its matter existing joined together and everywhere the same. And it is *completely* known through this one thing, namely, that it is extended. And such is the nature of Cartesian matter'. Barrow noticed that this matter 'can exist only in a two-fold manner, namely in motion and at rest'. Ironically, Barrow remarked that 'beyond this there seems nothing more subtle to note concerning the nature of motion and rest'. However, it was necessary to consider 'that there are primary and secondary causes of motion: God is thought to be the primary cause of motion and rest who, in the

¹⁵ Harrison, 'Laws of Nature in Seventeenth-Century England. From Cambridge Platonism to Newtonianism', 133–36.

¹⁶ Henry, 'The Reception of Cartesianism', 117–18.

¹⁷ Stewart, "Fleshy Books", 39. Cf. Kargon, *Atomism in England from Harriot to Newton*, 79.

beginning, created matter at the same time as motion and rest'. From the divine immutability, it followed that 'he moved the parts of matter in diverse ways when he created them in the beginning. And surely he would conserve all this matter in the same manner and, for the same reason, would conserve the amount of motion the same'. Because of this 'he could form and establish certain rules and laws that, in turn, effect secondary causes and particular motions similar to those motions that appear in individual bodies'. Barrow clearly grasped Descartes' causal function of laws. Next, Barrow succinctly enunciated the three laws of nature, following closely the Cartesian wording. The first section of the *Oratio* concludes remarking that 'These [the laws of nature] being supposed and conceded, Descartes thought that whatever changes, generations, alterations are observed in the nature of things, and whatever phenomena are observed in Nature, they could all be sufficiently explained'. However, Barrow's target was to show that 'this is not the case'.¹⁸

Barrow's reconstruction of the reasoning connects the nature of matter as extended, the rejection of vacuum and the divine immutability to the causal 'laws of nature'. Barrow carefully extracted the core of the Cartesian reasoning in order to criticise it. Nevertheless, Barrow never developed a natural philosophy and 'laws of nature' did not figure in his subsequent works. Because his reconstruction of the Cartesian hypothesis grasps the inextricable connection of the idea of laws with other substantial claims, Barrow did not feel that 'laws of nature' were worthy of pick up; his most famous student felt similarly.

Other English authors resumed Descartes' laws and incorporated them into their works. Walter Charleton inserted Cartesian laws in his monumental *Physiologia* in 1654. In it, he followed Gassendi, not Descartes. This is significant because Descartes had openly rejected the basic assumptions of atomism—atoms and void. However, Charleton reviewed the Cartesian laws and used them as premises in building his own atomistic explanations of natural phenomena. When enquiring for the 'elaters or springs' of air, causing 'its suddain restitution to its natural constitution', his answer was 'that, as it is the most catholique Law of Nature, for

¹⁸ Barrow, 'Oratio', 83–85; Stewart, "'Fleshy Books'", 70–72.

every thing, so much as in it lies, to endeavour the conservation of its originary state', So in the case of the spring of air 'it is the essential quality of the Aer, that its minute particles conserve their natural Contexture, and when forced in Rarefaction to a more open order, or in Condensation to a more close order, immediately upon the cessation of that expanding, or contracting violence, to reflect or restore themselves to their due and natural contexture'.¹⁹ The first law of Descartes is interpreted as explaining condensation and rarefaction. But apart from the references to specific laws, Charleton placed laws at the centre of his physiological project; his view of motion differed from Descartes, to a large extent as a consequence of his reconciliation of atomism with Helmontianism.²⁰ Like most of his English fellows, Charleton conceived matter as endowed with principles of activity. For Charleton, 'Because, Motion being the Heart, or rather the Vital Faculty of Nature, without which the Universe were yet but a meer Chaos; must also be the noblest part of *Physiology*'. This meant that 'if Motion and Quiet be the principal modes of Bodies Existing, as *Des Cartes (in princip. philosoph. part. 2. sect. 27.)* seems strongly to asserts if Generation, Corruption, Augmentation, Diminution, Alteration, be only certain species, or more properly the Effects of Motion'. However, the knowledge of these effects of motion 'results from our perception of the Impulses made upon the organs of our senses, by their species thither transmitted'. In consequence, 'the Physiologist is highly concerned to make the contemplation of *Motion*, its *Causes*, *Kinds*, and *Universal Laws*, the *First* link in the chain of all his Natural Theorems'.²¹ This characterisation of motion as the subject of physiology exhibits two characteristics I have mentioned before: Cartesian laws are detached from the main premise about the nature of matter as extended and inert; in its place, Charleton claimed that 'in Nature there is no Faculty but what is *active*', that is, that every 'Compound Bodie is naturally endowed' with a 'Motive virtue'.²² In addition, these laws are presented as 'universal' and put next to 'causes' and 'kinds' as the main subjects of the 'physiologist'.

¹⁹ Charleton, *Physiologia Epicuro-Gassendo-Charltoniana*, 34; Wang, 'Rarefaction and Condensation'.

²⁰ Henry, 'Occult Qualities and the Experimental Philosophy', 341.

²¹ Charleton, *Physiologia Epicuro-Gassendo-Charltoniana*, 435.

²² Charleton, 271.

Another recurring English view of the Cartesian philosophy was its hypothetical nature. Most *virtuosi* saw in this ‘hypothetical physics’ an important slap in the face to the anti-dogmatic style championed by the Royal Society associated to its fact-gathering, Baconian approach. Joseph Glanvill, the ‘most skilful apologist of all virtuosi’,²³ praised Descartes, ‘the Grand Secretary of nature’, for ‘giving a particular and *Analytical* account of the *Universal Fabrick*’. However, Glanvill regretted that ‘he intends his principles but for *Hypotheses*, and never pretends that things are really or necessarily as he hath supposed them: but that they may be admitted pertinently to solve the *Phenomena*, and are convenient supposals for the *use of life*’.²⁴ Against the dogmatism attributed to the Aristotelians, the syllogistic demonstrations and the physics of matter and form, Glanvill acclaimed the project of the Royal Society for ‘the credit which the mathematics have with you, your experimental way of enquiry, and mechanical attempts for solving the phenomena’. Although Glanvill claimed that it was not in the interest of the Society to refute other doctrines, ‘some of you ... publicly own the Cartesian and atomical hypotheses; these, I say, are arguments of your no great favour to the Aristotelian’. The ‘doctrine of matter and form’ was ‘of no accommodation’ to the ‘design’ of the Society which is ‘improving the minds of men in solid and useful notices of things, helping them to such theories as may be serviceable to common life, and the searching out the true laws of matter and motion, in order to the securing of the foundations of religion against all attempts of mechanical atheism’.²⁵

Charleton identified the ‘true laws of matter and motion’ as the core of the ‘designs’ of the Royal Society. He was not alone in this. This assessment is part of the intricate process of defining the objectives and approaches of the Society encompassing private and public interests.²⁶ Indeed, the communications by Wallis, Wren and Huygens in the late 1660s on the laws of impact and the subsequent portrayal by Oldenburg of the Royal Society as concerned with ‘the Principles and Laws of

²³ Westfall, *Science and Religion in Seventeenth-Century England*, 18.

²⁴ Glanvill, *Scepsis scientifica*, 211–12; Cf. Kargon, *Atomism in England from Harriot to Newton*, 112–13.

²⁵ Glanvill, *Scepsis scientifica*, a2.

²⁶ Hunter and Wood, ‘Towards Solomon’s House’; Hunter, ‘The Royal Society and the Decline of Magic’; Dear, ‘Totius in Verba’; Hall, ‘Mechanics and the Royal Society’.

motion'²⁷ are just the best known parts of the story, but hardly the only ones. In any case, Charleton remarked that instead of engaging in disputes against the dominant philosophy, even having the resources against Aristotle—mathematics, experiments and the ‘mechanical attempts to solve phenomena’—, the Royal Society opted for improving ‘the minds of men’ for useful purposes, and for finding the ‘laws of matter and motion’ that secure the foundations of religion against ‘mechanical atheism’. Notice that Charleton remarked that these laws were the ‘true’ ones as part of the growing feeling by the mid-1650s that Cartesian mechanical philosophy may lead to atheism.

The talk of laws in England in the Cartesian sense was not restricted to the abstract collision of bodies. Before the submissions of Wallis, Wren and Huygens in 1668-1669, Cartesian laws had made their way in England. The evidence suggests that this debate resembles a fruit rather than the root of an ongoing process.²⁸ Moreover, the use of Cartesian laws was not restricted to individual, isolated works. In a meeting of the Council of the Royal Society, in 9 May 1666, Oldenburg ‘produced’ a discourse by Wallis concerning the flux and the reflux of the sea. Wallis opened his hypothesis claiming

How much the World, and the great Bodies therein, are manag’d according to the *Laws of Motion*, and *Statick Principles*, and with how much more of clearness and satisfaction, many of the more abstruse *Phenomena*, have been solved on such Principles, within this last Century of years, than formerly they had been; I need not discourse to you, who are well versed in it.²⁹

Wallis’ hypothesis postulated that the motion of two interacting bodies must be calculated around the centre of the system; in the particular case of the tides, Wallis claimed that it is not the centre of the Earth that describes an orbit around the sun, but the centre of gravity of the Earth and the Moon. Thus, the inequality in the Earth’s distance from the Sun was the cause, for example, of the annual variation in the height of the tides.³⁰ This novel hypothesis also explained the diurnal and

²⁷ Oldenburg, *Correspondence*, 1968, 5:512.

²⁸ Cf. Steinle, ‘From Principles to Regularities: Tracing “Laws of Nature” in Early Modern England and France’, 221.

²⁹ Wallis, ‘An Essay of Dr. John Wallis....about the Flux and Reflux of the Sea’, 264.

³⁰ Oldenburg, *Correspondence*, 1966, 3:109.

monthly variation in the tides. Wallis' hypothesis was an attempt to provide a mathematical explanation correcting Galileo's theory of tides in the *Dialogo*.³¹ However, the point of departure of the mathematical hypothesis was the first 'law of nature':

Now in order to the giving account of these three Periods [diurnal, monthly and annual], according to the *Laws of Motion* and *Mechanick Principles*; We shall *first* take for granted, what is now adayes pretty commonly entertained by those, who treat of such matters; *That a Body in motion is apt to continue its motion, and that in the same degree of celerity, unless hindered by some contrary Impediment*; (like a Body at rest, to continue so, unless by some sufficient mover, put into motion).³²

Notice that the first Cartesian law is assumed as a principle of 'Mechanick'. This hypothesis on tides was originally presented in a letter to Boyle in 25 April and afterwards read at the meeting of the Society. Oldenburg wrote back to Wallis in 5 May—before the public discussion—concerning the possibility of printing this paper in the *Philosophical Transactions*, to which Wallis replied that he 'was not averse from it, if first it be approved by (...) ye Society'.³³ Wallis then accepted and the discourse circulated in the *Philosophical Transactions*. Wallis was utterly interested in developing this hypothesis, and Oldenburg asked to a considerable number of correspondents living by the sea in Germany, Italy, England, Ireland, Scotland and America to provide information to advance in the hypothesis.³⁴ Oldenburg forwarded to Wallis objections and suggestions from members of the Society and from readers of the *Transactions*. Some of these objections, including Wallis' rebuttals were also printed in the journal.³⁵ The centrality and importance of the first Cartesian law, in a discourse that circulated widely indicates the acceptance of the idea of laws as principles of explanation of physical phenomena, not restricted to the debate of the collision of bodies. Wallis remarked that the idea of laws, and particularly the first law, 'is now adayes pretty commonly entertained by those, who treat of such matters'. As far as I have been able to track, nobody objected to Wallis' use of the

³¹ See Bonelli and Russo, 'The Origin of Modern Astronomical Theories of Tides'.

³² Wallis, 'An Essay of Dr. John Wallis....about the Flux and Reflux of the Sea', 268.

³³ Oldenburg, *Correspondence*, 1966, 3:107–9.

³⁴ Evidence of Oldenburg interest in a wide circulation of the hypothesis is his request of a Latin version Wallis, *Correspondence*, 440.

³⁵ Oldenburg, *Correspondence*, 1966, 3:xxiv.

Cartesian laws.³⁶ However, his use of laws was not connected with uncovering the physical causes of the tides; Wallis' laws did not operate as causes. Wallis remarked that central to the history of tides was the connection between the Earth and the Moon explained 'whether by any Magnetick, or what other Tye'. Put otherwise, 'the connection being known', he does not need to provide any explanation

as to this purpose; as that the motion of the one follows of the other (the Moon observing the Earth as the Center of its *periodick* motion) may well enough be looked as *one Body*, or rather one Aggregate of Bodies, which have one common center of Gravity ; which Center (according to the known *Laws of Staticks*) is in a streight Line connecting their respective Centers, so divided as that its parts be in reciprocal proportion to the Gravities of the two Bodies.³⁷

Wallis provided a mathematical explanation in terms of laws of motion ('staticks') with experimental outcomes not requiring the specification of the immediate cause generating the phenomenon. Wallis claimed that mathematical laws were principles explaining quantitative properties of motion, although their cause was unknown. In so doing, Wallis was transforming in a radical way the Cartesian use of law. This way of dealing with natural philosophical problems was plausible for English natural philosophers, as we shall see next.

5.3. 'Dead and thoughtless principles'

The recourse to active principles and occult qualities in England opened possibilities for new uses of (Cartesian) laws including experiments and a varied range of mathematical techniques. This is visible in the integration of Cartesian laws into the 'elliptical astronomy' that practiced mathematical astronomy as bounded by and connected with natural philosophy, inspired by the 'magnetical philosophy' of Bacon and Gilbert.³⁸

³⁶ The initial reactions to Wallis' hypothesis are summarised in Birch, *The History of the Royal Society*, 1756, 2:89: 93. Wallis received a summary of these Wallis, *Correspondence*, 250–51.

³⁷ Wallis, 'An Essay of Dr. John Wallis....about the Flux and Reflux of the Sea', 271–72.

³⁸ Wang, 'Francis Bacon and Magnetical Cosmology'; Wang, "'Though Their Causes Be Not yet Discover'd'".

The reception of Descartes in England was critical since its first inception.³⁹ English natural philosophers quickly gathered that the Cartesian suppositions concerning the inactivity of matter and the resulting reduction of all activity in nature to motion generated by collision ‘satisfies by no means the principle phenomena of nature’ as Barrow claimed in 1652.⁴⁰ In part, this critical attitude was a consequence of the different agendas that English natural philosophers were pursuing, inspired by the experimental programmes of Bacon and Gilbert. In spite of the limitations and flaws that they attributed to the Cartesian laws and the potential dangers for religion, Cartesianism was not rejected *tout court*. Cartesian philosophy may be defective, but it was a promising substitution for the discredited Scholastic philosophy. The Cartesian hypothesis ‘concerning matter and motion’ may be problematic, but ‘the scope and power of the Cartesian system, and its easy intelligibility, could hardly fail to make a huge impression on educated consciousnesses’.⁴¹ ‘Laws of nature’ came to stand for this intelligibility. Instead of Scholastic-style definitions—which Descartes himself debunked, English natural philosophers saw in laws ‘catholique’ and ‘indispensable’ statements that may be compatible with active principles and therefore susceptible of experimental ‘verification’.

Active principles and occult qualities were embedded in the experimental philosophy of seventeenth century England.⁴² The constant reference to active principles at least since the 1630s up to and including Newton indicates ‘that there may be a clear tradition of active principles in English matter theory at this time’.⁴³ This tradition of active principles is a relevant background on which the ‘laws of nature’ were considerably transformed in England. If Descartes considered that his laws of nature were sufficient to ‘explain all the phenomena of nature’,⁴⁴ making powers and

³⁹ Rogers, ‘Descartes and the English’; Henry, ‘The Reception of Cartesianism’.

⁴⁰ Barrow, ‘Oratio’.

⁴¹ Henry, ‘The Reception of Cartesianism’, 123.

⁴² Henry, ‘Occult Qualities and the Experimental Philosophy’. Henry has further developed details of this general claim in Henry, ‘Robert Hooke: The Incongruous Mechanist’; Henry, ‘Boyle and Cosmical Qualities’; Henry, ‘The Fragmentation of Renaissance Occultism’. See also, Hutchison, ‘What Happened to Occult Qualities in the Scientific Revolution?’; Feingold, ‘The Occult Tradition in the English Universities of the Renaissance: A Reassessment’; Wang, “‘Though Their Causes Be Not yet Discover’d’”.

⁴³ Henry, ‘Occult Qualities and the Experimental Philosophy’, 342.

⁴⁴ Descartes, *AT*, 1897, I:70; Descartes, *CSM*, 1984, III:7.

qualities in matter redundant, his English fellows disagreed. In the hands of the English *virtuosi*, 'laws of nature' made necessary some other principles of activity in matter. Accordingly, the *virtuosi* disseminated the Cartesian idea of law with some radical transformations.

Two aspects of this tradition are central to this transformation of laws: (1) that passive matter could not 'save the phenomena' of everyday experience and therefore (2) some active principles which inhere in matter superadded by God account for them, although their cause is unknown. In this way, occult qualities or principles of activity in matter 'so important for understanding the true nature of God's creation, could only be evinced, it was claimed, by experimental procedures'.⁴⁵ The commitment to the establishment of matters-of-fact made reasonable to postulate occult qualities whose effects could be uncovered experimentally, although their cause remained hidden. These views were underpinned by religious and social assumptions. The Cartesian world-view, seen through the English lens, implied that the world did not require divine intervention to operate after its initial creation. If the world was set in motion and could endure in this way indefinitely by the endless transmission of the same amount of motion by the laws of nature, it was easy to imagine that God was not required at all.

The criticisms of 'laws of nature' for their inadequacy in accounting for phenomena and for their dangerous theological implications are prominent in Barrow's *Oratio*. After the thorough summary of the 'Cartesian hypothesis' already mentioned, Barrow claimed that 'this hypothesis is not able to give a suitable cause of many phenomena with which the philosophical man could be content'. The 'consensus' of philosophers, scholars and even the alchemists show how wrong is the solitary Descartes in giving birth to 'all this Theatre ... only from the brain of a single man'.⁴⁶ In Barrow's view, the singularity of opinion against the consensus was a powerful argument to invalidate his opponent. Among the various phenomena that the Cartesian hypothesis could not explain, Barrow emphasised the animal motion,

⁴⁵ Henry, 'Occult Qualities and the Experimental Philosophy', 338.

⁴⁶ See Stewart, "'Fleshy Books'". Cf. Levitin, *Ancient Wisdom in the Age of the New Science* esp. Ch 4.

particularly ‘certain natural spirit ... joined to the heart or brain ... Without it the body would putrify most horribly in two days’. But Descartes ‘has not ... nobly set this forth, in saying body is a mere bulk or mass of homogeneous nature, divided into parts of various shape, in which there are diverse motions, flowing by a clearly blind law, with no presiding spirit, no directing wisdom in the actions, no endeavour to a goal and no other efficient intervening thing than the ordinary maintenance of God’.

The general assessment of Descartes hypothesis, is negative:

For [Descartes] all things are established by God, devoid of innate power. All effects occur by force and with affliction, and the internal principles, thus far admitted by a vast consensus of Philosophers, are rejected by him, together with the idea of striving for ends [*intentione finium*]. And in place of these they [Plato and Aristotle] will exclaim, that Descartes has laid out dead and thoughtless principles; nature without soul; the outer shell without the nucleus.⁴⁷

The omission of active principles in matter was a peculiar feature of this problematic hypothesis against which, ‘the general consensus of philosophers’ disagreed. Barrow not only opposed the authority of the Ancient philosophers to the inadequacy of the Cartesian hypothesis, accusing Descartes’ of leaving aside ‘two of the four causes’; he also invoked that of ‘our Baron of Verulam’. The point here was to stress that Descartes recognised ‘no ends or goals of natural agents, no desires, no instincts in things inherent from the beginning, but utterly banishes the entire lot of affections, hostilities, contrarieties, sympathies and antipathies’. All these elements are the life, soul or nucleus of nature. Barrow put forth that an important layer of the natural world, on which there was vast agreement among philosophers, naturalists, alchemists, could not be explained by the Cartesian hypothesis. This blatant omission made manifest that the hypothesis failed to grasp the core of the natural world.

Instead, Descartes delivered ‘dead and thoughtless principles’. Simultaneously,

He thinks unworthily of the Supreme Author of things who asserts that God has created in a single manner only one, homogeneous matter, dull and inanimate, extending through all the acres of immense space, and that, by the single instrument of motion, he directs these festive games and the whole mundane comedy, like a woodworker or artificer, who repeats and displays his single art *ad nauseam*. Rather, it seems he thinks more worthily of God who believes God has, out of his immense goodness and kindness,

⁴⁷ Stewart, “Fleshy Books”, 74; Barrow, ‘Oratio’, 88.

imparted to innumerable species each its own particular essence, bestowing on each one its own peculiar desire and individual means of acting, distributing all things in a fitting order in their proper degree and station for their mutual aid, and subordinating some to others according to a preordained order of things.⁴⁸

In Barrow's view, the 'laws of nature' ('the single instrument of motion') were inevitably linked with an impious cosmology, based on 'dull and inanimate' matter, and making of God some kind of puppet master doing the same trick once and again forever. A view of the world in which matter is endowed with principles of activity, according to their 'own particular essence' in 'a fitting order in their proper degree and station for their mutual aid', that is, sympathies, 'it is better piously'.⁴⁹ While some English natural philosophers praised the universality ('catholique') of Cartesian laws, Barrow saw in them a 'general, loose and indeterminate' feature, so that 'one could with greatest justice think that whatsoever appears according to these laws, happened by a certain accident or by chance'. The generality was further evidence of their uselessness.

However, not all those who rejected the Cartesian exclusion of activity in matter got rid of laws of nature; on the contrary, the opposite seems to be the case. However, laws were not conceived as causes. From Glanvill and Charleton to Boyle and Hooke, 'laws of nature' were reworked into programmatic statements and into explanations of phenomena. Perhaps the most famous programmatic statement concerning the enquiry of the natural world in the seventeenth century is Boyle's characterisation of the 'mechanical philosophy' in the *The Origin of Forms and Qualities* (1666-7), which generations of historians made the golden standard of the strict mechanical philosophy, for reducing all explanation to matter in motion.⁵⁰ Boyle agreed 'with the generality of Philosophers so far as to allow, that there is one Catholick or Universal matter common to all Bodies, by which I mean a Substance extended, indivisible and impenetrable'. Following the Cartesian reasoning, Boyle explained that because this matter was 'in its own Nature but one' the diversity we

⁴⁸ Barrow, 'Oratio', 89–90; Cf. Stewart, "'Fleshy Books'".

⁴⁹ Stewart, "'Fleshy Books'", 74; Barrow, 'Oratio', 104.

⁵⁰ Boas, 'The Establishment of the Mechanical Philosophy'; Dijksterhuis, *The Mechanization of the World Picture*; Westfall, *The Construction of Modern Science: Mechanisms and Mechanics*.

see in bodies arose from motion that ‘must have various tendencies’. In order to clarify this conception of motion, ‘hotly disputed’, he reviewed that ‘Antient Corpuscularian Philosophers, not acknowledging an Author of the Universe, were thereby reduc’d to make Motion congenite to Matter’. Boyle provided two arguments against this view: first, motion should not be included in the definition of matter because ‘the nature of Matter, which is as much Matter, when it rests, as when it moves’. Second, because when ‘a portion of matter may from Motion be reduc’d to Rest’ remained at rest unless ‘some external Agents be set a moving again’. Next, Boyle praised ‘the Excellent *Des cartes*’ for having revived the opinion of the Greeks that ‘the Origine of Motion in Matter is from God’, specifically through laws of *motion*:

and not onely so, but that thinking it very unfit to be believ’d, that Matter barely put into Motion, and then left to it self, should Casually constitute this beautiful and orderly World: I think also further, that the wise Author of Things did by establishing the laws of Motion among Bodies, and by guiding the first Motions of the small parts of Matter, bring them to convene after the manner requisite to compose the World, and especially did contrive those curious and elaborate Engines, the bodies of living Creatures, endowing most of them with a power of propagating their Species.⁵¹

Boyle’s appropriation of ‘laws of nature’ entailed a revaluation of them to make laws of motion compatible ‘with our Hypothesis’. Boyle criticises the idea that ‘these *Cartesian Laws* ... could bring meer Matter into so orderly and well contriv’d Fabrick as This World’. In Boyle’s view, God created the world and established laws, but mainly he also did

contrive some portions of that Matter into Seminal Rudiments or Principles, lodg’d in convenient Receptacles, (and as it were Wombs,) and others into the Bodies of Plants and Animals: one main part of whose Contrivance, did, as I apprehend, consist in this, That some of their Organs were so fram’d, that, supposing the Fabrick of the greater Bodies of the Universe, and the Laws he had establish’d in Nature, some Juicy and Spirituous parts of these living Creatures must be fit to be turn’d into Prolifick Seeds, whereby they may have a power, by generating their like, to propagate their Species.⁵²

⁵¹ Boyle, *Works*, vols 5, 305–6.

⁵² Boyle, vols 5, 352–3.

Boyle introduced principles of activity in matter to account for some varieties of motion in nature and to correct the undesirable theological consequences of the ‘laws of nature’ stating that the same quantity of motion remained in the world. The validity of laws, as laws of motion, was subsidiary of ‘the contrivance’ of nature which contained ‘seminal rudiments or principles’ causing matter to move: these seeds, not the laws, generated motion.

5.4. ‘Elliptical astronomy’

Perhaps the soil on which the Cartesian laws were most fruitful in England was the ‘magnetical cosmology’. The movement was inspired by Gilbert’s attempts to provide a physical explanation of the motion of the Earth. In *De magnete* (1600) Gilbert formulated experiments on the magnetical properties of the *terrella*—small spherical loadstone—in order to treat ‘of the globe of earth as a loadstone’. Gilbert concluded that ‘magnetic energy (...) exists in the earth just as in the *terrella*’.⁵³ The magnetic nature of Earth accounted for terrestrial gravitation, the Earth’s constant orientation in space, its daily rotation generated by the magnetic influence of the Sun and the compact between the earth and the moon. The effect of the magnetical influence was not limited to the Earth but thanks to the Sun’s conception as a giant magnet, it extended all over the Solar System.⁵⁴ Gilbert’s efforts to provide a physical explanation of the motion of the Earth, as implied by the Copernican hypothesis, gave rise to a tradition that ranges up to the end of the century.

Arguably, Gilbert’s most enduring contribution to the development of English astronomy was the analogy between magnetism and gravity. His conception of magnetism rejected the possibility of *actio in distans* and explained the mutual attraction of magnets as the outcome of their capacity of self-motion in virtue of their souls.⁵⁵ Nevertheless, in the English natural philosophy that reached up to Hooke, magnetism and gravity were regarded as paradigmatic cases of action-at-a-distance and thus non-reducible to contact action. Thanks to Bacon’s appropriation of Gilbert, magnetism and gravity were conceived in terms of action-at-a-distance. This

⁵³ Gilbert, *De Magnete*, 313–14.

⁵⁴ Bennett, ‘Cosmology and the Magnetical Philosophy 1640-1680’, 165–66.

⁵⁵ Henry, ‘Animism and Empiricism’; Wang, ‘Francis Bacon and Magnetical Cosmology’.

influence is visible in John Wilkins, Walter Charleton, John Wallis, Christopher Wren and Robert Hooke.⁵⁶

One important stimulus for the development of this tradition came from Kepler's *physica cœlestis*. Kepler had formulated an astronomy based on natural philosophy which, in order to explain the orbital motion, drew explicitly from Gilbert's magnetical philosophy.⁵⁷ English astronomers and natural philosophers of the first half of the seventeenth century, who considered Gilbert as 'the founder of the experimental method', interpreted Kepler's work as part of the same endeavour.⁵⁸ English astronomers were already analysing the motion of planets in terms of physical causes under Gilbert's influence, so Kepler's astronomy did not depart from their assumptions. At least since the 1630s, astronomers associated with the 'magnetical cosmology' enthusiastically incorporated Kepler's astronomical and cosmological developments into their own work, in contrast with their Continental counterparts who explicitly rejected Kepler's natural philosophy and cosmology for transgressing the boundaries of astronomy.⁵⁹ The 'magnetical cosmologists' or 'elliptical astronomers', as some of them referred to themselves once Kepler was widely accepted, did not see the entire Cartesian philosophy as challenging their developments.

The reception of Kepler in England was mainly associated with the names of William Crabtree, Jeremiah Horrocks and William Gascoigne. Horrocks, a young Cambridge scholar, was dissatisfied with the work of the Dutch mathematician Philip van Lansberge, particularly with his tables. Thanks to the advice of Crabtree, who recommended buying Kepler's works, Horrocks turned to Kepler between 1636 and 1637.⁶⁰ When Horrocks got the *Tabulæ Rudolphinæ* in May 1637, he wrote to Crabtree that it is 'a most absolute piece of work'.⁶¹ Largely influenced by Kepler's

⁵⁶ Wang, 'Francis Bacon and Magnetical Cosmology'.

⁵⁷ See chapter 3.

⁵⁸ Bennett, 'Cosmology and the Magnetical Philosophy 1640-1680', 166.

⁵⁹ Russell, 'Kepler's Laws of Planetary Motion: 1609-1666'; Hatch, 'Boulliau, Ismaël (1605-1694)'.

⁶⁰ Russell, 'Kepler's Laws of Planetary Motion: 1609-1666', 12; Stancliffe, 'An Early Astronomical Manuscript of Jeremiah Horrocks', 456.

⁶¹ Stancliffe, 'An Early Astronomical Manuscript of Jeremiah Horrocks', 458.

astronomy, Horrocks revised Kepler's tables—from which he predicted the 1639 transit of Venus—, developed a theory of the moon and based on harmonic reasoning, calculated the planetary distances and the size of the universe.⁶² In his short life, Horrocks provided a fundamental link between the work of Kepler and the English cosmology.

Following Gilbert, Horrocks was concerned with the explanation of planetary motion in terms of magnetic attraction. In the same spirit, Horrocks critically revised Kepler's major tenets.⁶³ Central to Kepler's account of planetary motion was the idea that magnetic fibres within the Earth maintained a constant direction with respect to the zodiac, despite the Earth's rotation, and so caused the Earth to be alternately attracted to and repelled from the Sun. Horrocks' dissatisfaction with this explanation led him to find alternative explanations in conical pendula: the bob of the pendulum moved in an oval, and the line of apsides of the oval advanced just as in planetary motion.⁶⁴ In this model, the planet was attracted to, but never repelled by the Sun. However, the oval described by the conical pendulum was concentric, while Kepler had calculated an eccentric path. Horrocks conjectured that, in the case of the pendulum, a wind could cause the orbital oval of the conical pendulum to become eccentric. In the case of the planets, it would be due to an internal propensity, a tendency of the planets to rest in the place where they were first placed, their aphelia. Horrocks explained this in detail that:

y^e eccentricity of y^e planets is caused by the contention between y^e Suns magneticall (and always attractive) virtue, and y^e planets dulnes naturally desiring to rest unmoved, which dulnes, while y^e Suns circular motion carrys y^e planet from y^e aphelium, is conquered, and so y^e planets motion incraseth in fastnes; but when y^e Suns circular revolution doth recarry it backe toward the aphelium, the natural torpor and dulnes increaseth, by y^e presence and neerness of y^t place where it would rest.... Keplers Astronomy differes from mine...: He gives y^e planets a divers nature (good and bad) y^t they may eyther come to y^e Sun or fly away at their pleasure, or at least (as his second thoughts are) so dispose themselves (inspite of all y^e Suns magneticall power) y^t y^e Sun is

⁶² Wilson, 'On the Origin of Horrocks's Lunar Theory'; Chapman, 'Horrocks, Crabtree and the 1639 Transit of Venus'; Applebaum, *Horrocks [Horrox], Jeremiah (1618–1641), Astronomer*.

⁶³ Horrocks, *Opera posthuma*, 277–78; Wilson, 'On the Origin of Horrocks's Lunar Theory'.

⁶⁴ Horrocks, *Opera posthuma*, 313–14.

bound to attract or expel them, according to y^t position, w^{ch} themselves defend against all y^e Suns labouring to incline the fibers. I on y^e contrary, make the planet naturally to be averse from y^e Sun, and desirous to rest in its own place, caused by a material dulnes naturally opposite to motion, and averse from y^e Sun, without eyther power or will to move to y^e Sun of itselfe.⁶⁵

Kepler's explanation of planetary motion postulated magnetic properties in the Earth and the Sun; Horrocks moved in a different direction. In his view, the magnetic virtue of the Sun was compensated by what he calls 'material dullness naturally opposite to motion', a natural tendency in the Earth to find its own place. The interaction of these components explained the eccentricity of the oval orbit. The place where the Earth 'desires' to rest was the aphelion which, in Horrocks' cosmology, was the point where God placed it in the fourth day of the creation. In place of Kepler's repulsion, Horrocks introduced an 'aversion' to the Sun as the natural desire of Earth to find its own place. In *Astronomia Kepleriana*, Horrocks borrowed the term 'inertia' from Kepler to characterise 'this natural dulnes': 'The cause of the eccentricity is, in consequence, the very essence of the Planet, that is, its natural *inertia ad motum*, which tends to keep it in its place against the abduction of the Sun'.⁶⁶

Although Horrocks' papers remained unpublished until the 1670s, some members of the Royal Society had access to them in the 1660s, particularly Wallis, who met Horrocks in Cambridge.⁶⁷ An important stimulus in the development of the magnetical cosmology, initially gathered around the Gresham College in London, came from the move to Oxford by the mid-century;⁶⁸ figures such as Seth Ward, John Wallis and John Wilkins emerged and interacted here. Writing in the 1650s, Ward, the Savilian Professor of Astronomy reported that 'not one man here, who is so farre Astronomicall, as to be able to calculate an Eclipse, who hath not received the Copernican System, (as it was left by him, or as improved by *Kepler*, *Bullialdus*, our own Professor, and others of the Ellipticall way) either as an opinion, or at

⁶⁵ Horrocks, Jeremiah, *Philosophicall Exercises*, Part I, paragraph 26, fols 71v-73v in Wilson, 'On the Origin of Horrocks's Lunar Theory', 92-93.

⁶⁶ Horrocks, *Opera Posthuma*, 92-93. Kepler's use of inertia in Kepler, *KGW*, 1953, 7:7:296.

⁶⁷ Applebaum, *Horrocks [Horrox], Jeremiah (1618-1641), Astronomer*.

⁶⁸ Bennett, 'Cosmology and the Magnetical Philosophy 1640-1680'.

leastwise, as the most intelligible, and most convenient Hypothesis'.⁶⁹ In this context, Christopher Wren learned about Kepler, probably through Seth Ward. Wren, and Hooke after him, integrated the laws coming from Descartes with the elliptical astronomy by conceiving planetary motion as a compound of an impressed motion and an attractive force from the Sun. In so doing, they transformed the idea of law by encompassing it with a conception of motion as generated by forces, rather than by collision and impact.

In his inaugural address as Professor of Astronomy at Gresham College in 1657, Wren recalled the major achievements of astronomy and placed Gilbert as the founder of a new philosophy. In a draft of this address, Wren stated that thanks to the correspondence between the *terrella* and the 'great *Magnet* of the Earth', Gilbert found out 'a new Science' upon which '*Cartesius*' built. At the same time, Gilbert's experiment gave 'occasion to Kepler (...) of introducing Magneticks into the Motions of the Heavens, and consequently of building the elliptical astronomy'.⁷⁰ It is noteworthy that Wren sees Gilbert as founding a new philosophy and Descartes as building on it. Wren's 'new philosophy' is not the 'mechanical philosophy' of Boyle, revived by the 'Excellent Descartes', but the 'new Science' of Gilbert. Wren's approach to Descartes' laws of nature is framed within his commitment to the 'new philosophy'.

A key to understand this subordination of Descartes to Gilbert appears in a letter. The King had accepted an invitation to visit the Royal Society and Oldenburg wrote to some members about the experiments that should be performed for that occasion.

Wren replied that:

Experiments for the establishment of natural philosophy are seldom pompous. It is upon billiards and tennis-balls, upon the purling of sticks and tops, upon a vial of water, or a wedge of glass, that the great Des Cartes hath built the most refined accurate theories, that human wit ever reached to; and certainly nature, in the best of her works, is apparent enough by obvious things, were they but curiously observed. The key, that opens treasures is often

⁶⁹ Ward, *Vindiciæ academiarum Containing, Some Briefe Animadversions upon Mr Websters Book, Stiled, The Examination of Academies.*, 29.

⁷⁰ Wren, *Parentalia*, 204.

plain and rusty; but unless it be gilt, the key alone will make no shew at court.⁷¹

Wren's account inscribed Descartes' in the context of the experimental practises. Descartes' 'accurate theories' were the outcome of experiments inadequate for the royal visit. Interestingly, Wren mentioned that 'certainly nature, in the best of her works, is apparent enough by obvious things, were they but curiously observed'. While Barrow saw laws of nature as insufficient given their ultimate foundation in a problematic metaphysics, Wren focused on the innumerable Cartesian references to experiments and considered that his 'accurate theories' may be subject of experimental verification, particularly the rules of motion that spelt out the third Cartesian law. The experiments that Descartes presented here and there were regarded as underpinning his theories and thus improving these experiments should lead to better conclusions.

Sprat attributed the Royal Society's interest in the laws of motion, which ended up in the debate on the collision of bodies by the end of the 1660s,⁷² to the personal interests of Christopher Wren. Perhaps the search for the 'true' laws of motion exhibits, like no other topic, the dynamics of the early Royal Society, for it lies at the crossroads of personal and collective concerns. At the same time, the quest for laws was constant subject of disagreement between those who defended an orientation towards some forms of antiquarianism (fact-gathering) without any theoretical direction and those who stood for a more organised, systematic agenda, including the experimental validation of hypotheses.⁷³ It is no coincidence that the debate on the collision of bodies, including all the experimental displays concerning pendula—with trials on top of St. Paul's cathedral—, added to the correspondence with a Continental figure of the stature of Huygens, occurred during an identity crisis of the Royal Society.⁷⁴ In this context appeared Sprat's *History of the Royal Society of London* (1667). There, Sprat remarked Wren's role in single-handedly introducing into the Society his interest in the laws of motion, as part of a personal interest that

⁷¹ Wren to Oldenburg, July 30 1663 in Birch, *The History of the Royal Society*, 1754, 1:289.

⁷² Hall, 'Mechanics and the Royal Society'.

⁷³ Hunter and Wood, 'Towards Solomon's House'; Dear, 'Totius in Verba'; Hall, 'Mechanics and the Royal Society'; Jalobeanu, 'The Cartesians of the Royal Society'.

⁷⁴ Levitin, *Ancient Wisdom in the Age of the New Science*, 270.

had been going for some years.⁷⁵ Sprat's presentation of Wren as the main fellow concerned with the 'doctrine of motion' provides interesting elements. First, Sprat claimed that the 'doctrine of motion' established 'the principles of philosophy' by 'geometrical demonstration'; that is, the idea that motion is the centre of natural philosophy is not challenged here, but it is precisely in this sense that the laws of nature were dubbed as laws of motion. Second, Sprat mentioned that Descartes made some experiments on which he established his entire system. Again, the view that Descartes had proceeded in a similar way to that championed—or said to be championed—by the Royal Society allowed Sprat to explain that, because some of the Cartesian conclusions seemed flawed, Wren developed more sophisticated experiments including the construction of instruments for this purpose.⁷⁶ Descartes was read in line with experimental practices and, in consequence, his project was a valid enterprise for the fellows. Sprat highlighted the importance of this 'doctrine of motion' which, in the hands of Wren, achieved 'true Theories' validated by 'hundreds of *Experiments*'. Rather than founding principles on conjectures, Wren was said to offer experiments. In the last place, and more interestingly, Sprat characterises the principles of this doctrine with the traditional terms in which the English appropriated Descartes' laws: as 'fundamental and universal'. Beyond that, Sprat points to application of these principles as bringing up phenomena traditionally associated with active principles such as generation, corruption, alteration but in general 'all the Vicissitudes of Nature' that arise from the 'meeting of little Bodies'. From the reference it is not possible to infer whether these 'little Bodies' have self-motion, active principles or operate only by contact action.

Furthermore, it is hard to say that Sprat's account of the Royal Society provides accurate historical information, largely, because it appears in the context of the crisis that I have mentioned. However, we can corroborate Wren's early interest in the topic from an episode triggered by the central importance of the laws of motion in the Royal Society. Hooke had been performing 'experiments of motion' at least since 1662 and, as required by his obligations as demonstrator, he needed the approval to

⁷⁵ Sprat, *History of the Royal Society of London*, 311–12.

⁷⁶ Dear, 'Totius in Verba'.

pursue these experiments as part of the activities of the Society.⁷⁷ In the meeting of 22 October 1668, the president, The Viscount Brouncker, ‘desired, that it might be considered, whether it were so proper or necessary to try this sort of experiments, since Monsr. Huygens and Dr. Christopher Wren had already taken great pains to examine that subject, and were thought to have also found a theory to explicate all the phaenomena of motion’.⁷⁸ Brouckner was present when Huygens met Wren and performed these experiments. From his correspondents, Oldenburg already had gathered some information about the activities of Huygens during his visit to London in April 1661. Spinoza had enquired of Oldenburg in 1665 whether it was true, as Huygens presumed, that ‘all his former discoveries concerning motion ... had subsequently been verified by experiments in England, which I can hardly believe’.⁷⁹ In trying to uncover the story, Oldenburg wrote to Robert Moray, a Scottish diplomat and natural philosopher. Moray’s reply provides some interesting details:

When Mr Hugens came first over wee were with him at his lodging at the end of Newstreet in Convent Garden, where hee told us amongst other things of what he had done in the businesse of motion, as hee hath done to your friend of late. At that time Mr Rook & Dr Wren had made diverse experiments with balls of wood & other stuff hanging by threads whereof you may remember to have seen some, upon Mr Hugens undertaking to solve all questions of motion according to his rule Dr Wren did propose some which he had tryed by experiments, and Mr Hugens did in a very short space solve them so as it was concluded by all the solution did agree with the experiments that had been made. The L. Brouncker was there present too & perhaps your self to. of this you may assure your friend. So that Mr Hugens is sure to have the better of des Cartes in those things which have been determined by Experiments.⁸⁰

Apart from depicting the image of how the meetings went, Moray’s description shows that by that time, the experimental investigation on the laws of motion was conducted by experiments on pendula (‘with balls of wood & other stuff hanging by threads’); the same instrument to advance on astronomical examinations. According to this evidence, it is clear that Wren was actively concerned with trying experiments on the laws of motion when he met Huygens in 1661 (‘Dr Wren did propose some

⁷⁷ Hunter and Wood, ‘Towards Solomon’s House’; Shapin, ‘Who Was Robert Hooke?’

⁷⁸ Birch, *The History of the Royal Society*, 1756, 2:315.

⁷⁹ Oldenburg, *Correspondence*, 1965, 2:541.

⁸⁰ Oldenburg, 2:560–61.

which he had tried by experiments'). Huygens was probably trying to amuse his hosts with conclusions from his unpublished *De motu corporum ex percussione* (1658), in which he had established some new rules for collision which he communicated to the Royal Society in 1668-9. It is not surprising, then, that when Wallis, Wren and Huygens submitted their papers on the laws of motion by 1668-9, Wren's and Huygens' were equivalent.⁸¹

So far, we have seen that by the time Wren met Huygens in London he already had put forward some conclusions on the laws of motion including experiments that he shared with Huygens. These studies on the laws of motion occurred as Wren was putting forward his results on 'elliptical astronomy'. The studies on the 'laws of nature' were integrated into his wider intellectual interests which ultimately were framed within the 'new philosophy' put forward by Gilbert (Bacon) and invigorated by the work of Kepler. Indeed, Wren had conceived orbital motion in terms of a central, attractive force and some 'imprest motion'. When the dispute over the priority for the reciprocal inverse-square law surfaced in 1686, Newton wrote to Halley asking him whether Wren had knowledge of this proportion. After meeting Wren, Halley reported to Newton that:

According to your desire in your former, I waited upon Sr Christopher Wren, to inquire of him, if he had the first notion of the reciprocall duplicate proportion from Mr Hook, his answer was, that he himself very many years since had had his thoughts upon making out the Planets motions by a composition of a Descent towards the sun, & an imprest motion; but that at length he gave over, not finding the means of doing it. Since which time Mr Hook had frequently told him that he had done it, and attempted to make it out to him, but that he never satisfied him, that his demonstrations were cogent.⁸²

Noteworthy, the subject of controversy here is not the explanation of planetary motion by the composition of two forces, but whether 'Sr Chr. Wren knew ye duplicate proportion wⁿ I gave him a visit, & then Mr Hook (by his book *Cometa* written afterward) will prove ye last of us three yt knew it'.⁸³ Halley and Newton are not unbiased sources. However, this reinforces the idea that neither Halley nor

⁸¹ Hall, 'Mechanics and the Royal Society'.

⁸² Halley to Newton, 29 June 1686 Turnbull, *Correspondence*, 1960, 2:441-42.

⁸³ Newton to Halley, 20 June 1686 Turnbull, 2:435.

Newton made any further remark on the idea that planetary motion was composed of ‘a descent towards the sun’ and ‘an imprest motion’.⁸⁴ Wren avoided the controversy by claiming that he did not find ‘the means of doing it’, but he claimed that he knew the composite nature of the orbital motion.

Meanwhile, Wren was not the only fellow interested in laws before the Society incorporated them into their agenda. As early as 1662, Hooke’s experiments included Cartesian laws when investigating the free fall of bodies. Given Mr Hooke’s lower social status, in comparison with Dr Wren’s, it seems implausible that Sprat would introduce a controversial topic such as the ‘doctrine of motion’ backed up by a lower authority.⁸⁵ However, in early records of the Royal Society, Hooke appears as invoking Cartesian laws and presenting them in formalised ways as part of his experimental trials. In the mentioned experiments on the free fall, the entry reads:

Now as exact trials of this kind may be very useful in mechanics, so could they be made with bodies perfectly solid, would they be for the establishment of one of the chiefest philosophical principles, namely, to shew the strength, which a corpuscle moved has to move another; and though Des Cartes’, put it as a: principle, that *si corpus C plane quiesceret, essetque paulo majus quam B, quacunque cum celeritate B moveretur versus C, nunquam ipsum C moveret sed ab eo repelleretur in contrariam partem*: yet these experiments do seem to hint, that the least body by an acquired celerity may be able to remove the greatest ; though how much of its motion is imparted to the bigger, body, and how much of it is recoiled into the smaller, be not determined by these experiments.⁸⁶

Leaving aside the omission in Sprat’s history, from the story of Wren and from the record of Hooke’s experimental activity the appropriation of laws of nature, as laws of motion, was not restricted to mechanics. Hooke was in fact enquiring whether there was some loss of motion in the collision of bodies and then the total quantity of motion in the universe was not eternally preserved.⁸⁷ Both Wren and Hooke incorporated laws in enquiries falling beyond the traditional problems of the science

⁸⁴ Gal, *Meanest Foundations*; Henry, ‘Gravity and De gravitatione’.

⁸⁵ Shapin, ‘Who Was Robert Hooke?’

⁸⁶ Birch, *The History of the Royal Society*, 1754, 1:197.

⁸⁷ Gal, *Meanest Foundations*; Bennett, ‘Robert Hooke as Mechanic and Natural Philosopher’; Bennett, ‘Hooke and Wren and the System of the World’.

of mechanics as a mathematical discipline concerned only with the five manual powers or the classical machines.

In the experiments that Hooke ‘demonstrated’ to the Society on the collision of motion, it is possible to appreciate topics that he will incorporate later into his own natural philosophy. Hooke’s first significant activity before the Society concerning laws in February 1662 referred to Descartes’ third law in connection with experiments on the free fall of bodies. In a meeting of 16 January 1667, Oldenburg reported that the Council ‘had thought fit that the experiments for making out a theory of the laws of motion formerly begun by Dr. Wren, Dr. Croune, and Mr. Hooke; as also those about the magnet formerly begun by Mr. Balle and Mr. Hooke, should be prosecuted’. The Society decided to consult Wren for his experiments but he argued that ‘the account of them was at Oxford’ so ‘Mr Hooke was desired to bring his, as also that Mr. Hooke should prosecute the experiments of the loadstone’.⁸⁸ Hooke suggested experiments showing ‘that the motion of the celestial bodies might be represented by pendulums’.⁸⁹

Hooke greatly contributed to the tradition of the magnetical cosmology, including his appropriation of active principles and occult qualities as crucial elements in explaining the totality of phenomena.⁹⁰ Hooke incorporated the Cartesian laws with his own view dominated by active principles and his experimental approach. In so doing, Hooke transformed the meaning of laws in the sense that they now referred to forces generating motion. The existence of these forces may be ‘demonstrated’ by experiments and represented by mathematics, although their causes remained occult.

While the discussion on laws and the collision of bodies was gaining strength in the Royal Society, Hooke presented in 1666 a paper concerning ‘the inflection of a direct motion into a curve by a supervening attractive principle’, which situated the laws of nature in the context of the magnetical cosmology. ‘I have often wondered—Hooke

⁸⁸ Birch, *The History of the Royal Society*, 1756, 2:187.

⁸⁹ Birch, 2:89.

⁹⁰ Henry, ‘Robert Hooke: The Incongruous Mechanist’; Henry, ‘Occult Qualities and the Experimental Philosophy’; Bennett, ‘Cosmology and the Magnetical Philosophy 1640-1680’; Wang, ‘Francis Bacon and Magnetical Cosmology’; Wang, “‘Though Their Causes Be Not yet Discover’d’”, chaps 3, 5.

claimed—why the planets should move about the sun according to Copernicus’s supposition, being not included in any solid orbs (..) nor tied to it, as their center, by any visible strings; and neither depart from it beyond such a degree, nor yet move in a strait line, as all bodies, that have but one single impulse, ought to do’.⁹¹ This was the central problem of the magnetical cosmology: why the planets move around the Sun if there was no visible mechanism holding them. The planets revolve around the Sun in circular or elliptical motions, but ‘a solid body ... must persevere in its motion in a right line, and neither deflect this way nor that way from it’. Assuming in this way Descartes’ first and second laws, Hooke concluded that the motion of planets ‘must have some other cause, besides the first impressed impulse, that must bend their motion into that curve’. Hooke offered two hypotheses to explain the ‘inflection’: the first was a varying density in the medium in which planets move, but he quickly discarded it. The second hypothesis was ‘an attractive property of the body placed in the center; whereby it continually endeavours to attract or draw it to itself’. If this principle was assumed then, ‘all the phenomena of the planets seem possible to be explained by the common principle of mechanic motions’. And following this path, this ‘may give us a true hypothesis of their motion, and from some few observations, their motions may be so far brought to a certainty’.⁹² The ‘common principle of mechanics motion’ was some laws concerning the motion of bodies in inclined planes which Hooke applied to the motion of a circular pendulum, from which he concluded that the ‘conatus of returning to the center in a pendulum is greater and greater’. According to the records of the meeting, Hooke ‘demonstrated’ two experiments ‘with a large wooden ball of lignum vitæ fastened to the roof’. Interestingly, Hooke considered that the analysis of circular motion ‘compounded of an endeavour by a direct motion by the tangent, and of another endeavour tending to the center’ could explain ‘the phaenomena of the comets as well as of the planets (...) and the motions of the secondary, as well as of the primary planets’.⁹³ The solution to the problem did not include an explanation of the nature of the attractive power of the Sun. Hooke relied on the tradition of the magnetical cosmology explaining this attractive power in terms of magnets, claiming that the Sun has an

⁹¹ Birch, *The History of the Royal Society*, 1756, 2:91.

⁹² Birch, 2:91.

⁹³ Birch, 2:92.

attractive power ‘whereby they may be said to attract in the same manner as the Load-stone hath to Iron, and the Iron hath to the Load-stone’.⁹⁴ In his ‘Lectures of Light’ of the same year, Hooke speculated on the nature of this attraction, claiming that ‘there may be many other motions and Operations of Bodies at a distance, and several other ways by which the Bodies of the World may influence one another, though it has pleased God not to give us Organs or Senses to discover them, and thereby many things that are accounted Sympathetick or Magical may be done by Natural Causes and Powers of which we have no Organs to make us sensible’.⁹⁵ These ‘natural causes and powers’ could be clarified by experiments, but not entirely unveiled. In finding natural knowledge we go ‘from the lowest and most sensible Effects, to highest and higher Steps of Causes, the nearer shall we be to the highest and utmost pitch that human Nature is capable of arriving at’. This ‘highest point’ is ‘the knowledge of the alterations’ of matter and motion that ‘flow from the Omnipotent Wisdom that ordered them to do so ... which we call the Laws of Nature’. But these laws, or principles of alteration, were not grasped by an act of the mind, but by a faculty that God had ‘implanted in Man’ by which ‘he has a Power of understanding and finding out, by and according to what Order, Rule, Method, or Law, they act, and produce the Effects that are produced by them’. This faculty was not the Cartesian reason, but the capacity of ‘understanding’ how powers act from ‘the lowest and more sensible effects’, that is, from experiments.⁹⁶

Hooke’s efforts were not restricted to the explanation of some phenomena. In the closing words of *An attempt to prove the motion of the Earth by Observations* (1674), he sketched a programme encompassing active principles, occult qualities and his unique mathematical approach. This programme would be the outcome of the improvement of instruments and experiments that shall correct astronomical tables, but also provide advances in navigation and geography. These instruments detected ‘the properties and effects of motions from prompting secret and swift conveyance and correspondence’—a reference to the explanatory approach of the experimental tradition. But Hooke claimed that he has also ‘discovered some new Motions even in

⁹⁴ Wang, ““Though Their Causes Be Not yet Discover’d””, 144.

⁹⁵ Hooke, *Posthumous Works*, 79.

⁹⁶ Hooke, 173.

the Earth', although he did not have enough experiments on them yet. Instead of uncovering the causes, Hooke stated that he would explain 'a System of the World differing in many particulars from any yet known', whose main feature would be that it would answer 'in all things to the common Rules of Mechanical motions'. The system was based on three suppositions: the first is 'That all Coelestial Bodies whatsoever, have an attraction or gravitating power towards their own Centers'. However, this power was not restricted to their particles but also to the entire universe; Hooke's asserts the mutual attraction of all particles and bodies 'that are within the sphere of their activity'. The second was that 'all bodies whatsoever that are put into a direct and simple motion, will so continue to move forward in a streight line, till they are by some other effectual powers deflected and bent into a Motion, describing a Circle, Ellipsis, or some other more compounded Curve Line'. This is one of the Cartesian first two laws concerning the rectilinear conservation of impressed motion. The third supposition is that 'these attractive powers are so much the more powerful in operating, by how much the nearer the body wrought upon is to their own Centers', that is, these powers were proportional to the distance. The idea was not new, for Bacon had already suggested that (terrestrial) gravity may vary according to the distance from the earth.⁹⁷ Hooke explained that this principle 'will mightily assist the Astronomer to reduce all the Cœlestial Motions to a certain rule' and that the key to understanding it was 'the nature of the Circular Pendulum and Circular Motion'. Following this path shall lead to appreciate that 'all the great Motions of the World to be influenced by this Principle, and that the true understanding thereof will be the true perfection of Astronomy'.⁹⁸

* * *

The critical appropriation of Cartesianism in England transformed the meaning and uses of laws in function of the background against which Descartes' postulates were evaluated. The core critique can be summarised in the words of Barrow, when claiming that Descartes had provided 'the outer shell without the nucleus', that is, that Descartes' view just scratched the surface of nature but could not reach to the

⁹⁷ Bacon, *The Oxford Francis Bacon*, XI:329; Wang, 'Francis Bacon and Magnetical Cosmology', 718.

⁹⁸ Hooke, *An Attempt to Prove the Motion of the Earth*, 27–28.

principles of activity which were seen by English philosophers as central in explaining the natural world. In Barrow's view, this omission was enough to abandon Cartesian philosophy. However, other English fellows kept the shell, so to speak, but incorporated nucleuses coming from their own background. The tradition of the magnetical cosmology provided a fine example of this. The *Principia Philosophiæ* brought the idea that bodies remained in their state if undisturbed and that the minimum motion was rectilinear. However, in the eyes of most English philosophers, the consideration of motion was not limited to the *translatio* depending on a relative frame of reference (the shell) but, as Hooke said, it should be seen as alteration: as the result of powers or forces interacting in nature (the nucleus). The actions of these powers were uncovered and clarified by experiments and their motions could be explained mathematically by laws of motion. In this way, 'laws of nature' were deprived of their causal role in natural philosophy, which was now attributed to powers and forces; however, the content of these laws was reworked in the mathematical practices of mechanics.

6. *Mechanica sive motu geometria*: Wallis and Barrow

Any magnitude (among which I esteem even a Point ...) is moveable, that is, in what Manner soever we behold it, the same may be made to change its Place continually, according to the prescribed differences, viz. with a strait or circular Motion; equally swift or in any matter more accelerated, or retarded: I say, *Mathematicians* assume at pleasure any Motions of this kind as evidently possible, in order to find out and demonstrate what follows from thence.

Barrow, *Geometrical lectures*.¹

What need is there [in geometry] for the concepts of body or motion, since the concept of a line can be understood without them? (...) [This] is plainly physical, nor is it in any way connected with it, so to what purpose was any mention to be made of it in this mathematical definition of yours?

Wallis, *Elenchus Geometriæ Hobbianæ*.²

English natural philosophers criticised Descartes' reductionist approach and, in so doing, rejected Descartes' gambit to endow natural philosophy with the certainty of mathematics. Boyle, who appreciated mathematics, dismissed the idea of a natural philosophy governed by mathematics for considering it too selective, not to mention that mathematics ran at odds with the non-idealised approach championed by the experimental philosophy; mathematics was not an appropriate language for gentlemanly discussions on philosophical matters.³ In this light, while English natural philosophers incorporated the contents of Cartesian laws into their endeavours, the emphasis upon experiments and the probabilistic attitude towards knowledge discarded laws as *a priori* principles. Thus, 'laws of nature' were deprived of the causal role that Descartes' had attributed them and, in their place, English natural philosophers put the activity in matter, such as gravity, magnetism and fermentation whose existence could be postulated initially without stipulating any mechanism of action. The contents of 'laws of nature' were as well reworked as mathematical laws of motion in mechanics.

Although the experimental approach informed the developments in natural philosophy, the profound transformations in mathematics were also pivotal in

¹ Barrow, *Geometrical Lectures*, 9.

² Wallis, *Elenchus Geometriæ Hobbianæ*, 7, 10.

³ Shapin, 'Robert Boyle and Mathematics: Reality, Representation, and Experimental Practice'; Henry, 'Boyle and Cosmical Qualities'; Anstey, *The Philosophy of Robert Boyle*; Malet, 'Between Mathematics and Experimental Philosophy'.

reordering the disciplinary boundaries in the second half of the seventeenth century. The invention of the method of indivisibles, the emergence of algebra and the ‘new analysis’, the discovery of new curves, the growing importance of the infinite series and calculus cannot be isolated from the transformations in astronomy and mechanics.⁴ Problems such as the centres of gravity, the behaviour of bodies at the beginning of motion and the definition of instantaneous or mean velocity occurred in conjunction with these mathematical achievements.⁵ Furthermore, given the growing application of mechanical resources to astronomical problems, the boundaries and connections among experimental practices, mathematics, astronomy and mechanics were open-ended.

This chapter focuses on Wallis’ and Barrow’s ‘physicalisation’ of mathematical language, as a redrawing of disciplinary boundaries. I will highlight how mathematical language became aligned with the natural world and how this helps to understand the transformation of laws explaining motion, now relocated to mechanics, in the second half of the seventeenth century in England. Wallis’ *Mechanica* provides a respectable state of the art of mechanics at the time of its printing.⁶ He considered that the introduction of physical notions—such as motion—into mathematics was adequate only in mechanics, that is, in the geometry of motion as an application of the study of continuous magnitude in a particular domain. Physical notions were inadequate and unnecessary in (pure) geometry, not to mention arithmetic and algebra. In Wallis’ view, geometry was founded upon arithmetic so that even geometric results should be achieved by arithmetic calculation. Against Wallis, Barrow claimed that it is not the abstract nature of number but the ‘conditions of matter’ to which numbers were applied that ultimately validated the result of any mathematical procedure. In his *Lectiones*, Barrow put forward a long-scale reform of mathematics and developed a geometry of motion, in connection with other aspects of his intellectual interests.⁷ In this reform,

⁴ Cf. Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*; Malet, *From Indivisibles to Infinitesimals*; Jesseph, ‘The “Merely Mechanical” vs the “Scab of Symbols”’.

⁵ Bertoloni Meli, *Thinking with Objects*, 9; Malet, *From Indivisibles to Infinitesimals*.

⁶ Malet, ‘Between Mathematics and Experimental Philosophy’, 175.

⁷ Grafton, ‘Barrow as a Scholar’; Levitin, *Ancient Wisdom in the Age of the New Science*, 313–25.

mathematics was concerned with the generation of magnitudes and therefore geometry (of motion) became the model of knowledge. Because geometry primarily dealt with motion, the remaining mathematical sciences were but particular domains of geometry; natural philosophy was ‘coextensive’ with mathematics and mixed-mathematical sciences became branches of physics. The place of motion in mathematics implied different conceptions of what was explained in the laws of motion.

6.1. Wallis’ laws of motion as principles of mechanics

In November 1668, Wallis wrote to Oldenburg disclosing his ‘principles for determining motions’ at the request of the Royal Society, inaugurating the debate on the collision of bodies.⁸ Wallis started off by mentioning that he had sent already two papers to the Royal Society, both founded upon ‘the general principles of motion’. In the first, he presented the reason why ‘a man can raise at least a hundred pounds by blowing up a bladder with his breath’; in the second, he explained ‘several phenomena in the Torrcellian experiment upon the principles of hydrostatics’.⁹ Wallis explicitly located the discussion on the laws of motion within mechanics, rather than in natural philosophy or cosmology: the use of devices (in this case a bladder) in which ‘a great weight can be moved by a small force’ and hydrostatics.¹⁰ Wallis’ 1668 paper dealt with the collision of inelastic bodies, while Huygens and Wren studied the more complex situations of elastic bodies. Wallis assumed that the bodies were inelastic, equating elasticity with less than perfect hardness.¹¹ Consistent with his approach, Wallis’ model for collisions derived from the law of the lever, assuming that the products of forces and velocities were the same at their end, rather than producing any acceleration. The core of Wallis’ approach is stated in rule 8:

And this is the foundation of all machines for facilitating motion. For in whatever *ratio* the weight is increased, the speed is diminished in the same *ratio*; whence it is that the product of weight and the speed of a moving force is the same. Thus

⁸ Oldenburg, *Correspondence*, 1968, 5:164–67.

⁹ Oldenburg, 5:165.

¹⁰ Aristotle, ‘Mechanical Problems’, 330–32.

¹¹ Hall, ‘Mechanics and the Royal Society’, 29.

$$V:PC = V:mP \cdot \frac{c}{m} = PC^{12}$$

Given that impetus was the product of ‘weight’ and ‘speed’, Wallis proceeded to calculate the impetus of each body assuming that that other was at rest and then to sum algebraically the two impetuses derived for each body. For example, if the initial impetuses were equal there was no motion after the collision.¹³ However, if there is any difference, rule 8 makes possible to calculate forces algebraically, something that Wallis specified in rules 9, 10 and 11.

Wallis’ laws received critical comments by two anonymous fellows, although Huygens’ and Wren’s enquiries on elastic bodies attracted the most attention and praise.¹⁴ One reaction against Wallis’ terms came from William Neile (1637-1670), a young and talented mathematician friend of Wallis.¹⁵ Neile’s discomfort with the question appeared in a letter to Henry Oldenburg. Neile asked for ‘the nature of motion and the nature of quiet’; the knowledge of the outcomes of collisions ‘is good for use but it is not science or philosophye’.¹⁶ When Wallis’ paper was read at the Society in November 1668, Neile wrote to Oldenburg four questions addressed to Wallis. Neile asked: (1) Whether quiescent [resting] matter has any resistance to motion. (2) Whether motion may pass out of one subject into another. (3) Whether ‘no Motion in the World perish’, nor new motion be generated. (4) Whether different motions meeting, destroy one another.¹⁷ Wallis replied to Oldenburg in December 1668 to every question, without knowing who formulated them. To the first query, Wallis replied that it was taken for granted ‘by most of our moderns’ that matter was indifferent ‘as to rest or motion’ and also to any ‘direction of motion’, summarising the first two Cartesian laws. The reply to the second contrasts the experimental approach with the Scholastic philosophy. Wallis said that if the question concerned the issue of ‘migratio accidentis’, ‘it is onely to dispute of words’. All he could say

¹² Oldenburg, *Correspondence*, 1968, 5:165.

¹³ Hall, ‘Mechanics and the Royal Society’, 30.

¹⁴ Hall, ‘Mechanics and the Royal Society’; Murray, Harper, and Wilson, ‘Huygens, Wren, Wallis, and Newton on Rules of Impact and Reflection’.

¹⁵ A recent study has rescued Neil from oblivion: Kemeny, ‘What Motion Is’. See also Jalobeanu, ‘The Cartesians of the Royal Society’.

¹⁶ Oldenburg, *Correspondence*, 1968, 5:518.

¹⁷ Oldenburg, 5:220–21.

was that ‘ye Force or impetus whereby one body is moved, may, by percussion, cause another body against which it strikes, to be put into motion: &, withall, loose somewhat of its own strength or swiftness; experience tells us clearly inought yt it is so: And, in what proportion this is, my late Hypothesis teacheth’. Wallis disregarded the question for the transference and nature of motion that had been at the centre of the correspondence between More and Descartes by the late 1640s.¹⁸ His approach displays some distinctive features of the experimental philosophy: ‘it is inough yt it is so’ from experience, that the effects of the collision of bodies were clear enough from observations. Wallis did not commit himself to any precise terminology: bodies were moved by ‘force or impetus’ and once one body stroke the other, there was some change in their ‘strength or swiftness’. We observe this in experience. On the other hand, Wallis’ laws considered as ‘hypothesis’, ‘teacheth’ the proportion of these changes. Therefore, Wallis distinguished what could be gathered from experience from what could be explained by geometry (‘proportion’). Concerning the third and fourth questions, Wallis refused to provide a clear answer, arguing that ‘needs distinguishing also’. In a further communication, Wallis argued that when two bodies in motion mutually stopped each other (‘as contrary forces’), ‘ye motion of both is thereby extinguished & both remain at rest’. Concerning Neile’s replies on this point, Wallis commented that ‘whether this motion shall be sayd to be lost, or to be onely virtually preserved, is as men shall please to call it’, avoiding again to be carried into the Scholastic arena. Neile refused to accept these conclusions, because in his view motion and rest were opposed and different motions could not be mixed in one body.¹⁹

At the end of the questionnaire, Wallis made a general comment on these questions. Oldenburg seemed to have asked Wallis (according to Wallis’ quotation in the letter) that ‘*y^e Society in their present disquisitions have rather an Eye to y^e Physical causes of Motion, & the Principles thereof, than y^e Mathematical Rules of it*’. Wallis’ clarified that, according to his ‘doctrine of motion’, ‘the Hypothesis I sent, is indeed

¹⁸ Webster, ‘Henry More and Descartes: Some New Sources’; Leech, *The Hammer of the Cartesians : Henry More’s Philosophy of Spirit and the Origins of Modern Atheism*.

¹⁹ Oldenburg, *Correspondence*, 1968, 5:274; Kemeny, ‘What Motion Is’, 184; See also Hall, ‘Mechanics and the Royal Society’, 37.

of ye *Physical* Laws of Motion, but *Mathematically* demonstrated’ because ‘what is Physically performed, is Mathematically measured’. Thus ‘there is no other way to determine y^e Physical Laws of Motion exactly, but by applying y^e Mathematical measures & proportions to them’.²⁰ This final remark highlights the difference concerning disciplinary boundaries, between what constitutes a proper explanation of the phenomena at hand. Wallis felt that the questions came from a different shore and then addressed to the root of the problem. Laws of motion were mathematical measurements or the proportions accounting for physical phenomena. Mathematics and physics were complementary: ‘what is Physically performed, is Mathematically measured’. Because the question of the (physical) laws of motion belongs to mechanics, ‘there is no other way’ to determine them. Wallis’ laws were neither principles of an *a priori* physics, nor emanations of divine immutability operating as causes.²¹ Wallis’ paper also reflects that mechanics was no longer restricted to the study of the ‘violent’ motions or machines; it was extended now to topics that once belonged to natural philosophy, such as gravity or the principles of motion but from a different perspective.

6.2. Geometria Motu, sive Mechanica

Wallis’ *Mechanica, sive de motu. Tractatus Geometricus* was published between 1669 and 1671 in three voluminous books dealing with the algebraic solution of geometrical problems. The book is usually judged by its title, suggesting that it expanded significantly the scope of mechanics by including the study of natural motions. However, Wallis’ *Mechanica* barely dealt with any subject that his contemporaries would have recognised as mechanical such as the application of some demonstrations to astronomy or the study on gravity. Wallis’ greatest achievement was to systematise most of what was already known in mechanics that had been developed in loosely connected practices.²²

²⁰ Oldenburg, *Correspondence*, 1968, 5:221.

²¹ Cf. Richter, “‘Nature Doth Not Work by Election” John Wallis, Robert Grosseteste, and the Mathematical Laws of Nature’.

²² Scott, *The Mathematical Work of John Wallis*, 93.

Following Euclid's demonstrative way in the *Elements*, Wallis opened his *Mechanica* with definitions. The first definition established the nature of mechanics: 'I call *Mechanics* the geometry of motion'. Wallis' definition was historical and linguistic, and this argumentative style is prominent in the initial twenty-two definitions.²³ Wallis explored two uses of the term mechanics that he eventually rejected. In the first, mechanics referred to 'mechanical arts' as 'illiberal arts', that is to works concerning *labor* rather than *ingenium*. However, mechanics was not only differentiated from geometry but from the liberal arts requiring *ingenio*, and in consequence practised by the *liberi* in contrast with the illiberal arts relying on 'the hand' and accomplished by *servi*. The second definition was commonly found in geometry for non-constructive geometric solutions. 'But we do not say *Mechanics* in any of those meanings'—Wallis follows—'but we understand it as the part of geometry which considers motion and investigates by means of reasons and demonstrations the force by which any motion is effected'.²⁴ In this definition, Wallis considered mechanics as the branch of mathematics dealing with some properties of continuous quantity and therefore it is applied geometry. Its method of demonstration is axiomatic, hence the style of the book.

The subsequent definitions stated the main aspects of mechanics *tanquam* geometry of motion: its main subject was local motion. Wallis clarified that he did 'not wish to investigate here' other motions, such as generation, augmentation, alteration; he was only concerned with what Greeks called *φωρὰ* or *latio* in Latin. Because mechanics dealt with local motion, it entailed the quantitative consideration of force (*vis*), time, resistance, longitude, momentum, impedimentum, speed, gravity, weight.²⁵ Definitions 2 to 21 explained these terms. This set of definitions concluded with that of machines as 'the instruments to examine or to facilitate motion, applied from the outside'.²⁶ Force, in general, was 'the power that produces motion' (*potentiam efficiendi motum*) and resistance 'the power that is contrary to motion or resist

²³ See Beeley, 'Physical Arguments and Moral Inducements: John Wallis on Questions of Antiquarianism and Natural Philosophy'.

²⁴ Wallis, *Opera Mathematica*, 1:575.

²⁵ Wallis, 1:575.

²⁶ Wallis, 1:579.

motion' (*potentiam motui contrariam*). Force, together with time, were related with 'momentum', as generators of motion. Meanwhile, resistance and distance were associated with 'impedimentum', as preventing motion. In so doing, Wallis stated that motion and resistance to motion were outcomes of forces or powers. Meanwhile, this conception of force made possible the algebraic operation with them, as additive quantities. The central point here is that Wallis provided a definition of the cause of motion in an operative, quantitative way. Gravity as force appeared as a case of a broader class.

Wallis further studied gravity in a short experimental 'Discourse' requested by the Royal Society in 1674.²⁷ There, he defined gravity as '*vix motrix* downwards, that is, towards the Centre of the Earth'. In the discourse, Wallis explained the terms associated with this downwards motion:

This Motion downward, we call *Descent*; the Endeavour so to move, we call *Gravitation*; and the Principle from whence this Endeavour proceeds, we call *Gravity*. And things are said to be more or less Heavy, as they have more or less of Gravity: Which may be understood, either *Extensively*, according to the Quantity of it; as when we say a Pound is heavier than an Ounce, though that be Feathers, and this be Lead: Or *Intensively*, according to the Degree; as when we say, that Lead is heavier than Cork, or Quick-silver than Water; that is, gradually heavier, proportionably heavier (bulk for bulk) or (as it is now wont to be called) *Specifically* heavier.²⁸

The precision in the vocabulary sheds light on the conception of mechanics at work. In the 'Discourse', Wallis denominated gravity the principle of gravitation, while in the *Mechanica* he designated it as a motive force (*vis motrix, deorsum*), as the cause of a specific kind of motion, descent. However, the explanation of the definition is neither historical nor linguistic, but experimental. Both in *Mechanica* and in the 'Discourse', Wallis explained that the 'physical consideration' of the 'principle of gravitation' was not 'investigated here'. It was not of interest for mechanics whether gravity was a 'quality' or an 'affection of the body' or any other 'name' by which it was called. Neither was it of importance whether it was an innate quality making

²⁷ Wallis, *A Discourse of Gravity and Gravitation*,.

²⁸ Wallis, 2.

bodies heavy or a common inclination (*vergentia*) towards the centre; or an electric or magnetic capacity (*facultas*) of the Earth enticing (*alliciat*) bodies; or if an effluvium of the Earth attracts (*attrahat*) them. ‘It is enough that by the name of gravity we understand the meaning we gave before, a force of moving downwards (*vim deorsum movendi*)’.²⁹

Unlike Euclid’s *Elements* and similar to Guidobaldo, Wallis’ *Mechanica* did not contain postulates or axioms. However, two sets of propositions inserted immediately after the definitions perform a similar function: the first group concerns the numerical or algebraic theory of *ratios* preparing the mathematical groundwork (1-9). The second group brings back the laws of motion circulated in 1668 and other general principles connecting different aspects of motion in terms of the numerical theory of *ratio* (10-30). Proposition 11, for example, established that

if the momentum is greater than the resistance, motion will result. If there was none before, motion will begin; if it already existed, it will be increased. If the impedimentum is greater, it will oppose the motion if there was any, or at least check it. If the two are equal, motion will be neither started nor stopped, and the initial state of the body, either of rest or of motion, will persist.³⁰

This proposition and those dealing with the laws of motion, present similar results to those of the 1668 paper. However, there are two significant changes. First, Wallis formulated the proportions on the collision of bodies in a more general way, as a general theory of motion, by using the terminology he has just set out. Second, these propositions are presented within the framework of the theory of *ratios*. Proposition 12 moves from momentum to forces, and states that

A force will neutralise an equal and opposite force; if it is less, it will not even do that; if it is greater, it will move. And the opposite: if it moves, the force is greater; if it does not move, then or the force is less, or they are at least equivalent or there is some impediment.

$$\begin{array}{lll} +A - A = 0 & + 2A - 3A = -A & + 3A - 2A = +A \\ +S - D = 0 & + S - D = - & + S - D = + \end{array} \quad ^{31}$$

²⁹ Wallis, *Opera Mathematica*, 1:576.

³⁰ Wallis, 1:585.

³¹ Wallis, 1:586.

This approach to forces and motion related mathematics to physical motions. Some scholars have seen in it the reason why Newton did not claim any originality of his laws, not even of the third that seems to be implied here.³² But the Proposition 12 was an example of the general statement that Wallis had established in Proposition 7, claiming that ‘Effects are proportional to their adequate causes’. In the Scholium to this Proposition, Wallis noted that ‘this universal proposition should be set out at the beginning, since it opens the way by which, from purely mathematical speculation, one may move on to physical [speculation], or rather that the one is connected to the other’.³³ This proposition made it possible to infer from observable effects the proportion of the—not always observable—causes and from the proportion of the causes the expected and necessary effects. Wallis’ mathematical approach was arithmetical and algebraic, rather than geometric and representational as may be assumed from the title. The reasons behind this lies in Wallis’ view on the nature of mathematics. This was presented in detail in the *Mathesis Universalis*, but the Propositions 1-9 of the *Mechanica*, summarised the theory of *ratios* in order to build mechanics on it.³⁴

The theory of *ratios* appeared in Euclid’s *Elements* in order to make it possible to compare magnitudes of the *same kind*.³⁵ A ‘*ratio* is a sort of relation in respect of size between two magnitudes of the same kind’.³⁶ In this sense, magnitudes ‘*have a ratio* to one another which are capable, when multiplied, of exceeding one another’. For example, we can say that one angle is in proportion to another depending on their size. This *ratio* is normally expressed as a numerical relation (5:3, 2:1, for example). So far, so good. However, definitions 5 and 6 allow the comparison between *ratios*

³² ‘It becomes more and more evident as we read through the pages of these [Wallis’] letters that when Newton enunciated the famous laws which bear his name he was merely giving utterance to what had been in Wallis’s mind for many years’ Scott, *The Mathematical Work of John Wallis*, 105. See. Section 7.1

³³ Wallis, *Opera Mathematica*, 1:584.

³⁴ My view on Wallis’ algebraic theory of ratio is based on Jesseph, ‘Ratios, Quotients, and the Language of Nature’; Neal, *From Discrete to Continuous*, 151–56.

³⁵ Heath, *A History of Greek Mathematics. Volume 1: From Thales to Euclid*, 84–90; Klein, *Greek Mathematical Thought and the Origin of Algebra*; Grattan-Guinness, ‘Numbers, Magnitudes, Ratios, and Proportions in Euclid’s *Elements*. How Did He Handle Them?’; Thorup, ‘A Pre-Euclidean Theory of Proportions’; Drake, ‘Medieval Ratio Theory vs Compound Medicines in the Origins of Bradwardine’s Rule’; Jesseph, ‘Ratios, Quotients, and the Language of Nature’.

³⁶ Euclid, *The Thirteen Books of Euclid’s Elements*, 2:114.

of magnitudes of different kind. In other words, definitions 3 and 4 define a *ratio* as a relation between magnitudes of the *same* kind, but definition 5 says that

magnitudes are said to *be in the same ratio*, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

And definition 6: ‘Let magnitudes which have the same *ratio* be called *proportional*’.³⁷ The sameness of *ratio* in definition 5 is defined by the preservation of order relations under arbitrary equimultiples. For example, lines and angles are different kind of magnitudes and under definitions 3 and 4 we cannot establish any *ratio* between, say, a line and an angle. Nevertheless, if we stick to definitions 5 and 6 we can claim that two lines are *proportional* to two angles if they have the same *ratio*.

The puzzling definition 5 has led to different interpretations. Wallis, for example, followed what Jesseph has called the ‘numerical’ theory of *ratios*, as opposed to the ‘relational’ theory.³⁸ In the relational theory, *ratios* are not quantities but just relations between two quantities. In this sense, it is meaningless to claim that *ratios* are ‘greater than’ or ‘divisible by’; *ratios* are not magnitudes. Barrow championed this view. On the contrary, the numerical approach assumes that the comparability of *ratios* implied a general domain of magnitudes (‘*ratios*’) for which it was possible to determine their size—‘denomination’ or ‘exponent’. According to the numerical theory, *ratios* may be expressed as quotients, in the sense that the *ratio* a/b is proportional to c/d if $a * d = c * b$.³⁹ The problem here is that the quotients of *ratios* are the result of incommensurable magnitudes. Quotients were traditionally understood as fractions arising from the division of integers, in the sense that *quoties* means ‘how many’. This view of *ratios* as quotients implied the expansion of the traditional Greek notion of number to make ‘sense of expressions such as

³⁷ Euclid, 2:114.

³⁸ Jesseph, ‘Ratios, Quotients, and the Language of Nature’, 162.

³⁹ Wallis, *Opera Mathematica*, 1:184 as quoted in; Jesseph, ‘Ratios, Quotients, and the Language of Nature’, 172.

$\sqrt{7\pi}/\sqrt[3]{11}$ '.⁴⁰ The numerical theory required all *ratios* to be homogeneous in order to make a direct comparison with one another, leading to the difficult question of the status of irrationals.

The idea that *ratios* are magnitudes and susceptible of direct comparison was rooted in Wallis' views on the priority of arithmetic over geometry. Wallis followed the Aristotelian idea according to which arithmetic is more abstract and universal than geometry, for unlike points, units do not imply position. Thus, units are simpler than points.⁴¹ The assertion that arithmetic was more general than geometry is explained by the view that 'arithmetical truths are of a higher and more abstract nature than those of geometry. For example, it is not because *a two foot line added to a two foot line makes a four foot line* that *two and two are four*, but rather because the latter is true, the former follows'.⁴² Arithmetical principles were seen as special cases of principles of algebra, the so-called 'arithmetic of species'.⁴³ In Wallis' view, the foundation of mathematics is a *mathesis universalis*, conceived as a pure science of quantity completely abstracted from matter. Geometry deals with continuous quantity, while arithmetic is concerned with discrete quantity or number. 'But of these one is indeed more and the other less pure': the subject of arithmetic is 'purer and more universal' and their 'speculations' are applicable 'to geometry'.⁴⁴ Meanwhile geometry depended on the principles of arithmetic to perform its operations. In Wallis' view, 'someone asserts that a line of three feet added to a line of two feet makes a line five feet long, he asserts this because the numbers two and three added together make five; yet this calculation is not therefore geometrical, but clearly arithmetical, although it is used in geometric measurement'. The equality of the number five with two and three 'is a general assertion' and it may be applied to any object, geometrical or even to angels, 'for also two angels and three angels make five angels'.⁴⁵

⁴⁰ Jessephe, 'Ratios, Quotients, and the Language of Nature', 164.

⁴¹ Neal, *From Discrete to Continuous*, 153.

⁴² Wallis, *Opera Mathematica*, 1:53.

⁴³ Jessephe, 'The "Merely Mechanical" vs the "Scab of Symbols"'.
⁴⁴ Wallis, *Opera Mathematica*, 1:18.

⁴⁵ Wallis, 1:56; Cf. Jessephe, 'Ratios, Quotients, and the Language of Nature', 166.

The priority of arithmetic over geometry informs Wallis' conception of mechanics as geometry of motion. In Wallis' view, mechanics fell within the boundaries of the distinction between pure and mixed mathematics. In this sense, mechanics was the application of geometrical—and ultimately, arithmetical—principles to the study of motion. In other words, it was a mathematical study of the proportions of motions and forces and thus mechanics does not take the place of natural philosophy. On the contrary, in mechanics, Wallis' assessment of the superiority of the abstract character of arithmetic over geometry was underpinned by the recurrent refusal to deal with physical causes coming from his experimental approach to nature. In other words, Wallis' rejection of dealing with the cause of physical phenomena in mechanics depended on his experimental commitments and on his view of mechanics as a mathematical science, rather than as a part of physics. As he replied to Neile's objections, 'what is Physically performed, is Mathematically measured'.⁴⁶

6.3. Barrow's reply to Wallis: magnitude and motion

Barrow's conception of mathematics is one of a piece with his criticism of Descartes and it is shaped by his views on authority, the ancients and the importance of the textual traditions.⁴⁷ Barrow's rejection of Cartesianism, and particularly of laws of nature, stemmed from rejection of the (metaphysical) framework that validated the laws, the cosmology of the *Principia Philosophiæ* and their methods. Barrow read in Descartes an insufficient account of the activity of nature and, therefore, a misleading view of providence. The tone of Barrow's criticism is part of an anti-metaphysical stance informing Barrow's views on principles and definitions of mathematics that departed from the Euclidian tradition by considering them, not as self-evident and necessary, but as acceptable or true hypotheses, provided that they are not inconsistent or self-contradictory.⁴⁸

⁴⁶ Oldenburg, *Correspondence*, 1968, 5:221.

⁴⁷ Mahoney, 'Barrow's Mathematics: Between Ancients and Moderns'; Stewart, "Fleshy Books"; Grafton, 'Barrow as a Scholar'; Malet, 'Isaac Barrow on the Mathematization of Nature'; Malet, 'Mechanics in the 17th Century'; Levitin, *Ancient Wisdom in the Age of the New Science*; Kargon, 'Newton, Barrow and the Hypothetical Physics'.

⁴⁸ Malet, 'Mechanics in the 17th Century', 52; 54.

Although Barrow never elaborated a natural philosophy nor a mechanics, his answer to Descartes was a reform of mathematics based on geometry. However, it would be a mistake to consider this reform as a departure from classical traditions, for Barrow's judgment tends to favour the ancients, although his practice clearly moved off from Greek mathematics.⁴⁹ For example, when claiming that arithmetic was subordinate to geometry, he remarked that his intention was to 'restore it into its legitimate place, as being removed out of its proper Seat, and insert and unite it again into its native geometry'.⁵⁰ This reform was sketched out in his *Lectiones Mathematicæ* delivered between 1663 and 1666 as first Lucasian professor of mathematics at the University of Cambridge; subsequent developments appeared in the *Lectiones Geometriæ* and in the *Lectiones Opticæ*. Barrow's reform provided a new understanding of mathematics as a language aligned with nature, capable of accounting for change in causal and quantitative terms.⁵¹

The first three of Barrow's *Lectiones Mathematicæ* are devoted to the nature and division of mathematical sciences. The first lecture presents an overview of the conceptions of mathematics based on philological reflections on the name and then on a historical review of the divisions of mathematics in Greek and Roman. The name initially just meant 'sciences acquired by discipline' without any determination of its object so he moved on to investigate what the object of mathematics was. The answer, again, came from the Greeks and Romans. In the same discursive style, Barrow remarked that 'whatsoever becomes of the general object, it is plain the *Mathematics* is conversant about two things especially, *quantity* strictly taken and *quotity*, or, if you please, *magnitude* and *multitude*. By others, they are called *continued* and *discontinued quantity*'.⁵² Magnitude and number could be considered either as abstracted from matter, circumstances and accidents or as they 'inhere in some particular subject', from which Plato, for example, affirmed that some

⁴⁹ Mahoney, 'Barrow's Mathematics: Between Ancients and Moderns'; Guicciardini, *Newton on Mathematical Certainty*, 169–79; Stewart, "'Fleshy Books'"; Kargon, 'Newton, Barrow and the Hypothetical Physics'.

⁵⁰ Barrow, *The Mathematical Lectures*, 29; Barrow, *Mathematical Works*, 46.

⁵¹ Panza, 'Isaac Barrow and the Bounds of Geometry'; Malet, *From Indivisibles to Infinitesimals*; Malet, 'Mechanics in the 17th Century'; Mahoney, 'Barrow's Mathematics: Between Ancients and Moderns'; Guicciardini, *Reading the Principia*; Guicciardini, *Newton on Mathematical Certainty*.

⁵² Barrow, *The Mathematical Lectures*, 10.

disciplines were ‘pure and prime’ (arithmetic and geometry) and other disciplines were ‘impure, secondary and even less exact’. Because these last groups contemplated magnitude and number ‘as applied to certain bodies and particular subjects’, Aristotle called them physical and sensible, and others physico-mathematical.

Barrow concluded that there were two main branches of mathematics: ‘pure and primary parts’ conformed by geometry and arithmetic and ‘four mixed and subaltern’, optics, mechanics, music and astronomy with further ramifications.⁵³ The point here is that there seems to be a *fundamental* distinction between two irreducible kinds of pure mathematics generating a diversity of fields depending on their applications to sensible objects. Nevertheless, Barrow put forward that *all* sciences could be reduced to geometry, because all of them have the same subject, magnitude, whose most general properties were investigated by geometry. Against Wallis and the modern analysis, Barrow dedicated the third lecture to explaining that arithmetic was to be included in geometry, for its calculative capacity derived from the nature of magnitude rather than from the abstract nature of number. His claim was that ‘there is really no Quantity in Nature different from what is called *Magnitude* or *continuous quantity*’ and therefore ‘this alone ought to be accounted the object of Mathematics; whose most general Properties it enquires into and demonstrates’.⁵⁴ Put otherwise, the identification of quantity with continuous magnitude rather than with number, entailed the primacy of geometry over all other sciences. Moreover, Barrow’s approach to magnitude was empirical rather than essentialist. Instead of providing a definition based on one fundamental attribute of magnitude, Barrow appealed to experience to claim that the subject of mathematics was not different from the quantities that we find in nature. In the ninth lecture, Barrow put forward some ideas concerning termination (limits), extension, composition and divisibility of magnitude implied in his geometry of motion. The main targets were Descartes and Hobbes, for they provided paradoxical definitions based on metaphysical assumptions about space. Barrow brought up the famous words of St. Agustin

⁵³ Barrow, 20; Barrow, *Mathematical Works*, 39–40.

⁵⁴ Barrow, *The Mathematical Lectures*, 20; Barrow, *Mathematical Works*, 39–40.

concerning time: ‘if none seek of me what Time is, I know; if any ask me I don’t know’ and extends it to extension, space and motion.⁵⁵ The bottom line was a rejection of a metaphysical definition of these notions. Rather than claiming that he found the essence of bodies, such as Descartes and Hobbes, and that from there he derived ‘the common affections and properties of magnitude’, he opted for a definition ‘more accommodate to common Sense than to metaphysical conceptions; and so far only as I judge useful to my Purpose, i.e. as they are subservient to *Mathematical Hypothesis*’.⁵⁶

The anti-metaphysical approach to magnitude allowed the reduction of arithmetic to geometry and aligned all other sciences with mathematics. The so-called mixed mathematical subjects were in fact natural sciences or ‘branches of Physics’ (*Physicæ partes*). In Barrow’s view, the reverse thesis was also valid, for physical sciences were no less entitled to be called mathematical than the traditional mixed mathematics.⁵⁷ In his view, ‘there is no Branch of natural Science that may not arrogate the Title to itself; since there is really none, from which the Consideration of Quantity is wholly excluded, and consequently to which some Light or Assistance may not be fetched from Geometry’. The reason was that ‘Magnitude is the common Affection of all physical Things, it is interwoven in the Nature of Bodies [*corporum naturae penitus illigata*], blended with all corporeal Accidents, and well nigh bears the principal Part in the Production of every natural Effect’. Because of these, ‘All Bodies obtain their own Figures, and execute their own local Motions; by which means, if not all, yet the most and chieftest Effects (whatsoever admits of a philosophical Explication) are performed, for the determining and comparing of which the Theorems of Geometry do often conduce’.⁵⁸ Therefore, geometrical theorems could account for the main cause of natural effects, that is, local motion. In this sense, physics was largely a branch of geometry. But Barrow’s position developed other ramifications: he agreed that different sciences have varying degrees of mathematisation, depending on the nature of their principles.

⁵⁵ Barrow, *The Mathematical Lectures*, 139–40.

⁵⁶ Barrow, 141.

⁵⁷ Barrow, *Mathematical Works*, 40.

⁵⁸ Barrow, *The Mathematical Lectures*, 21; Barrow, *Mathematical Works*, 40.

Barrow was aware that his view challenged Wallis' approach, particularly the definition of number and the subsequent classification of sciences. Nevertheless, from a more general perspective, Barrow was not only opposing to Wallis' specific developments but to modern tendencies of the new analysis in which mathematics dealt with relations of magnitudes rather than with magnitudes and, in consequence, these approaches favoured the abstract nature of number and the operations with them as performed in algebra.⁵⁹ In Barrow's conception, there was no need to formulate the separate existence of numbers and things counted. Numbers were symbols representing collections of units whose effectiveness depended on 'the condition of things they are attributed to', that is, the units. Put otherwise, numbers were not entities separable from the magnitude to which they were applied, for their operative capacity depended on the matter. In geometry, the connection between numbers and units was evident, because geometry was concerned with measurement. However, Barrow claimed that the use of numbers in arithmetical calculation implied the homogeneity of magnitude: 'If the things which the numbers are brought to denote be homogeneous, of the same name, and when compared have a mutual proportion to one another, and consequently the one can be increased by the addition or diminished by the subtraction of other; then they impart the same attributes, proportions, and increments or decrements to the numbers by which they are denominated.' Otherwise, operations would be impossible, given the incommensurability of units. Barrow did attribute any abstract property to numbers.⁶⁰ 'Number (at least that treated of by mathematicians) differs nothing from *continued magnitude* itself, nor seems to have any other property (composition, division, proportion and the like) than either from, or in respect to it, as it represents or supplies its place'. Because of this, Barrow reminded his students that 'the writer of the *Elements*' was correct in including arithmetical speculations in his work, 'assigning to arithmetic a suitable place in geometry'.

⁵⁹ Mahoney, 'Barrow's Mathematics: Between Ancients and Moderns', 185–91.

⁶⁰ Barrow, *The Mathematical Lectures*, 36–37; Barrow, *Mathematical Works*, 52–53; Cf. Mahoney, 'Barrow's Mathematics: Between Ancients and Moderns', 185–87.

6.4. Geometria Motu, sive Mathematica

Instead of assuming magnitudes as given, Barrow's mathematician was concerned with 'the several ways whereby the various species of magnitudes may be conceived to be generated or produced'.⁶¹ If geometry dealt with the generation, then it was possible to provide certain and causal demonstrations. These demonstrations were certain because they were rooted in principles proved by 'induction or multiple experiences';⁶² and they were causal, because they provide the generative process, insofar as geometry determined the precise conditions of motion generating a magnitude.⁶³

In the *Lectiones geometricæ*, Barrow explained that local motion was the 'primary and chief' way of generating magnitudes. While philosophers 'argue with great subtilty on the nature of motion', for the mathematician it was enough taking 'for granted what is allow'd by common sense, and proved by obvious experience, that is, that any magnitude is moveable; that is, in whatsoever manner we behold it, the same may be made to change its place continually, viz. with a straight or circular motion; equally swift, or in any manner accelerated or retarded'.⁶⁴ In so doing, Barrow aligned the fundamental concepts of mathematics with experience, in the sense that it was enough for the mathematician to rely on experience to understand the definitions of magnitude and motion. In Barrow's view, who would deny that objects of our experience change place in 'straight' or 'circular' motion and that they do it with different speeds? Although the understanding of basic notions derived from experience, geometrical investigation was not limited to experience, for the mathematician 'assume[s] at pleasure any motions of this kind as evidently possible, in order to find out and demonstrate what follows from thence'.⁶⁵

Barrow's geometry applied the terminology of local motion to geometrical objects, explaining in this way their origin as 'generation' in terms of 'differences of motion', particularly direction and speed. For solving any geometrical problem, it was enough

⁶¹ Barrow, *Geometrical Lectures*, 2; Barrow, *Mathematical Works*, 159.

⁶² *Mathematical Works*, 78.

⁶³ 172.

⁶⁴ Barrow, *Geometrical Lectures*, 2–3; Barrow, *Mathematical Works*, 160–61.

⁶⁵ Barrow, *Mathematical Works*, 159–60.

to know: (1) *ipse modus lationes* and (2) *quantitatis vis motivæ*. The *mode of 'lation'* or *manner of bearing* was the determination of the direction of motion. Meanwhile, the *quantity of motive force* determined the speed at which the motion was performed and it could be 'swifter, slower or equally swifter (accelerated)'. The direction and the speed of motion presuppose some notions of space and time. Indeed, the time is represented by a straight line symbolising its constant flow and direction by geometric figures. Barrow's demonstration of Galileo's motion uniformly accelerated illustrates this geometry of motion at work:

Let's suppose the triangle AEY, wherein the side AE denotes the time, and the parallel lines BY, CY, DY, EY applied to the points thereof, the several degrees of velocity, answering to every instant of time, equally increasing from point A, (representing rest, or the least degree of velocity) to a given degree represented by the greatest line EY, or decreasing back again from the said line EY, to the point A (...) The triangles ABY, ACY, ADY, AEY will represent the spaces moved through, from the beginning of the motion, in the respective times ⁶⁶

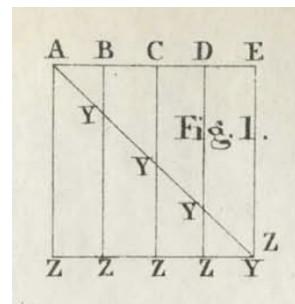


Figure 8. Barrow's representation of uniform and uniformly accelerated motion.

From these initial representations, Barrow expected to demonstrate all the remaining properties of this motion, following the synthetic method.⁶⁷ In the case of motion uniformly accelerated, 'that the spaces moved through by a motion uniformly accelerated from rest, are to one another, as the squares of the times (or in the duplicate *ratio* of the time)'.⁶⁸

This geometry founded upon magnitude as 'interwoven in the Nature of Bodies' entailed concepts of space and time. From the mathematical point of view, space and time were necessary to determine direction and speed—the components of motion. In his view, his approach was different from that of the philosophers, for he was concerned solely with the particular way of conceiving space and time proper to mathematicians.⁶⁹ However, given the physical implications of Barrow's conception

⁶⁶ Barrow, 171.

⁶⁷ Barrow, 45.

⁶⁸ Barrow, 171–72.

⁶⁹ 160.

of motion, space and time were not only operative definitions for abstract exercises; they had physical and metaphysical implications. Indeed, Barrow had rejected the Cartesian reduction of matter to extension (as we saw) and the subsequent elimination of space by reducing it to the place filled by matter. In its place, Barrow developed concepts of space and time that have not received considerable scholarly attention. The resemblance of his approach with Newton's has proved a hard topic for historians.⁷⁰

Time is connected to quantity of motive force and closely related to speed, for the determination of speed requires a referent. In Barrow's view, 'Time is the continuance of anything in its own being. But some things continue longer in their being than others; those *were* when these *were* not, and *are* when these *are* not ... Time absolutely therefore is a quantity, as admitting in some manner the chief affections of quantity, equality, inequality and proportion'.⁷¹ Time can be measured. Although this view apparently implies a relational conception, in which time is nothing in itself but just a relation of equality and inequality arising from the difference of 'continuance of being' of magnitudes,⁷² Barrow explained that space and time existed before 'the world was created'.⁷³ The argument is far from clear, but Barrow's point is that since 'there was space before the world was created, and that there now is an Extramundane, infinite space (where God is present)' in which there are also bodies that do not exist now, 'time existed before the world began, and does exist together with the World in the extramundane space'. The reasoning shows, in Barrow's view, that time does not depend on things for being and that, in fact, 'time does not imply an actual existence, but only the capacity or possibility of the continuance of existence'.⁷⁴ Put otherwise, space and time do not depend on the world and they constitute capacities or possibilities of existence (of 'continuance in being' and 'position') of things.⁷⁵ The argument is that it is possible to conceive that

⁷⁰ Grant, *Much Ado About Nothing*, 236–38; Thomas, 'Space and Time in Isaac Barrow: A Modal Relationist Metaphysic'.

⁷¹ Barrow, *Geometrical Lectures*, 4–5.

⁷² Thomas, 'Space and Time in Isaac Barrow: A Modal Relationist Metaphysic'.

⁷³ *Mathematical Works*, 161.

⁷⁴ Barrow, *Geometrical Lectures*, 5–6.

⁷⁵ The obscurity of the passage and of Barrow's reflection on this topic in general, lies in that he is not interested in providing an exhaustive definition of space and time, but he only needs to postulate their

something existed before the existence of the world (this world), and that thing may continue existing. Therefore, there is no necessary connection between the world and the existence of time. At the same time, the view that space and time were possibilities implied that they are prior to things and make them possible.

Barrow did not specify what kind of existence time has, but his point was to affirm the independent existence even if the knowledge of time was commonly associated to something else (time to motion and rest to bodies). ‘The quantity of time, in itself, depends not on either of them [motion or rest]; for whether things move on, or stand still; whether we sleep or wake, Time flows perpetually with an equal tenor’. Time itself is a magnitude that could be measured or determined: ‘But as magnitudes themselves are absolute *Quantums* independent on all Kinds of Measure, tho’ indeed we cannot tell what their Quantity is unless we measure them; so Time is likewise a *Quantum* in itself, tho’ in Order to find the Quantity of it, we are obliged to call in motion to our assistance’.⁷⁶ If time is independent of things moved, how can we measure and represent it? It was a mistake to think that time is measured by motion. On the contrary, the quantity of motion is determined by time. However, to ‘determine and show’ the quantity of time, we need something flowing constantly. Barrow found the answer in the divine will of the Creator ‘who pronounced as follows: *Let there be Lights in the Firmament of the Heaven, to divide the day from the night, and then let them be for Signs, and for seasons, and for days, and for years*’.⁷⁷ The quotation from Genesis was evidence indicating that celestial bodies, particularly the Sun and the Moon, were the adequate signs or units to measure time. The use of sundials as representation of the uniform motion of the Sun confirmed this. These serve as the standard to establish the precision of any other ‘time keeper’, such as mechanical clocks or even water or sand clocks. Thus, measurement of time

existence as universal referents of motion, in the sense that they are stable and independent of things. Although this may be a metaphysical view, Barrow does not develop it and he is only concerned with an attempt to show that the possibility of a philosophical view of space and time as derived from his mathematical definitions is, at least, non-paradoxical. But this does not amount to develop a ‘doctrine’ of space and time (as Newton did not do either).

⁷⁶ Barrow, *Geometrical Lectures*, 7.

⁷⁷ Barrow, *Mathematical Works*, 162; Barrow, *Geometrical Lectures*, 9–10.

and the determination of motion was nothing else but the typical geometrical procedure of establishing a *ratio* (in the sense of the theory of *ratios*):

We first assume Time from some Motion, and afterwards judge thence of other Motions, which in Reality is no more than comparing some Motions with others, by the Assistance of Time; just as we investigate the *Ratios* of Magnitudes by the help of some Space. For Example, he who computes the Proportion of Motion, by the Proportion of Time, does no more than get the said *Ratio* of Motions from Clocks, Dials; or from the Proportion of solar Motions performed in the same Time.⁷⁸

Time is measured by a uniform flux. Therefore, the determination of speed is a *ratio* comparing homogenous magnitudes: the motion of one body in proportion to a uniform motion standing for time. Consequently, time was represented in geometry by a straight line according to its properties: consisting of parts altogether similar, endowed with one dimension, and conceived as ‘the simple addition of rising moments, or the continual flux of one moment’.⁷⁹

Meanwhile, space was characterised in opposition to magnitude. As time was usually mistaken for motion, so space was usually mistaken for magnitude or extension.⁸⁰ However, magnitude is what occupies or fills the space, not space: ‘Everything subsists of itself, or is an accident to another thing, but neither of these seems to agree with space’.⁸¹ Space had not the ‘dignity’ of a substance, but it was not an accident either because it was extrinsic to all subjects, not the accident of a particular object. Instead, space was a ‘receptacle of immense capacity’ or ‘an immoveable vessel’, infinitely extended, without limits, perfectly penetrable, easily admitting ‘everything within itself’, receiving the successions of moveable bodies, determining the velocities of motions and measuring the distances of things. Space ‘has no actual but only potential figures, dimensions and parts consentaneous to its nature; by which means the capacity of admitting a body includes the capacity of admitting lines and superficies’.⁸² In Barrow’s story, the confusion between space and extension had its roots in Aristotle and more recently in Descartes who ‘has failed in

⁷⁸ Barrow, *Mathematical Works*, 165; Barrow, *Geometrical Lectures*, 13.

⁷⁹ *Mathematical Works*, 166.

⁸⁰ 165.

⁸¹ 150.

⁸² 150–51; 162.

this case' to distinguish between place and space. In order to distinguish space from magnitude, Barrow offers a series of arguments directed towards Descartes' specific claims, which cannot be detailed here but which were echoed in Newton's *De gravitatione* and in the 'Queries' of the *Opticks*.⁸³ First, since matter may be finite but God is infinite in essence, he must subsist beyond the bounds of matter; otherwise, he would be enclosed within his limits. Therefore, something is beyond, that is, some sort of space. Second, God can create other worlds beyond this, and we can imagine them to be created somewhere. Therefore, there has to be some space when these worlds could be created and exist. And, such as it happens in our world, God will also be present in these new worlds, 'yet without being the least affected with motion itself'.⁸⁴

The idea that geometry dealt with motion not only implied a change of subject; it also had consequences for the demonstration that geometry could achieve and how it was connected to natural philosophy. If geometry established the most general properties of magnitude, and magnitude was the subject of any science, geometrical demonstrations provided guides to explain nature. However, the co-extension of mathematics and natural philosophy was not a reduction of the former to the latter. For Barrow, mathematical demonstration was concerned with *possible* existence, not with *actual* existence. Physics dealt with the motion of bodies actually existing in this world, while mathematics was concerned with motions that *may* be generated. Forces actually generating motion in this world, adumbrated by geometry, could be postulated in physics at least in formal terms, although the efficient cause remained beyond understanding.

As mentioned above, Barrow's mathematics was said to be causal because 'the affections' or properties of bodies, arose from the definitions or 'forms', ultimately founded on experience.⁸⁵ Barrow rejected that any property or affection had primacy in definition; resembling Galileo, any affection 'may be rightly supposed or assumed

⁸³ Hall and Hall, *Unpublished*, 89–156; Ruffner, 'Newton's *De gravitatione*: A Review and Reassessment'; Henry, 'Gravity and *De gravitatione*'.

⁸⁴ *Mathematical Works*, 154–55.

⁸⁵ On Barrow's definitions see Malet, 'Mechanics in the 17th Century'; Malet, 'Isaac Barrow on the Mathematization of Nature'.

in defining the subject, since they are connected together with such an essential, close, reciprocal tie, that if anyone be supposed, the rest must necessarily follow'. A circle may be defined as the figure generated by the rotation of a right line, but also by the drawing of perpendiculars or by the affection of right angles. Any of these properties 'supply the place of a cause, in respect of the rest; because by the intervention of it, as a mean, the rest do necessarily follow'. This connection between different properties 'may be called a *formal causality*, because the remaining affections do result from that one property, which is first assumed, as from a Form'.⁸⁶ In the natural world, 'there is any other *causality* wherein a necessary consequence can be found', because there could not be a necessary 'connection of an *external, ex. gr. Efficient cause* with its *effect* (at least not such can be understood by us) through which, strictly speaking, the *effect* is necessarily supposed by the supposition of the *efficient cause*; or any *determinate* cause by the supposition of the *effect*'. In summary, 'there can be no efficient cause in the nature of things of a philosophical consideration which is altogether necessary'.⁸⁷

This particular view on causation and the resulting limitations of human knowledge are underpinned by Barrow's commitments with the unrestricted will of God when creating and ruling the world.⁸⁸ God could produce *any* effect by *any* means, and the order already known to us could not restrict his will. For example, when we see ashes or smoke, Barrow exemplified, we not only infer that a fire took place but also that some kind of fuel was required to keep it, as we have been told by '[natural] history'. However, God could immediately create ashes or smoke or produce them by any other mean, unknown to us, without requiring fuel. No efficient cause is necessary from the point of view of the omnipotence of God.

The dismissal of efficient causation has consequences for mathematics and physics. For Barrow, any demonstration assumed 'the existence of God', because all the 'possible existence or the effect of the things' implied a reference to the possible or actual cause of the effect at hand. In all cases, the cause is ultimately 'the infinite

⁸⁶ Barrow, *The Mathematical Lectures*, 86–88.

⁸⁷ Barrow, 88.

⁸⁸ Malet, 'Isaac Barrow on the Mathematization of Nature'.

and incomprehensible power of God'.⁸⁹ From here, he derived that non-contradiction was enough to claim a mathematical demonstration as true, as a demonstration on possible motions: 'Hence also it follows, that Demonstrations may be made of Things, which never had Existence anywhere; because it is sufficient for a Demonstration to assume true Hypotheses, *i.e.*, such as imply no Inconsistence in themselves'.⁹⁰ However, these mathematical demonstrations of the possible causes of motion did not imply any *necessary* reference to our world; the connection had to be established by experience. Barrow illustrated this with examples from mechanics and astronomy. Galileo—Barrow claimed—supposed that heavy things were naturally carried towards the centre of the Earth with a motion uniformly accelerated. In the realm of mathematics, this may be assumed as true if it satisfied the principle of non-contradiction, because 'such a motion may exist at the Pleasure of God, as implying nothing in it contrary to possibility'. However, because Galileo was saying something about this world, his supposition had to be referred to 'the Sense': 'Every particular Science of this Kind [mixed] produces its own Hypotheses, which require no other Proof, but to be explained and demonstrated by some example or Experiment more intelligible (...) and their possibility made evident as far as it may be done'. From astronomy, Barrow presented that before proving that planets moved in ellipses, Copernicus and Ptolemy were not wrong in the mathematical sense by supposing the perfect circular movement of stars: 'God may create such a World, where the Stars will exactly agree with such motions'. Therefore, 'their astronomy [is] true, not indeed of this World, but of the other, which is supposed capable of being created by God'. God endowed humans with the 'power of creating innumerable imaginary worlds in our thoughts', which himself, if he pleased, could 'cause to be real'.⁹¹ Imagination was associated with God's unrestricted capacity to create anything he wanted and this may be outlined by geometry, while sense was concerned with the world that God actually created and is the subject of physics.

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⁸⁹ Barrow, *The Mathematical Lectures*, 109–10.

⁹⁰ Barrow, 110.

⁹¹ Barrow, 111. For the importance of imagination in geometry and similar theological underpinnings, see Iliffe, *Priest of Nature*.

The two different approaches on mechanics put forward by Wallis and Barrow entailed divergent views on the nature of mathematics, particularly the role of motion in knowledge. At the same time, these assumptions sketched contrasting disciplinary settings, values and practices. While Wallis' approach distinguished between quantity and matter and connected nature and numbers *via* computation of discreet and abstract quantities, Barrow's reform of mathematics held that quantity was continuous magnitude and, therefore, geometry was essentially concerned with the generation of figures by motion rather than with computation of given figures; in this view, mathematics and nature were coextensive. The transformation of 'laws of nature' into mechanical laws of motion was embedded within local discussions on the connection between quantity, nature and God's action on the world.

7. 'The present frame of nature': Newton on the laws of motion

The business of experimental philosophy is [only] to find out by experience and observation not how things were created but what is the present frame of nature.

Newton, ULC, Ms. Add 3970, f. 242^v.

The idea that Newton's 'laws of motion' were in some way laws of nature was current in Newtonian circles at least since the early eighteenth century.¹ Historians have assumed that Newton inherited the idea of laws of nature from Descartes and incorporated them into his own terminology. In this view, Newton's laws of motion and the resultant law of gravitation would be his own version of the 'laws of nature' and therefore laws of nature. After all, the *Principia* seems modelled after Descartes' *Principia Philosophiæ* and accordingly it seems reasonable that Newton would have re-fashioned the Cartesian laws into his own laws of nature. This reading is not absent in accounts of seventeenth-century mechanics and is also assumed in Newton's scholarship. Recently, it has been claimed that Newton held a neo-Aristotelian approach to 'the metaphysics of laws'.² In this way, Newton's laws are said to operate as secondary causes in the sense of formal causes, by which the 'things themselves' are formed: laws constituted and informed bodies and forces.³

However, Newton did not denominate his achievements 'laws of nature'. As in the case of his English fellows, Newton's choice was bounded by the disciplinary transformations of mathematics, natural philosophy and mechanics connected with his theological and religious concerns. Similarly, causation was considered in terms of active principles rather than of immanent laws. In this context, I will argue that

¹ Charrak, *Contingence et nécessité des lois de la nature au XVIIIe siècle*; Shank, *Before Voltaire*; Snobelen, 'On Reading Isaac Newton's *Principia* in the 18th Century'.

² Biener and Schliesser, 'The Certainty, Modality...' For recent treatments making similar assumptions, Domski, 'Laws of Nature and the Divine Order'; Giere, *Science without Laws*, 85.

³ 'Laws of nature are prior to bodies and forces, since they are the principles by which the 'things themselves' are formed' Biener and Schliesser, 'The Certainty, Modality...', 319.

Newton considered that ‘laws of nature’ were part of a cosmology that he rejected as mistaken and harmful. The expression laws of nature was not neutral for Newton; it was embedded within a network of connections with determinism, necessity and a negative meaning of hypotheses. His preference for laws of motion over laws of nature points to his commitment with certainty and locates his enquiries in the context of the English appropriation of Cartesian philosophy presented in previous chapters, in which Descartes’ ‘laws of nature’ were reworked as laws of motion in mechanics. I will show that the only references to laws of nature in Newton’s printed works occurred in the context of controversies in which he broke the limits of the barriers that he had erected between different aspects of his studies.⁴

In order to substantiate this claim, this chapter has three sections. In the first, I present Newton’s appropriation of ‘laws of nature’ in his early intellectual explorations. Newton assessed ‘laws of nature’ in the restricted context of mechanics and, in this sense, as laws of motion. As it appears from the *Waste Book*, Newton engaged with the content of ‘laws of nature’ in a mathematical way, as it was customary in the mechanical study of the collision of bodies. Following Barrow, Newton also formulated some philosophical criticism of Descartes in mechanical exercises, including the identification of matter with extension, the existence and infinite character of space and the relationship between God and nature. In the second section, I show how Newton’s solution to Halley after the visit of August 1684 triggered a process in which Newton redefined the boundaries of mathematics, mechanics and natural philosophy in connection with values and ideas of his studies in religion and chronology. In solving Halley’s question, Newton realised that he had in his hands not only a mechanical problem but an insight into the true system of the world, the one represented in the Vestal temples of the true—and lost—religion. The mechanical laws of motion became, in this process, mathematical principles of natural philosophy. In the final section, I analyse the reactions against the universal gravitation and its law as controversial products of the new disciplinary order of the *Principia*. The conclusion that gravity was a force operating in nature transgressed the disciplinary practices of the Continent. Newton’s use in print of laws of nature

⁴ See Iliffe, *Priest of Nature*, 315–54.

was part of his strategies of response to the Continental charge of reintroducing occult qualities. Newton publicly—although cryptically—mobilised in his defence ideas concerning the foundations of natural philosophy and outcomes of his theological and historical enquiries.

7.1. Master of the whole

In his last years, Newton recalled that in the 1660s he bought Descartes' geometry and 'when he got over 2 or 3 pages he could understand no farther than he began again & got 3 or 4 pages farther till he came to another difficult place, than he began again & advanced farther & continued so doing till he made himself Master of the whole without having the least light or instruction from anybody'.⁵ However, during his first years at Cambridge he not only became a master of the whole *Géométrie* but also of the *Principia Philosophiæ*, the *Meditationes* and the *Discours de la méthode*. The attention devoted to Descartes became a constant and evolving influence on different aspects of Newton's thought, from mathematics to metaphysics. However, his approach to Descartes was critical since the beginning.⁶

In the so-called *Waste Book*, Newton dealt with the contents of 'laws of nature' as principles of the mechanical analysis of the collision of bodies. Entries on the topic are registered from January 1665.⁷ Newton's systematic examination of Descartes' laws occurred in mechanics, not in natural philosophy, although he was familiar with the *Principia*.⁸ By dealing with 'laws of nature' as principles of mechanics Newton aligned his work with the traditions presented in previous chapters, detaching them from the metaphysical and natural philosophical implications in which Descartes initially formulated them. In the *Waste Book*, the contents of 'laws of nature' were reworked into an axiomatic structure mainly concerned with the quantitative definition of abstract concepts for mathematical computation with the intention of

⁵ Conduitt, 'Anecdotes', fol. 1r. For different versions of Newton's encounter with Descartes' *Géométrie* see Westfall, *Never at Rest*, 98–101.

⁶ Westfall, *Force in Newton's Physics*, 323–34; Westfall, *Never at Rest*, 66–104; McGuire and Tamny, *Certain Philosophical Questions*; Iliffe, *Priest of Nature*, 84–122; Guicciardini, *Isaac Newton and Natural Philosophy*, 42–75.

⁷ Herivel, *Background*, 120.

⁸ McGuire and Tamny, *Certain Philosophical Questions*.

solving mechanical problems such as the impact of bodies or circular motion—which Newton reduced to a case of impact. In opposition to Descartes, these results are susceptible of contrast with experience. Aligned with the English practice of mechanics, Newton used some of these results to make some astronomical computations such as a deduction from Kepler’s proportion that planets’ ‘endeavours to recede from the sun will be reciprocally as the squares of their distances from it’ or a comparison of the endeavour of the Moon to recede from the centre of the Earth with the force of gravity at the surface of the Earth.⁹

The *Waste Book* contains Newton’s efforts at formulating a general theory of collisions. Newton assumed ‘laws of nature’ without any explicit reference to their author; furthermore, they were not denominated laws but definitions or axioms without any visible criteria. Moreover, Newton transformed the contents of these laws as he faced the solution of specific problems. In this sense, the approach is piecemeal and indirect, although the jacket of axiomatic structure gives the impression of a deductive, coherent exercise.

The first important change related to laws occurred when Newton introduced the idea that motion in collisions is a directed quantity, that is, that the direction is an inseparable variable in accounting for the outcome of impact. Descartes was aware of the importance of direction, but his treatment suggested that motion could be dealt independently of direction and then that the conservation of motion did not entail it. Explaining the collision of two inelastic bodies (‘have noe *vis elastica*’) in which one has less motion, Newton indicated that the quantity of motion was directed by the use of a negative sign, determinant in the resulting computation.¹⁰ After some propositions on inelastic collisions, Newton sketched some definitions and two sets of axioms intended to cover elastic bodies. In Definition 5, he stated that ‘reflection’ was the loss of determination (direction) of motion by rebounding. When bodies meet and rebound ‘they are parted either by some springing motion in themselves or in the matter crowded betwixt them’. Newton conceived impacts as springs, a

⁹ Newton, ‘Geometrical Tract’, fol. 87v. See Herivel, *Background*, 183–91; Westfall, *Never at Rest*, 150–53; Whiteside, *MP*, 6:4–5.

¹⁰ Descartes, *CSM*, 1985, I:242–43; Fraser, ‘The Third Law in Newton’s *Waste Book*’, 48–49.

powerful resource that Boyle had exploited in his *New Experiments Physico-Mechanicall* (1660).¹¹ The comparison indicates that as the spring is more ‘dull or vigorous’ the bodies will be reflected with more or less force, ‘as if it endeavour to get liberty to inlarge itself’. An initial effort to refine this resultant idea of force appears in Definition 9: ‘Force is the pressure or crouding of one body upon another’.¹²

This physical characterisation of force was afterwards connected—not without tensions—¹³ with axioms 1 and 2 in which Newton applied Descartes’ first two laws to bodies: ‘1. If a quantity once move it will never rest unlesse hindered by some externall caus. 2. A quantity will always move on in the same streight line (not changing the determination nor celerity of its motion) unless some externall cause divert it’. Associated to the first two laws, Descartes had sketched a theory of collisions in his third law and in seven rules. The key concept in this theory was the force of bodies (*vis corporis*) or quantity of motion, the product of size per speed, which determined the outcome of impact.¹⁴ For example, Descartes’ rule 2 established that after impact two bodies, being one larger than the other and travelling at the same speed but in opposite directions, shall travel in the direction of the larger without any change in speed.¹⁵ Collision appears as a ‘contest’ between forces in bodies resulting in losers and winners, depending on the ‘forces of motion’ of bodies involved.¹⁶ In the following axioms, Newton defined force as the ‘externall cause’ referred to in the first two axioms and thus presented impact under a different light. Newton switched the point of reference in collisions from the force in the bodies to the external cause (also called force) that ‘reduce[s] the body to rest’ or ‘put[s] it upon motion’. Although in his definitions Newton used the expression ‘quantity of motion’, the context implies a shift in the meaning; for in assuming the viewpoint of the external cause Newton ended up talking of the ‘generation’ or

¹¹ McGuire and Tamny, *Certain Philosophical Questions*, 24.

¹² Newton, ‘Waste Book’, fol. 10v.

¹³ For details see Westfall, *Force in Newton’s Physics*, 346–47; Fraser, ‘The Third Law in Newton’s Waste Book’.

¹⁴ Descartes, *AT*, 1905, VIII:66–67. See section, 4.4. above.

¹⁵ Descartes, VIII:68.

¹⁶ Gabbey, ‘Force and Inertia in Seventeenth-Century Physics’, 20–29.

‘destruction’ of motion, of change of motion. Axiom 4, for example, reads: ‘Soe much force as is required to destroy any quantity of motion in a body, so much is required to generate it; and soe much as is required to generate it soe much is alsoe required to destroy it’.¹⁷ After the definition of force, Newton quantified the change of motion by stating that when unequal forces move equal bodies, their velocities are proportional to the forces (axiom 5) and when equal forces move unequal bodies, their velocities are inversely as their quantities (axiom 6).¹⁸ Finally, this initial set of axioms on motion as generated by forces led to the postulation of the conservation of motion in elastic collisions: ‘the difference of theire [two elastic bodies] motion shall not bee lost nor loose its determination’. According to the definition of force as pressure, when bodies collide ‘they presse equally upon one another and therefore one must loose noe more motion than the other doth; soe that the difference of their motions cannot bee destroyed’.¹⁹

In the next page, a second set of axioms appears as the foundation of the previous series. In the axiom 100, Newton reformulated Descartes’ first law: ‘Every thing doth naturally persevere in that state in which it is unlesse it bee interrupted by some externall cause, hence axiome 1st and 2^d (...) A body once moved will always keepe the same celerity, quantity and determination of its motion’. This formulation is more general than axioms 1 and 2; perseverance is said to be natural and its implications are translated into a quantifiable form by claiming that this axiom amounts to the conservation of celerity, quantity and determination of motion. In this new sequence of axioms, Newton resumed his functional idea of force as change of motion. After explaining some possible outcomes of collision, Newton claimed that

it appears how and why amongst bodyes move some require a more potent or efficacious cause others a lesse to hinder or helpe their velocity. And the power of this cause is usually called force. And as this cause useth or applieth its power or force to hinder or change the perseverance of bodyes in their state, it is said to Indevour to change their perseverance.

¹⁷ The importance of this change of perspective is studied by Westfall, *Force in Newton’s Physics*, 345–49; Herivel, *Background*, 4–9; Whiteside, ‘Before the Principia’; Fraser, ‘The Third Law in Newton’s Waste Book’.

¹⁸ Newton, ‘Waste Book’, fol. 10v.

¹⁹ Newton, fol. 11r.

Notice that this conception of force has no metaphysical connotation and is limited to name the generic cause of changes of motion in impacts. Meanwhile, in axiom 106, Newton characterised the cause ‘which hindereth the progression’—the condition in the body also contemplated in axiom 100 opposing to the external cause—as a power of the body ‘to persevere in its velocity or state and is usually called the force of the body’.²⁰ In summary, force was the cause of the change of motion from the perspective of the colliding body and, at the same time, was the name of the power of the body by which it reacts against the external cause. Newton’s view of impact in terms of an external cause implied that when two bodies met the tendency to persevere in their state was a power or force internal to them, while in Descartes, the continuation of matter in its state was the necessary consequence of their definition as extended.

This dual conception of force appeared again in an unfinished tract on hydrostatics, dating from the 1670s, commonly known as *De gravitatione*. This time Newton was aware of employing the term in two different forms: ‘Force is the causal principle of motion and rest. And it is either an external one that generates or destroys or otherwise changes impressed motion in some body; or it is an internal principle by which existing motion or rest is conserved in a body, and by which any being endeavours to continue in its state and opposes resistance’. The ‘resisted force’ or ‘force in so far as it is resisted’ is named *conatus*, and the impressed, *impetus*. Newton denominated *inertia* the force in the body, ‘lest its state should be easily changed by an external exciting force’, making clear that the perseverance in a state is produced by a power in the body.²¹ In summary, forces considered in mechanics were: the acting force, the resisting force and the reacting force, this last generated by the interaction between the first two. Based on this conception of force, Newton derived the definition of density and rarity of bodies, central to hydrostatic. Not only the idea of force was reorganised in *De gravitatione* but also the ‘mutual pressing’ of bodies that in the *Waste Book* Newton seemed to have obtained from the initial caveat of force as a pressure or ‘crouding’. In the axioms, Newton succinctly stated

²⁰ Newton, fol. 12v.

²¹ Newton, ‘De gravitatione’, fol. 32.

that ‘bodies in contact press each other equally’, equating in magnitude the action and the reaction.

This tract on hydrostatics advanced a criticism to the foundations of Descartes’ natural philosophy in a long excursion inserted between definitions 4 and 5. *De gravitatione* elaborated a sustained rejection of Cartesian principles and traced ambitious alternatives. Newton did not mention the ‘laws of nature’ which seem to be restricted to mechanics, even if the main target here was the conception of motion on which the *Principia Philosophiæ* relies. Newton’s definition of motion as change of place entailed a distinction between bodies and space against Descartes. In fact, Descartes had reduced matter to extension and space to the place occupied by these three-dimensional bodies and thus ‘there is no real difference between space and corporeal substance’.²² Thus motion was determined by the change of position of one part of matter ‘from the vicinity of other bodies’; the ‘vicinity’ was always relative to the observer rather than a fix referent. Descartes further explained that by emphasising upon the idea of motion as transfer he showed that the ‘motion is always in the moving body’ as opposed to the force of action which brings about the movement. We saw that Newton forged a variant of the latter position in the mechanics of the *Waste Book*.²³ In *De gravitatione*, Newton claimed that motion has to be determined ‘with respect to the parts of space, and not with respect to the position of neighbouring bodies’.²⁴ This disagreement led Newton to the nature of space and its relation to matter. In the first part of his criticism Newton put forward some arguments against Descartes’ conception of motion, particularly concerning the contradictions implied in the relativity of the referent, such as that ‘motion can be generated where there is no force acting’. Newton took advantage of the paradoxes and contradictions of this definition, including that it is always possible to say that the planets do not move if we assume that their ‘vicinity’ is their vortex, even if God impressed some motion directly to celestial bodies.²⁵

²² Descartes, *AT*, 1905, VIII:46.

²³ Descartes, VIII:54–55.

²⁴ Newton, ‘De gravitatione’, 3; Hall and Hall, *Unpublished*, 123.

²⁵ Newton, ‘De gravitatione’, 7; Hall and Hall, *Unpublished*, 127–28.

Once Newton has criticised the concept of motion, he moved to the foundation of the Cartesian physics: ‘what extension and body are, and how they differ from each other’. In his view, this required correction ‘in order to lay truer foundations of the mechanical sciences’.²⁶ The core of Newton’s criticism is that the identification of extension and body, from which the erratic conception of motion sprung, leads to atheism. In contrast, Newton claimed that extension defined as space where there is no matter is an ‘emanent effect of God’; in his terms, extension does not exist as an accident or as a substance, but as an infinite and eternal necessary consequence of divine existence.²⁷ Newton considered that he had shed light on the necessary connection between God and matter that Descartes had removed by reducing bodies to extension. Newton’s strategy was to argue that bodies are parts of space which God, by his mere will, endows with ‘active powers’; these powers entail causal properties, such as mobility according to laws and impenetrability, but also the powers to generate sensations in human organs. The differentiation between matter and space also provides Newton with the occasion to advance a conception of space under Barrow’s influence. In a wording strongly resembling the *Mathematical Lectures*, space is said to be infinite and eternal. Thus, it provides the frame to determine the action of forces generating motion.²⁸ The core of Newton’s criticism coincides with Barrow’s arguments, in the sense that Descartes had excluded the necessity of God in the world by making matter self-sufficient for motion.

7.2. The force of human wit

Sometime during the 1670s, Newton turned his attention to antiquity. The idea that some remote forms of religion and natural/moral knowledge were originally pristine but were corrupted over history was available to Newton in a wide range of literature

²⁶ Newton, ‘De gravitatione’, 11; Hall and Hall, *Unpublished*, 131. Some historians consider that in this extensive note Newton is no longer interested in hydrostatics. This passage suggests otherwise. See, Stein, ‘On the Notion of Field in Newton, Maxwell, and Beyond’, 274–75; Palter, ‘Saving Newton’s Text: Documents, Readers, and the Ways of the World’, 1987; Dobbs, *The Janus Faces of Genius*, 139. Indeed, the lack of attention to this central aspect has misled some recent historians to claim that Newton cultivated a discipline called ‘metaphysics’ and *De gravitatione* would be the earliest and most important record of it, see Janiak, *Newton as Philosopher*; Schliesser, ‘Newtonian Emanation’; Kochiras, ‘Force, Matter, and Metaphysics in Newton’s Natural Philosophy’.

²⁷ On the details of these thorny topics see McGuire, ‘Space, Infinity and Indivisibility’; McGuire, ‘Existence, Actuality and Necessity’; Henry, ‘Gravity and De gravitatione’.

²⁸ Barrow, *Mathematical Works*, 149–65.

from alchemy to mathematics going through theology and chronology.²⁹ Following Neoplatonic philosophers and his Cambridge fellows' interpreters of the biblical prophecies, Newton studied the corruption of primitive Christianity that furnished in him—as in others—the idea of an even more remote form of uncorrupted religion identified with Noah and his sons after the Deluge. This corruption over history fascinated the Lucasian Professor.³⁰ After the long and exhausting controversy generated by the publication of his theory of light, Newton went through some 'years of silence' in the late 1670s until the early 1680s. During this time, alchemy and theology received his full attention, except for the intromission of some unwanted correspondents, some scattered work in mathematics and the appearance in the skies of one or two comets.³¹

In studying the *Book of Revelation* and the history of the Church, Newton thought that diverse passages of the prophecy had been fulfilled. Most Protestant readers interpreted the Apocalypse as the persecution of the first Christians under the Romans, the establishment of the Catholic doctrines and the later persecution of Protestants by Roman Catholics. This prophetic view of history concluded with the second coming of Christ in the millennium, in which the elected will finally reign. Newton advanced in strategies to interpret the words and images of revelations first following the works of Joseph Mede and Henry More and later, as usual, by construing his own and unique approach.³² The prophecies of the *Revelation* referred to the 'Great Apostasy', identified with the period in which the establishment of the main doctrines of the Catholic Church occurred during and after the Council of Nicaea in 325 CE. Mede defined the core of the great apostasy as the institution of idolatry, that is, of the adoration of false gods. This covered the adoration of the

²⁹ Feingold, 'Isaac Newton, Historian', 525; Buchwald and Feingold, *Newton and the Origin of Civilization*, 143. Guicciardini, *Newton on Mathematical Certainty*, 79–107. Guicciardini, 'Analysis and Synthesis in Newton's Mathematical Work', 316–17.

³⁰ The study of these topics have been recently invigorated by the publication of Iliffe, *Priest of Nature*. See also Manuel, *The Religion of Isaac Newton*; Westfall, *Never at Rest*, 335–401; Westfall, 'Newton's Theological Manuscripts'; Force and Popkin, *Essays on the Context, Nature, and Influence of Isaac Newton's Theology*; Schaffer, 'Comets and Idols: Newton's Cosmology and Political Theology'.

³¹ The phrase 'years of silence' comes from Westfall, *Never at Rest*, 335–401.

³² Full details of Newton's complex and evolving 'methodisation' of the Apocalypse are presented by Iliffe, *Priest of Nature*, 219–353.

Virgin Mary, the saints and their relics. Based on intense readings of the Church fathers, ancient writers and historians, Newton went beyond. The main corruption was the introduction of Trinitarism, ‘the cult of three equal gods’.³³ Newton found Athanasius, bishop of Alexandria, guilty of introducing and circulating these heresies. Consequently, studies turned to the intricate ways in which these views corrupted the early Christianity and spread over the course of history. In so doing, Newton opened the door to heterodoxy and heresy in the eyes of both Roman Catholics and Protestants.³⁴

Newton’s interest in idolatry and corruption acquired a new dimension by moving from the investigation of the early Christianity to the wider perspective of a pre-Christian religion dating back to Moses and Noah, which allegedly contained the core truths Adam possessed before the Fall. The idea of an original amalgamation of natural, divine and moral knowledge corrupted over history took different shapes in traditions that Newton studied fervently.³⁵ As early Christianity was corrupted in the fourth century, Newton adopted the idea present in other scholars that the original religion was corrupted, in the first place, by worshiping nature and later by identifying gods with kings and then building statues to venerate them. Idolatry was at the root of human corruption of knowledge and religion. Newton elaborated complicated and heavily-documented genealogies showing that Egyptians, Chaldeans, Phoenicians, Greeks and Romans had corrupted this religion/knowledge. However, some of them were still in possession of uncorrupted remains. The true, pristine worship was represented in temples that reflected the universe, the divine creation. The reason behind these Vestal temples, as Newton claimed in a sermon on 2 Kings 17:15-16 is that ‘y^e wisest of beings requires of us to be celebrated not so much for his essence as for his actions, the creating and preserving & governing all things according to his good will & pleasure’.³⁶ In fact, Newton came to believe that

³³ Newton, “Prooemium” and First Chapter of a Treatise on Church History’, fol. 7.

³⁴ Snobelen, ‘Isaac Newton, Heretic’; Iliffe, *Priest of Nature*, 132–88; Westfall, *Never at Rest*, 310–30.

³⁵ Iliffe, *Priest of Nature*, 189–218; Westfall, ‘Isaac Newton’s Theologiae Gentilis Origines Philosophicae’; Buchwald and Feingold, *Newton and the Origin of Civilization*, 151–63; McGuire and Rattansi, ‘Newton and the “Pipes of Pan”’; Knoespel, ‘Interpretative Strategies in Newton’s Origines Philosophicae Theologiae Gentilis’.

³⁶ Newton, ‘Exposition of 2 Kings 17:15-16’, fol. 2r.

Vestal temples reproduced the universe having a central fire accompanied by seven lamps that signified the planets. In his historical enquiries, Newton found records of the existence of these temples all around the world and thus assumed them as evidence of the spreading of this original religion that worshipped God for his power and dominion.

Newton's devotion to history, prophecy and alchemy in the late 1670s and early 1680s was occasionally interrupted by visitors and correspondents asking questions about astronomy, natural philosophy and mechanics. The questions dragged Newton to discussions outside his immediate concerns; he often made excuses avoiding further communication. Nevertheless, these exchanges provide clues on the evolution of Newton's early engagement with different problems and traditions in mathematics, natural philosophy and mechanics.

7.2.1. Interruptions

In November 1679, Hooke, recently in charge of the correspondence of the Royal Society of London, contacted Newton asking him to resume the 'former favours to the Society by communicating what shall occur to you that is philosophicall'.

Newton politely replied that at the moment he was 'unfurnished wth matter answerable' Hooke's expectations. In a confident tone, Newton claimed that 'for some years past been endeavouring to bend myself from Philosophy to other studies ... which makes me almost wholly unacquainted wth what Philosophers at London or abroad have of late been imployed about'.³⁷ However, in trying to engage his potential contributor, Hooke had sent in the initial letter a 'hypothesis and opinion of mine' according to which the motion of planets is compounded by 'a direct motion of the tangent & an attractive motion towards the centrall body' and other questions of natural philosophy posed by correspondents to the Society. From the manuscript evidence, we know that Newton was dedicated to other topics, so his insistence on his affection for Philosophy 'being worn out' was not incorrect. However, Newton penned a 'fancy' of his own concerning the Earth's diurnal motion in which a body falling to the centre of the Earth should travel slightly further than the place directly

³⁷ Turnbull, *Correspondence*, 1960, 2:300.

beneath its point of release, describing a spiral. The curve and its demonstration ended up being entirely flawed.³⁸ In a quick reply, Hooke politely pointed to Newton's mistake and offered a different result, in which the released body was supposed to describe an elliptical path, according to Hooke's 'theory of circular motions compounded by a direct motion and an attractive one to a centre'.³⁹ In a short and considerably less polite letter, Newton replied that Hooke was right, that the body would not descend to the centre and that it would circulate alternatively between an 'ascent & descent' but not describing an ellipse, as Hooke suggested. Newton replied that if gravity were assumed as uniform, the body would oscillate between gravity and its *vis centrifuga* and would describe a more complex curve. Newton provided a short proof. Hooke replied that his supposition was rather that the 'attraction always is in duplicate proportion to the distance from the center reciprocally, and consequently that the velocity will be in a subduplicate proportion to the attraction and consequently as Kepler supposes reciprocally to the distance'. In other words, Hooke was saying that this attractive force is as the square of the distance from which he derived Kepler's law of velocities—a formulation that Kepler initially considered equivalent to the proportionality of areas and times but that he finally rejected.⁴⁰ Hooke further adds that finding out the properties of this curve by these two principles 'will be of great concern to mankind', because it will make intelligible 'all the appearances of the Heavens'. In an attempt to advance into the matter, Hooke reported that he performed some experiments on falling bodies that Newton had implied in a former letter.⁴¹ Not receiving an answer, he wrote back to Newton insisting that his 'tryalls' succeeded under new conditions so that we was persuaded that the experiment was 'very certaine'.⁴² In response, Newton sent a couple of paragraphs recommending a book on fevers and his author, an Italian 'Doctor of Physick of the City of Lucca' and two lines concerning the trials: that not

³⁸ For details on this correspondence, see Koyré, *Newtonian Studies*, 221–60; Westfall, *Force in Newton's Physics*, 424–35; Westfall, *Never at Rest*, 382–88; Whiteside, *MP*, 6:8–17; Guicciardini, *Newton on Mathematical Certainty*, 237–38; Nauenberg, 'Robert Hooke's Seminal Contribution to Orbital Dynamics'; Gal, *Meanest Foundations*; De Gandt, *Force and Geometry*, 147–55.

³⁹ Turnbull, *Correspondence*, 1960, 2:306–7.

⁴⁰ Gal, *Meanest Foundations*, 8; Westfall, *Never at Rest*, 987; Cf. Thoren, 'Kepler's Second Law in England'.

⁴¹ Turnbull, *Correspondence*, 1960, 2:309.

⁴² Turnbull, 2:312–13.

having the opportunity to thank him ‘by word of mouth’ for them, he contented himself to do it by letter. Nothing more.

Newton did not send any proof to Hooke. A short paper traditionally considered as Newton’s unsent reply, found a place amongst many other exercises in Newton’s mechanics, a field that had barely attracted his interest in the last few years.⁴³ It is important to notice that the terminology and mathematical methods of this short paper remain within the boundaries of mechanics, in line with the *Waste Book*. On the other hand, Newton did not speak of planets in motion and there is no identification of this attraction with any physical force arising from the Sun. Newton was talking about bodies moving in ellipses under a tendency or attraction to the focus.

The simultaneous correspondence with the Astronomer Royal, John Flamsteed, on the 1680 comet and with Thomas Burnet, a theologian writing a sacred history of earth, involving the discussion of physical topics, reveals that Newton was constantly relying on celestial fluids revolving and carrying with it the planets and comets, sometimes referring to vortices and drawing upon tendencies to move *away* from the centre to account for phenomena.⁴⁴ Newton may have reworked some elements of Hooke’s views on the orbital motion based on his own achievements in mechanics. However, in natural philosophy and to some extent in astronomy when dealing with ‘philosophy’, Newton was ‘deeply enmeshed’ in the Cartesian vortices.⁴⁵

7.2.2. De motu

Another interruption shall prove more fruitful. In August 1684, Halley visited Cambridge in order to consult Newton’s opinion on a tough problem: to find the planetary orbit produced by an inverse-square central force. The problem was

⁴³ On the dating of this manuscript see Whiteside, ‘Before the Principia’, 53–54; Hall and Hall, *Unpublished*; Herivel, *Background*, 108–17; Westfall, *Force in Newton’s Physics*, 513–14; Westfall, *Never at Rest*, 387–88.

⁴⁴ Turnbull, *Correspondence*, 1960, 2:315–56; Westfall, *Never at Rest*, 390–91; Schaffer, ‘Comets and Idols: Newton’s Cosmology and Political Theology’; Ruffner, ‘Newton’s Propositions on Comets: Steps in Transition, 1681–84’; De Gandt, *Force and Geometry*, 155–58.

⁴⁵ Cohen, ‘Guide’, 15.

embedded in the conceptions of orbital motion of the ‘elliptical astronomy’ as a compound of a ‘descent towards the sun and an imprest motion’. Hooke and Wren had widely discussed this idea in private and public since the 1660s, as we saw in the chapter 5. Newton claimed to have proved that the answer was an ellipse, but he could not find the demonstration so he promised to send it later.⁴⁶ In November 1684, Halley received a short tract entitled ‘De motu corporum in gyrum’ (*On the motion of bodies in an orbit*), formed by three definitions, four hypothesis, three theorems and seven problems.⁴⁷ The opening definitions are Newton’s adaptations of Hooke’s components of orbital motion to his earlier work in mechanics. The first deals with ‘centripetal force’ (*vis centripeta*), a term Newton coined to denominate the force drawing a body towards the centre (as opposed to centrifugal introduced by Huygens in 1673 to denominate the tendency to recede from the centre). The second one is the ‘force of a body or the force inherent in a body’ (*vis corporis seu corporis insita*) by which the body tends to persevere in rectilinear motion. A third definition establishes resisting force as generated from the ‘uniform impeding medium’. Instead of providing a clear concept of force, these definitions outline the use of centripetal, inherent and resisting forces involved in the quantification of change of motion in order to elaborate some calculations. The hypotheses include: the assumption of the motion of bodies *in vacuo* for some of the problems; the claim that if a body moves only under the action of its ‘inherent force’ will move uniformly in a straight line unless something external hinders it; and an interesting consequence of Newton’s mathematical treatment of centripetal forces: that the space described by a body under the action of any centripetal force is proportional to the square of time at the beginning of its motion. In other words, that the *ratio* between the motion of a body under a centripetal force and its trajectory is only valid ‘at the beginning of motion’—an extension of Galileo’s results on the free fall to bodies to the motion under centripetal forces. In order to validate Galileo’s proportion under different conditions, Newton restricted the scope to the moment in which the centripetal force deviates the body from its rectilinear path, ‘the beginning of motion’. Based on these assumptions, the tract demonstrated that an inverse-square force to one of the focus

⁴⁶ Details of the visit, analysing contradictory evidence in Cohen, *Introduction to Newton’s Principia*, 47–54; Westfall, *Never at Rest*, 402–7; Whiteside, *MP*, 6:1–21; Herivel, *Background*.

⁴⁷ Herivel, *Background*, 255–92; Whiteside, *MP*, 6:30–75; Hall and Hall, *Unpublished*, 231–92.

entailed the elliptical shape of the orbit (Problem 3). However, it also put forward other important generalisations: the demonstration of Kepler's proportions between areas and times in orbital motion (Theorem 1); the demonstration of centripetal forces in circular motion (Theorem 2); the generalisation that if the orbit is an ellipse, a parabola or a hyperbola, the force is directed towards a focus and then the force is as the inverse square of the distance (Theorem 3); and the demonstration of Kepler's rule according to which the square of periodic times are as the cube of the transverse axes (Theorem 4). In other words, Newton showed that his mathematical treatment of orbital motion accounted for other widely accepted proportions of the planetary motion. At the same time, Newton's hypotheses made possible to sketch some calculations on the motion of bodies in resisting media, such as hydrostatical questions and the motion of projectiles.

There is no reason to think that these demonstrations from the 1684 autumn did not make an impression in the Lucasian Professor. However, it is important to stress that contrary to a frequent approach when studying the developments of 'De motu', Newton did not know at this point that he was going to be the author of the *Principia* and probably he had no idea of how this work was going to look in its final form. It is misleading to read 'De motu' as an intuition of the revolution or transformation that will take place in the *Principia* and its reception. Westfall, for example, attributed the impetus that encouraged Newton from 'De motu' to the *Principia* to 'the sheer grandeur of the theme'.⁴⁸ It is important to consider three aspects. First, in previous situations Newton was also aware of significant results in mathematics, mechanics and optics, such as the generalisation of the binomial theorem that led to some successful applications in geometry and that his early calculations of the tendency to recede from the centre that were used to solve mechanical and astronomical problems, not to mention the method of fluxions. However, he was only moved to circulate some of these results by external pressure and more often than not, he tended to step back from the press. Second, Halley played an important role as external stimulus by asking the initial question and by providing constant motivation in subsequent visits and letters. However, Newton had similar

⁴⁸ Westfall, *Never at Rest*, 407.

encouragements by Oldenburg, Collins and Barrow in the past and even though he withdrew from the press. Finally, from the historiographical perspective, the teleological reading of ‘De motu’ as root of the *Principia* renders invisible the complexity of the process by which Newton redefined the boundaries of mathematics, natural philosophy and mechanics and decided to call his book *Philosophiæ Naturalis Principia Mathematica*, not *Mechanicæ Principia Mathematica* or *Astronomiæ Principia Mathematica*. Had Newton opted for any of the last two options, his claims on gravitation may not have generated such a reaction in the Continent. He would have been working within the comfort of widely accepted disciplinary boundaries.⁴⁹

If we turn back to the first ‘De motu’ (November 1684), Newton was not doing any significant transformation of the boundaries of mechanics. Since the *Waste Book* and the *De gravitatione*, Newton had distinguished between two forces in the analysis of motion: an inherent force keeping the body at rest or in uniform straight motion and the external forces of impact responsible of changing the trajectory of a body moved (or resting) by its own inherent force. On the other hand, Newton had found the idea of a tendency to move to the centre in Hooke’s correspondence, but it was already explained in Hooke’s *Attempt to prove the motion of the Earth*; it had been discussed by Wren and Halley also, as we saw in the chapter 5. Newton’s achievement in the first ‘De motu’ was to understand the tendency to move to the centre—that he coined centripetal—as a force in the terms of his previous explorations in mechanics: as an instantaneous force like impact, whose (constant) action on the body deviated it from the rectilinear tendency tracing conical paths. From the point of view of the explanatory resources, Newton stuck to traditional forms of classical geometry ‘at least for those adept to Euclid and Apollonius’.⁵⁰ Some specific points of ‘De motu’ may hint at the use of infinitesimals or to a limits procedure. The hypothesis 4 extended Galileo’s proportion for free fall to motion under centripetal forces by limiting the scope to the space described by a body at ‘beginning of its motion’, suggesting a restriction problematic from the point of view of classical geometry.

⁴⁹ Cf. Cunningham, ‘How the Principia Got Its Name’; Gabbey, ‘Newton’s Mathematical Principles’; Gabbey, ‘Between Ars and Philosophia Naturalis’.

⁵⁰ De Gandt, *Force and Geometry*, 55.

However, the appeal to this physicalised language was not uncommon in the learned circles, for it circulated in the works of Galileo, Roberval, Hobbes and Barrow.⁵¹ Finally, the extension of mechanical procedures to tackle astronomical problems had been a defining characteristic of the English mathematicians of the ‘elliptical astronomy’ at least since the recension of Kepler’s work. Thus, the first ‘De motu’ moves within the blurry boundaries of mechanics and relies on the geometrical procedures accepted and employed by mathematicians as different as Wallis and Barrow.

7.2.3. Revisions

The first revision of ‘De motu’ dated in December 1684 exhibits a change of direction in Newton’s goals that probably occurred as the initial version was travelling down to London. In the hand of Newton’s amanuensis, this first revision now called ‘De motu sphaericorum in fluidis’ (*On the motion of spherical bodies in fluids*) seems ‘a faithful copy’ of the one sent to Halley with some substantial additions.⁵² The new title indicates that Newton already had an insight that under an inverse-square force a body behaves as though the mass were concentrated at its centre.⁵³ But it also points to a consideration of physical elements, for Newton here conceived that planets moved in a fluid celestial ether. Indeed, Newton’s first major transformation here concerns the interaction of bodies under centripetal forces in a more detailed manner. The three initial definitions remain intact, but Newton changed the order of the hypotheses that moved up to five and reworked some lemmas. The purpose of the new set of hypotheses was to advance in the quantitative treatment of forces from a more general perspective. To the previous hypotheses, Newton added one that substantiates the quantitative properties of force, specifying that ‘the change of motion in a body is proportional to the impressed force and takes place along the straight line in which that force is impressed’ (Hypothesis 2). Thus, Newton incorporated his earlier achievements concerning Descartes’ first and second

⁵¹ Malet, *From Indivisibles to Infinitesimals*.

⁵² Herivel, *Background*, 294; Different editions of this version has been printed in Herivel, 294–303; Whiteside, *MP*, 6:74–80; Hall and Hall, *Unpublished*, 249–67. Because these editions tend to overlap materials from other similar versions, I have followed the manuscript Newton, ‘De motu sphaericorum corporum in fluidis’.

⁵³ Whiteside, *MP*, 6:31.

laws by expressing force as a directed quantity. The next one dealt with the indifference of relative motions in a space that rests or moves ‘perpetually and uniformly in a straight line without circular motion’ (Hypothesis 3). Another one, backed up by Hypothesis 3, stated that the mutual action between bodies does not affect their common centre of gravity, that is, the common centre does not change its state of motion or rest (Hypothesis 4). The last hypothesis (5) contained a revision of Newton’s earlier definition of resistance as a joint proportion between the density of the medium, the speed and the spherical surface of the body moved. The previous Hypothesis 3 became Lemma 1, expressed this time in terms of forces: when two forces act simultaneously on a body, this describes the diagonal of a parallelogram in the same time as it would be the sides if the forces acted separately. The Lemma 2 opened the door to more advanced mathematics than the initial geometrical demonstrations of ‘De motu’. We had seen that in the initial Hypothesis 3 Newton limited the validity of the extension of the Galilean proportion of free fall to the space described by a body ‘at the beginning of motion’ in order to make it valid for the case of centripetal forces as impulses. Now, Newton generalised this proposition by claiming that ‘the distance a body describes from the beginning of its motion under the action of any centripetal force whatsoever is in the duplicate ratio of the time’. In the proof, Newton claimed that under uniform centripetal forces, the distance may be represented by the areas of a triangle and the demonstration proceeds ‘as shown by Galileo’. However, under non-uniform centripetal forces, the hypotenuse of the triangle becomes a curve. In order to calculate the areas under the curve, Newton introduced the idea of ‘the limiting *ratio* of the evanescent spaces or the first *ratio* of the vanishing spaces’.⁵⁴ The earlier treatment of the mechanics of impact and, by extension, of the orbital motion was explained with the resources of (classical) geometry. The quantification of non-uniform centripetal forces described complex curves which required the calculation of areas under them, so Newton began to mobilise procedures that he had developed when he transformed his method of fluxions into the geometrical determination of the *ratios* of ‘nascent and evanescent’ quantities.⁵⁵

⁵⁴ A detailed analysis of this proof is presented by De Gandt, *Force and Geometry*, 161–67.

⁵⁵ On this method see Guicciardini, *Reading the Principia*, 32–37; De Gandt, *Force and Geometry*, 221–24; Westfall, *Never at Rest*, 378–80.

The introduction of Newton's original geometrical developments is subordinated to a deeper change undergoing in this new 'De motu'. Newton enlarged the Scholium to Theorem 4—which demonstrated Kepler's rule according to which the square of periodic times is as the cube of the transverse axes.⁵⁶ According to the achievements of this Theorem, there was a correlation between the period and size of the orbit that may be used to calculate the elliptical path of the planets. For the new 'De motu', Newton realised that the whole space of planetary heavens either rests ('ut vulgo creditur') or moves in a straight line and in consequence, that the common centre of gravity of the planets either rests or moves along with the space of the planetary system according to the Hypothesis 4. However, in either case, by the Hypothesis 3, the relative motions of the planets remain the same and their common centre of gravity rests in relation to the whole space, so the centre of the whole planetary system *must* be considered at rest. From here, Newton claims, 'the true Copernican system can be demonstrated *a priori*'. Indeed, if the common centre of gravity can be proved to be at rest, then the motion of the planets is real, not apparent or relative. However, this conclusion implied another point: if the common centre of gravity is calculated for any position of the planets it will lay either in the body of the Sun or very near of it. However, this deviation of the Sun from the centre of gravity implied that centripetal forces did not always tend to the immoveable centre and then 'the planets neither move exactly in ellipse nor revolve twice in the same orbit'. The cause was the mutual interference of all the celestial bodies, 'so that there are as many orbits to a planet as it has revolutions' and thus 'the orbit of any one planet depends on the combined motion of all the planets, not to mention the action of all these on each other'. Newton was not making a light assumption. Between December and January 1684/5, he wrote to Flamsteed asking whether there was any important change in the speed of Saturn when approaching Jupiter, contrary to Kepler's tables. Flamsteed replied with scepticism to Newton's intuition of mutual perturbations but he sent some observations suggesting a variation in speed.⁵⁷ The mutual interference posed a major challenge: 'Unless I am much mistaken, it would exceed the force of

⁵⁶ Other readings of this Scholium can be found in Smith, 'Newton's *Philosophiae Naturalis Principia Mathematica*'; Cohen, 'Guide', 18.

⁵⁷ Turnbull, *Correspondence*, 1960, 2:413.

human wit to consider so many causes of motion at the same time, and to define the motions by exact laws which would allow of an easy calculation'.⁵⁸ From this perspective, the trigonometrical methods of calculating the ellipse based on three observations became invalid, for the three observations may correspond to different ellipses or revolutions. The complexity of nature does not mean the impossibility to understand it. Newton's ellipse 'ought to be the mean among all errors', and thus it should be calculated on the basis of 'very many observations' that may be assigned to 'a single operation which mutually moderate each other and display the mean ellipse both as regards position and magnitude'.

The major insight of December 1684 was that 'De motu' offered a glimpse into the authentic constitution of the world, beyond the boundaries of mechanics. Newton realised that his mathematical hypotheses could reach farther than the calculation of two points moving *in vacuo* satisfying Kepler's geometrical proportions.

Confidently, Newton claimed that his approach could *demonstrate* the Copernican system, the one represented in the Vestal temples. Revising his amanuensis writing, Newton crossed out the word 'Hypothesis', not only in the opening parts but in all other mentions—including those in the Scholium just analysed—and replaced it for 'Lex'. The hypotheses of the mechanical approach gave way to laws from which natural philosophical conclusions could be drawn (Figure 9). The Hypothesis 2, initially stating that 'the change of motion is proportional to the impressed force', was also revised in Newton's hand by deleting the word 'motion' and inserting above 'state of movement or rest'. In this revised form, motion and rest appear as states equally responsive to external forces.

The change of perspective is also manifest in Newton's inclusion of a closing Scholium claiming that he had considered the motion of bodies in non-resisting media in order to 'determine the motion of celestial bodies in ether'. The revised title (*On the motion of spherical bodies in fluids*), that may suggest to the modern reader that the text deals with hydrostatics, in fact the name of an all-encompassing study

⁵⁸ Newton says that 'superat ni fallor vim omnem humani ingenij'. Herivel has pointed to the irony that Newton's inscription in the statue of Newton in the chapel of the Trinity College reads 'Qui genus humanum ingenio superavit', coming from Lucretius. Herivel, *Background*, 303.

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De motu sphaericorum Corporum in fluidis.

Def. 1. Vim contripetam appello qua corpus attrahitur
vel impellitur versus punctum aliquod quod ut centrum spectatur.

Def. 2. Et vim corporis seu corpori impositam qua id conatur
perseverare in motu suo secundum lineam rectam.

Def. 3. Et resistenciam qua est modis regulariter impeden-
-tis.

Def. 4. Exponentes quantitatum sunt alia quavis quantita-
-tis proportionalis exponentis.

^{Lex} Hypoth. 1. Sola vi imposita corpus motu uniformi in linea
recta semper pergens si nil impediat.

^{Lex} Hypoth. 2. ~~mutacionem~~ ^{status motus vel quiescenti} proportionalis est vi impressa
et fieri secundum lineam rectam qua vis illa imprimatur.

^{Lex} Hypoth. 3. Corporum dato spatio inclusorum eodem esse
motus inter se sive spatium illud quiescat sive moveat id
perpetuo et uniformiter in directum absq. motu circulari.

^{Lex} Hypoth. 4. Mutuis corporum actionibus commune centrum
gravitatis non mutare statum suum motus vel quiescit. Constat ex
~~Lex 3~~

^{Lex} Hypoth. 5. Resistenciam modis est ut modis illius denfi-
-tatis et corporis moti sphaerica superficies & velocitas conjunctim.

Lemma 1. Corpus viribus conjunctis Diagonalem parallelo-
-grammi eodem tempore describere quo latera separatim.

Si corpus dato tempore vi sola M ferretur ab
 A ad B et vi sola N ab A ad C , compleatur
parallelogrammum $ABDC$ et vi utraq. ferre-
-tur id eodem tempore ab A ad D . Nam quoniam
vis M agit secundum lineam AC ipsi BD paral-
-lelam, hoc vis N mutabit auctoritatem accedendi ad lineam
illam BD vi altera impressam. Accedet igitur corpus eodem
tempore ad lineam BD sive vis AC imprimatur sita non,
atq. adeo in fine illius temporis reperitur alicubi in linea
illa BD . Eodem argumento in fine temporis ejusdem repe-
-rietur alicubi in linea CD , et proinde in utriusq. linea con-
-cursu D reperiri necesse est.

Lemma 2. Spatium quod corpus urgente quacumq. vi con-
-tripeta ipso motus initio describit, est in duplicata ratione temporis.

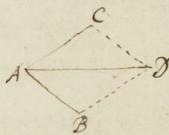


Figure 9 Newton's introduction of laws. ULC, Ms. Add 3965 f. 40

on the motion of bodies, including the celestial ones moving in an ether, ‘a fluid’ opposing no resistance. The importance of the move from Hypotheses to Laws is confirmed by Newton’s conclusions in this ur-General Scholium that the ‘Motions in the heavens are ruled therefore by the laws demonstrated’. Newton was not making reference to laws as a vague metaphor of divine or natural order, but to the specific statements that he had written down in his paper and assumed for mathematical demonstration.⁵⁹ The quantitative correlations between motion and forces—including the conditions establishing their interaction denominated ‘laws’—adumbrated the actual constitution of the planetary system. In this same spirit, Newton declared that ‘gravity is one kind of centripetal force’, suggested by the similarity between the force holding the Moon ‘in her monthly motion about the Earth’ and the force of gravity at the surface of the earth being nearly as the reciprocal square of the distance from the centre of the Earth. The motion of projectiles had to be referred now ‘to the immense and truly immobile space of the heavens, and not to the moveable space which turns around with the earth’. Newton’s laws of motion left the domain of mathematical mechanics and operated now as mathematical principles of natural philosophy.

Some revisions after December 1684/5 considerably enlarged the definitions (18 now), including absolute time and space in order to determine actual motions; the impenetrability of matter, the difference between place (as the space occupied by a body) and the absolute space, revisiting arguments from *De gravitatione*; a more detailed characterisation of forces, resistance and the ‘moments of quantities’ as their principles of generation or alteration in function of time. The new set of definitions was now directed to the determination of quantitative elements to distinguish actual from apparent motions. A note immediately after the definitions called the attention of the reader to the importance of being ‘freed from certain vulgar prejudices’ in order to understand ‘distinct principles of mechanics’. Here Newton introduced an argument from his Biblical studies: given that ‘ordinary people ... fail to abstract thought from sensible appearances’, it would be absurd for wise men or even

⁵⁹ In the original version of this Scholium in Newton’s hand appears the term law, suggesting that the Scholium was added once he realised of the scope of what he had at hand.

Prophets to speak to them in terms of the ‘distinct principles of mechanics’. ‘Hence, the sacred writings and theological writings are always to be understood in terms of relative quantities, and he who would on this account bandy words with philosophers concerning the absolute motions of natural things would be labouring under a gross misapprehension’.⁶⁰ The laws moved in number from 5 to 6 and the most important novelty is that Newton included a new third law stating that ‘as much as any body acts on another so much does it experience in reaction’, a principle sketched in the *Waste Book* and that now acquired a new dimension.⁶¹ Newton justifies this new law by definition 12 (stating the perseverance in direction of motions under their inherent forces) and 14 (presenting the varieties of impressed force, including centripetal forces).⁶² The first significant revised version composed between Winter and Spring 1684/5,⁶³ including the achievements of the previous papers on definitions and laws—now denominated ‘laws of motion’—got closer in the introductory sections to the first printed edition, particularly three laws that remained almost unaltered until 1687. The election of ‘laws of motion’ indicated the advance in the transformation of the scope of disciplines from the initial ‘De motu’: Newton used an expression common in English (mathematical) mechanics as principle of mathematical explanation of the actual motion of bodies in nature. After the laws, Newton introduced some lemmas and presented his method of ‘prime and ultimate ratios’ in order to deal in more satisfactory ways with the varieties of curves generated by different forces considered mathematically.⁶⁴ At the same time, this version included 47 propositions and a diversity of Lemmas and Problems. Nevertheless, all these additions did not emerge *ex nihilo*. The new project motivated to revisit previous elaborations in astronomy, mechanics, mathematics and natural philosophy. Newton began to rephrase past developments and to integrate them, *quantum in se est*, in his new project. The absorbing process that reached to conclusion with the first printed

⁶⁰ A wider analysis of this point, its appearance in the *Principia* and its importance in Newton in Iliffe, *Priest of Nature*, 240–45.

⁶¹ Cf. Cohen, ‘Guide’, 62–66; Fraser, ‘The Third Law in Newton’s Waste Book’.

⁶² These versions are edited in different ways by Herivel, *Background*, 304–20; Whiteside, *MP*, 6:188–94.

⁶³ Whiteside, *MP*, 6:92.

⁶⁴ On the introduction of this method into the *Principia* see Whiteside, 6:92–94; Guicciardini, *Newton on Mathematical Certainty*, 217–22; De Gandt, *Force and Geometry*, 159–68.

edition of the *Principia* in 1687 is, under this light, a synthesis.⁶⁵ However, it is a synthesis of Newton's piecemeal approach to a wide variety of fields reflecting typical interest of the seventeenth-century and, at the same time, his unique approach. Nevertheless, the idea of a synthesis does not imply that Newton put all this into a coherent, deductive whole. On the contrary, the successive revisions of 'De motu' display how previous works in conics, calculations on the motion of planets and the moon, a geometrical incarnation of the method of fluxions, analytical solutions to specific problems, computations on previous observations of comets, speculations on the actions of light, were reworked once and again in an attempt to point to something: that the laws of motion made possible to demonstrate principles indicating the actual structure of the world, the one represented in Vestal temples before the original knowledge was corrupted.

Between December 1684/1685 and the autumn 1685, the project took the form of two books denominated *De motu corporum*.⁶⁶ The *liber primus* shall contain Newton's mathematical principles concerning the properties of force in general, whose main aspects we have seen, while the *liber secundus* will present 'the application of this Mathematical part, to the System of the world', to the explanation of the (true) celestial motions and other phenomena such as the motion of comets and the 'flux and reflux' of the sea. The surviving evidence of this version of the philosophical part of the project is scattered and problematic,⁶⁷ but it is enough to make my point. The fundamental change of perspective that invigorated the enlargement of 'De motu' came from Newton's glimpse into the possibility of proving *a priori* the 'Copernican system'. In the *liber secundus*, Newton projected his natural philosophy back into the knowledge of the ancients. Newton opens the book claiming that it was an

⁶⁵ Koyré, *Newtonian Studies*, 3–24; Westfall, 'The Scientific Revolution Reasserted'; Dobbs, 'Newton as Final Cause and First Mover'.

⁶⁶ Dating these manuscripts is speculative but important information comes from the correspondence with Halley in June 1686, see Turnbull, *Correspondence*, 1960, 2:434–40; Cohen, *Introduction to Newton's Principia*, 113.

⁶⁷ Details of this in Cohen, *Introduction to Newton's Principia*, 109–15; 327–35; and in the introduction to Newton, *A Treatise of the System of the World*. I have used this edition and the manuscript in Latin (different from this edition).

ancient opinion of not a few in the earliest ages of philosophy that the fixed Stars remain immoveable in the highest parts of the world; that under the Fixed Stars the Planets were carried about the Sun; that the Earth, as one of the Planets, described an annual course about the Sun, while by a diurnal motion it was in the mean time revolved around its own axe; and that the Sun, as the common fire which served to warm the whole, was fixed in the centre of the Universe.⁶⁸

This Copernican arrangement of the universe was the philosophy of '*Philolaus*, *Aristarchus* of *Samos*, *Plato* in his riper years, and the whole sect of *Pythagoreans*'. Also, it was the 'judgment of *Anaximander*' and of '*Numa Pompilius* who, as a symbol of the figure of the World with the Sun in the centre, erected a temple in honour of *Vesta*, of a round form, and ordained perpetual fire to be kept in the middle of it'. This knowledge can be traced back to the 'ancient spirit of Egyptians', who performed vestal ceremonies, though concealed them under 'the veil of religious rites and hieroglyphic symbols'. Meanwhile, *Anaxagoras* and *Democritus* explained that the motions of celestial bodies 'were performed in spaces altogether free, and void of resistance', although philosophy 'began to decline' with the introduction of the solid orbs by *Eudoxus*, *Calippus* and *Aristotle*. The *Chaldeans*, 'the most learned astronomers', considered comets as 'particular sort of planets which describing very eccentric orbits'. The ancients knew the Copernican arrangement of the celestial bodies including the comets. However, *Newton* claims that we do not know how the ancients explained 'that the planets came to be retained within any certain bounds in those free spaces', that is, that the celestial bodies deviated from their rectilinear path 'into regular revolutions in curvilinear forms'.⁶⁹ Solid orbs were introduced to fill this gap. However, more recent philosophers, such as *Kepler* and *Descartes* formulated the existence of vortices or '*Borelli*, *Hooke* and other of our countrymen' introduced 'some other principle whether of impulse of or attraction'. *Newton* presented his own natural philosophy in this way: 'From the first law of motion it is very certain that some force is required. Our purpose is to bring out its quantity and properties and to investigate mathematically its effects in moving bodies; further, in order not to delimit its type hypothetically, we have called by the general name

⁶⁸ *Newton*, *A Treatise of the System of the World*, 1; *Newton*, 'Liber Secundus', fol. 1r.

⁶⁹ *Newton*, *A Treatise of the System of the World*, 2–4; *Newton*, 'Liber Secundus', fol. 1r.

‘centripetal’ that force which tends towards the centre’.⁷⁰ Newton attributed to the ancients a precise knowledge of the constitution of the universe and pointed to some moments of history in which this knowledge was corrupted. The remaining mystery was still how the ancients explained the motion of planets in free spaces assuming the first law, whose knowledge was evident in Anaxagoras, Aristotle and Lucretius.⁷¹ Given the truth of the first law, some force is required to explain orbital motion. The whole point of his enquiry was, then, to investigate possible forces and then, in the *liber secundus*, to show that the force of gravitation, a kind of centripetal force, filled this gap (or recovered this bit of lost knowledge) in the understanding of the system of the world.

However, this original plan of two books, including explicit references to the historical awareness that motivated Newton’s enquiry, changed radically as we know from his correspondence in 1686. Halley informed Newton that Hooke ‘has some pretension upon the invention of ye rule of the decrease of gravity’. ‘He says— Halley continues—you had the notion from him, though he owns the demonstration of the curves generated therby [sic] to be wholly your own’. Hooke’s pretension was that Newton ‘make some mention of him, in the preface’.⁷² Newton’s anger can be guessed considering the wider perspective in which he was locating his own achievements.⁷³ In one of the resulting replies to Halley, Newton mentioned two books on mathematics and one third on natural philosophy, including a theory of comets on which he has been working. ‘The Third I now designe to suppress’, famously claiming that ‘Philosophy is such an impertinently litigious Lady that a man had as good be engaged in Law suits as have to do with her’. Newton had been consumed for years replying to philosophical questions concerning the nature of light. He had found some peace of mind after cutting down this correspondence in the 1670s. The perspective of the publication of the *Principia* seemed to drag him again to a bitter dispute with Hooke. The suppression of the philosophical book

⁷⁰ Newton, ‘Liber Secundus’, fol. 2r; For the translation of this passage I have followed the manuscript and Cohen, *Introduction to Newton’s Principia*, 332.

⁷¹ Hall and Hall, *Unpublished*, 309–11.

⁷² Turnbull, *Correspondence*, 1960, 2:431.

⁷³ Westfall, *Never at Rest*, 444–52; Iliffe, *Priest of Nature*, 341–46.

would render the title *Philosophiæ Naturalis Principia Mathematica* ‘inadequate’, Newton claimed, so he thought that the version restricted to the mathematical properties of forces shall be called just *De motu corporum libri duo*. ‘But upon second thoughts I retain ye former title. Twill help ye sale of ye book wch I ought not to diminish now tis yours’. After all, we can conjecture, the first two books effectively contained mathematical principles of natural philosophy, although they did not explicitly contain their application to natural philosophy. Halley replied begging ‘not to let your resentments run so high, as to deprive us of your third book, wherein the application of your Mathematicall doctrine to the Theory of Comets, and several curious Experiments ... will undoubtedly render it acceptable to those that will call themselves philosophers without Mathematicks’.⁷⁴ Newton finally included a third book, but it was not written for ‘philosophers without Mathematicks’, as we suspect he may have presented the *liber secundus* to Halley. A scar of this story seems to be in the 1687 edition (and in all subsequent editions), in the opening of the book 3: ‘I composed an earlier version of book 3 in popular form, so that it might be more widely read. But those who have not sufficiently grasped the principles set down here will certainly not perceive the force of the conclusions’.⁷⁵

7.3. The effect of choice

The first edition of the *Principia* appeared in 1687. The work opens with a ‘Præfatio’ that remained unaltered in the subsequent editions. The main purpose of this intriguing section is to introduce the subject of the book that is unfamiliar for readers.⁷⁶ Newton was aware that his ‘way of philosophising’ reconfigured the boundaries of disciplines as they were widely practiced in his time, particularly of geometry, mechanics and natural philosophy. Indeed, what Newton called ‘mathematical principles of natural philosophy’ was hardly intelligible as such for his contemporaries. Because of this, the first part of the ‘Præfatio’ attempts to (re)establish the boundaries of geometry and mechanics and in the second, once these

⁷⁴ Turnbull, *Correspondence*, 1960, 2:431, 435, 443.

⁷⁵ Newton, *The Principia*, 793.

⁷⁶ Detailed analysis of the ‘Præfatio’ in Guicciardini, *Newton on Mathematical Certainty*, 293–99. Garrison, ‘Newton and the Relation of Mathematics to Natural Philosophy’; Ducheyne, *The Main Business*, 179–224.

fields are reorganised, Newton shows their legitimacy to account for natural phenomena. The most important outcome of this new way of studying nature was the law of universal gravitation.

7.3.1. Mechanics, geometry and motion

The ‘Præfatio’ announced that the book’s main interest was ‘on *mathematics* as it relates to natural philosophy’. Newton mentioned two important antecedents: that the ancients considered ‘*mechanics* to be of the greatest importance in the investigation of nature’ and that recent authors have tried to ‘reduce the phenomena of nature to mathematical laws’.⁷⁷ If we consider Wallis’ mechanics as some state of the art in the English context, Newton’s claim that mechanics deals with nature and provides the foundations of geometry is controversial. However, Newton’s argument attempts to shed light on two problematic notions associated with mechanics and geometry: (1) that mechanics is less exact and thus opposed to geometry and (2) that mechanics is restricted ‘to manual arts’.

(1) Newton claimed that the ancients divided mechanics into *rational*, which proceeds through rigorous demonstrations and *practical*, which deals with the manual arts—a distinction he most likely derived from Pappus’ *Collectiones*; this last branch was considered less exact than geometry and was mistakenly used as a synonym for mechanics in general, reducing the scope and nature of the rational part. Nonetheless, Newton argued that the attribution of less exactitude to mechanics is not an error of the art but of those ‘who practise the art’. In this way, ‘anyone who works with less exactness is a more imperfect mechanic, and if anyone could work with the greatest exactness, he would be the most perfect mechanic of all’. The distinction between mechanics and geometry has nothing to do with exactness *pace* Descartes.⁷⁸ In its place, Newton claimed that geometry is founded upon mechanics. The argument runs this way: mechanics is mainly concerned with the description of straight lines and circles, that is, with the generation (tracing or construction) of circles and lines. As for geometry, it ‘teaches how problems are solved by these

⁷⁷ Newton, *Principia Variorum*, 1972, 2:15–17; Newton, *The Principia*, 381–83.

⁷⁸ Domski, ‘The Constructible and the Intelligible in Newton’s Philosophy of Geometry’.

operations', that is, geometry shows how the construction of lines and figures solves geometrical problems. The very action of construction, Newton clarified, is not a geometrical problem because '*geometry* postulates the solution of these problems from *mechanics* and teaches the use of the problems thus solved'. In other words, mechanics provides the subject matter of geometry—actually, creates it. In consequence, geometry is that part of '*universal mechanics* which reduces the art of measuring to exact proportions and demonstrations'. What mechanics constructs is measured by geometry and based on these measurements, demonstrated. Geometry requires some kind of construction. The subordination of geometry to mechanics implied that geometry in general deals with the magnitude of motion, (the generation of motion). Geometry does not generate motion, but establishes its proportions based on mechanical production. Motion then is the raw material matter of geometry (and of the phenomena of the world, as we shall see).

(2) The distinction between rational and practical mechanics displays Newton's redrawing of the traditional distinction between the manual arts and a *scientia* of machines as it emerged in the late sixteenth century. In the first chapter, we saw that the science of mechanics emerged as an attempt to organise the mathematical principles of their functioning, while the manual arts were concerned with the construction and operation of machines. Newton traces different boundaries. 'Rational mechanics' is not restricted to uncovering the principles of the construction and operations of machines, that is, the proportions of manual powers required to produce a desired motion. Instead, '*rational mechanics* will be the science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever and of the forces that are required for any motion whatever'. Because mechanics deals with motions and geometry is concerned with magnitude, rational mechanics establishes the proportions (magnitudes) between forces and motions, on the one hand, and motion and forces on the other. Rational mechanics has been mistakenly reduced to manual arts because the ancients hardly paid any attention 'to gravity (since it is not a manual power) except in the moving of weights by these powers'.

Newton now turned to the subject of the book. Because mechanics can deal with natural powers (gravity, levity, elastic forces, resistance of fluids) according to the ancients and, against Descartes, geometry is founded upon mechanics, his work ‘sets forth mathematical principles of natural philosophy’. Both mechanics and natural philosophy deal with objects generated by motion that can be studied by geometry, whose main concern is the ‘generation of motion’. In this view, the ‘whole difficulty’ of (natural) philosophy is the discovery of ‘the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces’. The *Principia* advances in this direction. The first two books provide ‘general propositions’, while the third explains ‘the system of the world’ illustrating these propositions. While the first two books deal (mathematically) with forces and motions in general, the third book deduces from the first two the forces operating in our ‘system of the world’ and, assuming them, demonstrates other phenomena. The third book ‘deduces’ from celestial phenomena ‘the gravitational forces by which bodies tend towards the sun and towards the individual planets’. Once this force is deduced, he demonstrated phenomena such as the motions of the planets, the comets, the moon and the sea.

This complex reorganisation of mechanical and geometrical principles as foundations of natural philosophy explains the dual function of the laws in the *Principia*. Newton called them ‘Axioms or laws of motion’ [Axiomata sive leges motus].⁷⁹ Their function was to stipulate the most general conditions of interaction between forces and bodies in terms of motion. In other words, the axioms-laws provide the mathematician-natural philosopher the general framework to determine the motions generated by any force and the forces generating any motion. In the specific case of the natural philosopher, based on ‘very many observations’ and equipped with the mathematical principles, it was possible to determine the specific kinds of force generating certain motions and use them to explain some other phenomena.⁸⁰

⁷⁹ Newton, *Principia Variorum*, 1972, 1:54–56.

⁸⁰ On Newton’s gathering of experimental evidence for his *Principia* see Schaffer, ‘Newton on the Beach: The Information Order of the Principia Mathematica’.

Newton's most salient philosophical conclusion in the *Principia* was the 'deduction' of a law quantifying the effects of universal gravitation.⁸¹ In the first two books, he calculated the motion of bodies under centripetal forces (among many other things) and explained the motion of bodies when this force was proportional to the inverse-square of the distance between the objects involved. In the third book, Newton concluded that 'gravity exists in all bodies universally' and was directly proportional to the quantity of matter and inversely proportional to the square of the distances.⁸² The characterisation of gravity as a force acting at a distance did not specify its nature or its cause; the *Principia* did not provide any mechanism explaining how this attraction was performed. On the contrary, the entire point of the *Principia*, Newton insisted, was to show that this force existed and could be quantified in a law. This because assuming the force of gravity—on experimental evidence—explained with unprecedented accuracy a wide variety of phenomena different from those from which it was initially deduced.

7.3.2. Crossing boundaries

The most prominent aspect of Newton's *Principia* was, at the same time, the centre of criticisms. One of the first reviews, published in the *Journal des Sçavans*, addressed its concerns to the disciplinary aspects, emphasising the impossibility of postulating the nature of gravity from the means Newton had employed.

The work of Mr. Newton is a mechanics, the most perfect that one could imagine, as it is not possible to make demonstrations more precise or more exact than those he gives in the first two books on lightness, on springiness, on the resistance of bodies, and on the attractive and repulsive forces that are the principal basis of Physics. But one has to confess that one cannot regard these demonstrations otherwise than as only mechanical; indeed, the author recognizes himself (...) that he has not considered their Principles as a Physicist, but as a mere Geometer.

He confesses the same thing at the beginning of the third book, where he endeavours nevertheless to explain the System of the

⁸¹ On the complex structure of Newton's 'deduction' see Cohen, 'The Principia, Universal Gravitation, and the "Newtonian Style", in Relation to the Newtonian Revolution in Science'; Cohen, 'Newton's Concepts of Force and Mass, with Notes on the Laws of Motion'; Smith, 'Newton's Philosophiae Naturalis Principia Mathematica'; Westfall, *Force in Newton's Physics*, 467–91; Guicciardini, *Newton on Mathematical Certainty*; Harper, *Isaac Newton's Scientific Method*; Ducheyne, *The Main Business*.

⁸² Newton, *The Principia*, 810.

World. But it is done only by hypotheses that are, most of them, arbitrary, and that, consequently, can serve as foundation only to a treatise of pure mechanics. He bases the explanation of the inequality of the tides on the principle that all the planets gravitate reciprocally towards each other ... But this supposition is arbitrary and it has not been proved; the demonstration that depends on it can therefore be mechanics.

In order to make an *opus* as perfect as possible, M. Newton has only to give us a Physic as exact as his Mechanics. He will give it when he substitutes true motions for those he has supposed.⁸³

Read against the background of the previous sections, this review criticised what Newton esteemed as his greatest achievements, stated as a hypotheses what he saw as rigorous demonstrations and demanded from him what he considered his major success: the determination of true motions by forces. At the same time, the review uses mechanics, geometry and physics in meanings that Newton had radically modified in the *Principia*. Huygens, who admired Newton's mathematical skills, confessed that he neither agreed 'with a Principle according to which all the small parts that we can imagine in two or several different bodies mutually attract each other or tend to approach each other'. In his view, 'the cause of such an attraction is not explainable by any of the principles of Mechanics, or of the rules of motion'.⁸⁴ Leibniz was also perplexed by Newton's introduction of attraction, as he said to Huygens: 'I do not understand how he conceives gravitation or attraction. It seems that, according to him, it is nothing more than an inexplicable incorporeal virtue'.⁸⁵

Newton's strategies of defence were complex, intertwined and varied and a full study of them and their interactions would deserve separate studies. From the point of view of his intellectual itinerary after the 1687 *Principia*, Newton further emphasised his admiration for ancient geometry and enhanced a distinction between analysis and synthesis as methods of mathematics that should be employed in natural philosophy as well.⁸⁶ Meanwhile, he took shelter in the loose talk on 'experimental philosophy' that circulated in England since the 1660s and wrapped up his *Principia* and his

⁸³ Journal de Scavans, 2 August 1688, p.153 in Koyré, *Newtonian Studies*, 115.

⁸⁴ Huygens, *Oeuvres Complètes*, 22:471.

⁸⁵ Leibniz to Huygens, October 1690 Turnbull, *Correspondence*, 1961, 3:80.

⁸⁶ Guicciardini, *Newton on Mathematical Certainty*, 252–57; Guicciardini, 'Analysis and Synthesis in Newton's Mathematical Work'.

Opticks under different interpretations of the expression after 1706 until Leibniz's passing.⁸⁷ During the late 1680s and 1690s he deepened his theological studies and contrived industrious chronologies of religions, ancient kingdoms and different forms of corruption.⁸⁸ Also in the 1690s, Newton seems to have achieved important results in alchemy.⁸⁹ Furthermore, Newton began to create an 'inner circle', a group of followers that had access to private documents and had a view on some of his private ideas. These acolytes acted as emissaries and presented Newtonian ideas to different correspondents and audiences, arguably under the guidance of Newton. This circle included Fatio de Duillier, David Gregory, Samuel Clarke and William Whiston, just to name a few.⁹⁰ After 1696 Newton moved to London, acquired a position in the Mint and became more visible for the circles converging in the Royal Society. Finally, the scenarios in which these strategies take place were varied: the correspondence, circulation of manuscripts, public lectures (like the 'Boyle Lectures'), Newton's indirect and direct intervention in the controversies with Hooke and Leibniz, the reworking, edition and publication of previous developments (*Opticks*) and, the most famous, the insertions of the 'Queries' to the *Optice* (to the 1706 edition, the Latin translation by Samuel Clarke, that was expected to circulate abroad), Roger Cotes' 'Editoris Præfatio' to the 1713 *Principia*, and the 'Scholium Generale' introduced for the first time to this second edition.⁹¹

Newton's references to laws of nature in print are part of the strategies that he mobilised to defend his work against these Continental reactions to the *Principia*—specifically to the force of gravity and its law—that started as soon as the work was published and reached their peak in the 1710s with the dispute with Leibniz. The lack of understanding of the apologetic nature of Newton's references to laws of nature,

⁸⁷ Schaffer, 'Glass Works: Newton's Prisms and the Uses of Experiment', 91–96; Shapiro, 'Newton's "Experimental Philosophy"'; Feingold, "Experimental Philosophy".

⁸⁸ Schaffer, 'Comets and Idols: Newton's Cosmology and Political Theology'; Iliffe, *Priest of Nature*, 355–401; Buchwald and Feingold, *Newton and the Origin of Civilization*.

⁸⁹ Newman, *Newton the Alchemist*, 367–414.

⁹⁰ A general idea of Newton's changes after the *Principia* can be appreciated in Westfall, *Never at Rest*, 469–550; Cohen, *Introduction to Newton's Principia*, 227–64.

⁹¹ It would be impossible to list all works dealing with this, but see Westfall, *Never at Rest*, 698–780; Bertoloni Meli, *Equivalence and Priority*; Vailati, *Leibniz and Clarke: A Study of Their Correspondence*; Guicciardini, *Newton on Mathematical Certainty*, 329–84; Iliffe, *Priest of Nature*, 354–401; Shapiro, *Fits, Passions, and Paroxysms*.

including the audience to which these texts was directed, has misled scholars.⁹² In his works, Newton used consistently the expression ‘laws of motion’ to explain the connection between forces (active or passive principles) and motions as different from laws of nature, prevailing in Descartes’ and Leibniz’s approaches.⁹³ While active principles—inherited from his fellow countrymen—highlighted that the world was the effect of divine choice, the idea of ‘laws of nature’ was for Newton and his followers a reduction of the universe to ‘fate and necessity’.⁹⁴ Curiously, when the validity of active principles was questioned, Newton used the idea of laws of nature in order to defend them.

Newton introduced the expression laws of nature in 1706. A summary of events immediately before this set this reference in context. After the publication of the *Principia*, the Cambridge divine Richard Bentley was invited to inaugurate the ‘Boyle lectures’ in 1692, a series of sermons that according to Boyle’s will should prove ‘the Christian religion’. Bentley considered that Newton’s explanation of the universe may provide an excellent argument to ‘confute atheism’. In order to exploit Newton’s achievements, Bentley and Newton exchanged a few letters in 1692. At some point, Bentley seemed to have interpreted gravity as an inherent property of matter, a claim that Newton rejected in the *Principia*. In order to correct Bentley’s mistake, Newton explained that

Tis unconceivable that inanimate brute matter should (without the mediation of something else which is not material) operate upon & affect other matter without mutual contact; as it must if gravitation in the sense of Epicurus be essential & inherent in it. And this is one reason why I desired you would not ascribe {innate} gravity to me. That gravity should be innate inherent & {essential} to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by & through which their action or force {may} be conveyed from one to another is to me so great an absurdity that I beleive no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent {acting} constantly according to certain laws, but whether this agent be

⁹² Brading, ‘Three Principles of Unity in Newton’; Schliesser, ‘Newtonian Emanation’; Biener and Schliesser, ‘The Certainty, Modality...’; Domski, ‘Laws of Nature and the Divine Order’.

⁹³ Rutherford, ‘Laws and Powers in Leibniz’; Garber, ‘God, Laws, and the Order of Nature: Descartes and Leibniz, Hobbes and Spinoza’.

⁹⁴ Clarke IV Alexander, *The Leibniz-Clarke Correspondence*, 50.

material or immaterial is a question I have left to the consideration of my readers.⁹⁵

In this often misinterpreted, Newton claims that gravity is not an *essential* property of matter. Because gravity, according to the *Principia*, acts at a distance, the idea that matter can act at a distance in virtue of its essential properties is inconceivable. If gravity was not caused by the inherent properties of matter, it had to be caused ‘by an agent acting constantly according to certain laws’. This is all that Newton can claim based on the outcome of his book: whatever causes gravity has to act constantly—because gravity operates instantaneously—and according to the laws of motion and the law of universal gravitation.⁹⁶ Newton tried to explain in different ways how the *Principia* backed up these conclusions before the priority dispute broke out. In a letter to Leibniz in 1693, Newton expounded that vortices ‘contribute no to the regulation but to the disturbance of the motion of the planets’ and that ‘from nothing but gravity acting in accordance with the laws described by me’ it was possible to explain all the phenomena of the heavens and of the sea. ‘And since nature is very simple, I have myself concluded that all other causes are to be rejected and that the heavens are to be stripped as far as may be of all matter, lest the motions of planets and comets be hindered or render irregular’.⁹⁷ Leibniz disagreed. Rejecting vortices and ‘all other causes’ led to dead-ends: to claim that gravity was a property of matter amounted to reintroducing occult qualities in philosophy. Or to affirm that gravity was a perpetual miracle, because the only possible explanation was the direct divine intervention. In the *New essays* (1704), Leibniz reasoned that the distinction between what is natural and what is unexplainable and miraculous solves all difficulties. ‘To reject it would be to uphold something worse than occult qualities, and thereby to renounce philosophy and reason, giving refuge to ignorance and laziness by means of an irrational system which maintains not only that there are qualities which we do not understand’.⁹⁸

⁹⁵ Newton, ‘Original Letter from Isaac Newton to Richard Bentley’, fol. 7r.

⁹⁶ The debate on the reading of this passage and on Newton’s action-at-a-distance seems endless, see Newton, *Principia Variorum*, 1972, 1:149–63; Henry, ‘Gravity and De gravitatione’; Janiak, *Newton as Philosopher*; Ducheyne, ‘Newton on Action at a Distance’; Henry, ‘Action at a Distance’.

⁹⁷ Newton to Leibniz, 16 October 1693 Turnbull, *Correspondence*, 1961, 3:285–86.

⁹⁸ Leibniz, *New Essays on Human Understanding*, 66. On Leibniz criticism see the valuable insights of Psillos, ‘Laws and Powers in the Frame of Nature’.

7.3.3. Contradictions and will

Newton addressed these charges in additions to the 1706 Latin translation of the *Opticks*. The Latin translation, carried out by Samuel Clarke, intended to reach a wider audience than the 1704 English edition and particularly Continental readers. The ‘Queries’ added to the end dealt with the activity of matter, with speculations about the existence, action and quantification of forces generating a wide range of phenomena such as the motion of planets, the cohesion of matter and the motion of animals.⁹⁹ Indeed, Newton had established a strong connection between optical phenomena, active ethers and the properties of matter since his early experimental work.¹⁰⁰ In the ‘Queries’ these explorations were connected to the outcomes of the *Principia*. In the concluding query, the 23rd, the voluminous experimental evidence pointed towards a general conclusion: that the motion in the world could not emerge only from the inherent properties of matter codified in the laws of motion (the three opening laws of the *Principia*). There had to be other sources of motion and other laws, such as gravity and its law, which were not inherent to matter. The *Principia* provided Newton confidence to claim that the action of active principles, such as gravity, could be revealed by further experiments and explained by mathematics just as laws of motion explained the force of inertia. The three passive laws of motion widely accepted according to Newton (even by the ancients)¹⁰¹ were the premises from which he inferred another law describing the effects of an active principle, the force of gravitation. He was confident that other active principles might be postulated and explained in a similar way.

Against this background, Newton undertook the defence of the *modus operandi* and the conclusions of the *Principia* and remarkably of the universal force of gravitation. Facing Leibniz’s options, Newton’s strategy contended that gravity was an active principle backed up by experiments, although its cause remained unknown. Moreover, active principles were not Aristotelian occult qualities nor direct divine

⁹⁹ The literature on the ‘Queries’ is immense, but see McGuire, ‘Force, Active Principles, and Newton’s Invisible Realm’; McGuire, ‘Atoms and the “Analogy of Nature”’; Henry, ‘Enlarging the Bounds of Moral Philosophy’; Henry and McGuire, ‘Voluntarism and Panentheism’; McMullin, *Newton on Matter and Activity*.

¹⁰⁰ Newman, *Newton the Alchemist*, 114–35.

¹⁰¹ Newton, *The Principia*, 70–76; Hall and Hall, *Unpublished*, 310–11.

interventions. They may be thought of *as* laws of nature if by these we understand parts of a universe created by the free will of God. In other words, active principles, including gravitation, were *similar* to what his contenders meant by laws of nature—an expression that, as we have seen, he avoided in mechanics and natural philosophy. In so doing, Newton did not compare his mathematical laws of motion—whose contents he reworked from Descartes’ laws of nature—to laws of nature; his reply to Leibniz was that active principles could be understood as *universal* laws of nature if these were considered as ‘effects of choice’.

The only three references to laws of nature occur in Query 23: ‘Have not the small Particles of Bodies certain Powers, Virtues, or Forces, by which they act at a distance, not only upon the Rays of Light for reflecting, refracting, and inflecting them, but also upon one another for producing a great Part of the Phænomena of nature?’¹⁰² Newton described abundant experiments suggesting that these phenomena should be the effect of active principles. After having presented chymical, optical and mechanical experiments, Newton claimed that nature will be ‘very conformable to herself and very simple’ performing the motion of celestial bodies and of particles of matter by attractive and repulsive forces. According to these experiments, the force of inertia is central to the operations of nature, but were nature reduced to it, ‘the motion would constantly decay’. In fact, the motion of bodies ‘in oil or water, or some fluider matter’ can last for long, but unless matter is completely deprived of ‘all tenacity’ (which it is not), the motion would continually get lost. An account of the world restricted to inertial motions was typical of Descartes: ‘The Cartesians make God the author of all motion & its as reasonable to make him the author of the laws of motion. Matter is a passive principle & cannot move itself ... these are passive laws & to affirm that there are no other is to speak against experience’.¹⁰³ Consequently, Newton postulated the existence of active principles accounting for optical, chymical, magnetical, physiological and even celestial phenomena. In contrast with the cautious claims to Bentley two decades ago, Newton speculated here about a universe in which matter is moved by active

¹⁰² Newton, *Optice*, 322; Cf. Newton, *Opticks*, 375–76.

¹⁰³ Newton, ‘Hydrostatics, Optics, Sound and Heat’, fol. 619r.

principles. In so doing, Newton followed the English experimental philosophers and Barrow, criticising the Cartesian *Principia* as a description of the ‘outer shell without the nucleus’. Given the variety of motion observed in the world, Newton claimed that there was ‘a necessity of conserving and recruiting it by active principles’ such as the *causes* of gravity, fermentation and cohesion of bodies—whose experimental evidence was presented to the reader. A world governed only by the force of inertia would not look like the one we gather from experiments.¹⁰⁴

From the postulation of this world based on the experimental evidence and the conclusions of the *Principia*, Newton moved to a more speculative level, recurrent in the ‘Queries’, to deal with two aspects: the creation of matter and the nature of active principles ‘conserving and recruiting’ motion. The distinction between levels of reasoning is clearly demarcated by the way in which the topics are introduced: ‘All these things being considered, it seems probable to me (*illud mihi videtur denique simillimum veri*) that God in the beginning form’d Matter in solid, massy, hard, impenetrable, moveable particles, of such Sizes and Figures, and with such other Properties, and in such proportion to Space, as most adjusted to the End for which he form’d them’.¹⁰⁵ In Newton’s view, these particles were created so hard and solid that ‘no ordinary power’ is able to divide ‘what God himself made one in the first Creation’.¹⁰⁶ From experiments, we infer that the particles composing bodies are solid so it seems probable that God in the beginning created indivisible particles that do not ‘wear away’ and do not ‘break in pieces’. Accordingly, changes in nature do not emerge from the transformations or division of these particles but from ‘the various separations and new associations and motions of these permanent particles’.¹⁰⁷ The variety of phenomena arises from the different configurations of these particles: from the motions resultant from passive and active principles.

In the same speculative tone, Newton introduces his view on the causes of these motions:

¹⁰⁴ Newton, *Opticks*, 397–400.

¹⁰⁵ Newton, *Optice*, 343; Cf. Newton, *Opticks*, 400.

¹⁰⁶ On this thorny topic see McGuire, ‘Space, Infinity and Indivisibility’.

¹⁰⁷ Newton, *Optice*, 343–44; Cf. Newton, *Opticks*, 400.

[1] It seems to me farther, that these primitive Particles have not only a *Vis inertiae*, accompanied with such passive Laws of Motion as naturally result from that Force, but also that they are perpetually moved by certain active Principles, such as is that of Gravity, and that which causes Fermentation, and the Cohesion of Bodies. [2] I consider these Principles, not as *occult qualities*, imagined (*fingantur*) to arise from the *specific forms* of things, but as *universal* Laws of Nature, according to which things themselves are formed (*quibus res ipsae sunt formatae*). [3] For, that such Principles do really exist, appears from the phenomena of nature; though what the *causes* of them are, be not yet explained. [4] To affirm that every distinct Species of Things, is endued with specific occult qualities, by means whereof the Things have certain Active Forces; this indeed is saying *Nothing*. [5] But to deduce from the phenomena of nature, two or three general principles of motion; and then to explain how the Properties and Actions of all corporeal things follow from those Principles; this would be a great progress in Philosophy, though the *causes* of those Principles were not yet discovered.¹⁰⁸

The general structure of the argument runs this way: [1] particles of matter only have, in virtue of their nature, a force of inertia—whose properties were described by the three laws of motion of the *Principia*. However, these particles are also ‘perpetually’ moved by active principles, such as that of gravity, fermentation and cohesion of bodies. [2] He considers these principles as ‘*universal* laws of nature’, that is, as general causes of motion and in this sense as opposed to ‘*specific forms* of things’. Active principles are not singular qualities in matter generating concrete effects, such as the Aristotelian qualities [4]. That gravity and other active principles exist has been proved by phenomena (in the *Principia*, for example) [3]. Once active principles are ‘deduced from phenomena’, they can be assumed to explain properties and actions of things by their laws, such as the law of gravitation. Although the form in which these principles perform their action remained unknown, the quantitative explanation of their effects is a great contribution to ‘philosophy’ [5]. This last point was the focus of Leibniz’s charge of reintroducing occult qualities or supernaturalism. In order to defend his work from these accusations, Newton compared active principles to ‘*universal* laws of nature’, as opposed to particular occult qualities [2]. Next, he explains how active principles viewed in this way do

¹⁰⁸ Numbers added. Newton, *Optice*, 344–45; Cf. Cohen, ‘The First English Version of Newton’s Hypotheses non Fingo’, 386.

not entail supernaturalism. In fact, only if we make Newton's laws of motion (mathematical descriptions) equal to laws of nature (causes of motion) is there room for supernaturalism; but this is not the case, as I will show.

A close reading of the passage shows that Newton is making a *comparison* between active principles and 'universal laws of nature according to which things are formed', not claiming that active principles *are* laws of nature or that laws of motion are laws of nature. Put otherwise, Newton's active principles are similar to universal laws of nature, *in the sense* that they form things. This last sentence has been interpreted as a metaphysical stance about the composition of substances, that is, as a thesis on the ultimate nature of bodies.¹⁰⁹ Nevertheless, Newton used laws of nature to shake off the accusations against the universal principle of gravitation. From this perspective, Newton was not making a theory of matter but defending his views on active principles as non-essential to matter and generating motion in the world, although their cause remained occult. Newton had a local tradition behind him supporting this move.¹¹⁰ The expression laws of nature, coming from Descartes, is used to explain this point in a clearer way for his audience and to hit back at Leibniz's attack. Interestingly, the Latin phrasing of the comparison between active principles and laws of nature makes clear that Newton is not putting forward a metaphysical doctrine in which laws of nature are previous to bodies and enter into their composition, but rather that active principles are not the *specific* occult qualities of the Scholastics.

Newton claimed that particles are 'moved' by (rather than endued with) active principles by causes unknown and that the things themselves are formed by them [1,2,3]. Why does this expression avoid the proximity with the Scholastic doctrine of occult qualities? Newton had explained how the active principles of gravitation, fermentation and cohesion form 'things' (not substances) in connection with the experiments he had presented in the 'Queries'. His point is that active principles are 'associated' with the 'primitive particles' to form things. Active principles are

¹⁰⁹ McMullin, *Newton on Matter and Activity*; Brading, 'Three Principles of Unity in Newton'; Biener and Schliesser, 'The Certainty, Modality...'; Domski, 'Laws of Nature and the Divine Order'.

¹¹⁰ Henry, 'Occult Qualities and the Experimental Philosophy'. See, section 5.3.

necessary to generate the phenomena of nature, but this does not amount to a description of how these things were *initially* created. After all, Newton's bottom line is that the cause of these principles is unknown. Because motion arising from inertial force is passive,

There is a necessity of conserving and recruiting it [motion] by active principles, such as are the cause of Gravity, by which Planets and Comets keep their motions in their Orbs: and the cause of Fermentation, by which the Heart and Blood of Animals are kept in perpetual motion and Heat: the inward Parts of the Earth are constantly warm'd, and in some places grow very hot; bodies burn and shine, Mountains take fire, the Caverns of the Earth are blown up, and the Sun continues violently hot and lucid, and warms all things by his Light. And if it were not for these Principles, the bodies of the Earth, Planets, Comets, Sun, and all things in them, would grow old and freeze, and become inactive Masses; and all Putrefaction, Generation, Vegetation and Life would cease, and the Planets and Comets would not remain in their Orbs.¹¹¹

This passage vividly describes how 'things' are 'formed' by active principles based on the experiments presented in the first part of the 'Query' and including the active principle of gravitation explained in the *Principia*. If the expression 'things' sounds inadequate, it seems hard to think of a better word to put together the planetary system, the life of animals, fires in mountains, the heat of the Sun. The active principles moving matter are responsible of 'forming' these 'things', that is, of generating phenomena which cannot be reduced to the mere actions of the inertial force of the primitive particles. These 'things' require active principles in order to behave as they do and, in this sense, active principles were considered *as* universal laws of nature, not as singular qualities postulated to explain specific phenomena.

At this point, Newton turned to consider the 'first creation'. The reason behind this move seems to be that Descartes' 'laws of nature' were said to play a creative role that Newton's theology explicitly forbids—and that Boyle, as we saw, also rejected.¹¹² Immediately after comparing active principles to laws of nature [2-5], Newton claimed that 'by the help of these [active] Principles, all material Things

¹¹¹ Newton, *Opticks*, 399–400.

¹¹² See, section 5.3.

seem to have been composed of the hard and solid Particles above-mention'd, variously associated in the first Creation by the Counsel of an intelligent Agent'.¹¹³ God probably introduced active principles in the creation, but these were not responsible for creating things. Thus, it is 'unphilosophical to seek for any other Origin of the World', particularly suggesting that the world 'might arise out of a Chaos by the mere Laws of Nature'. Descartes had claimed that 'laws of nature' were such 'even if we were to assume [the existence of] the Chaos; we could still demonstrate that, by these laws, this confusion must gradually be transformed into the order which is at present in the world'.¹¹⁴ Cartesian 'laws of nature' were *sufficient* principles of creation. Newton, in line with his English fellows, rejected this view and explained that the origin of the world was *Deus Optimus Maximus*. However, once the world was formed, 'it may continue by those Laws for many ages': motion can be transmitted and recruited by active principles. Active principles considered as laws of nature cannot be taken for principles creating the world but as principles 'recruiting and conserving' motion for many ages.

Nevertheless, why active principles could not create the world from chaos such as 'laws of nature'? In order to answer this question Newton turned to theology. The move consists of two aspects: the argument from design and the inference that God not only created the world but also governs it by his unrestricted will: he is 'a powerful ever-living Agent' (*entis potentis semperq; viventis*).¹¹⁵ Against the claim that the world may be created by the mere course of necessary 'laws of nature', Newton argued that the singular configuration of the planetary system, including the 'very excentric orbs' in which Comets move and the contrivance of the bodies of animals 'must be allowed [to be] the effect of choice'. The constitution of the world and of living beings reveal that they are intentional actions of God. An important aspect is that God created this world at will but, given that he is omnipresent, he 'is more able by his Will to move the bodies within his boundless uniform sensorium [space], and thereby to form and reform the parts of the universe, than we are by our

¹¹³ Newton, *Optice*, 343.

¹¹⁴ Descartes, *AT*, 1905, VIII:101–3.

¹¹⁵¹¹⁵ On the theological references of this 'Query' see McGuire, 'Space, Infinity and Indivisibility'; Henry and McGuire, 'Voluntarism and Panentheism'.

will to move the parts of our own bodies'.¹¹⁶ God is everywhere and this means that he exists all over the infinite space in which, if he wanted, he could act according to his free will. The analogy between human motions at will and God's action over the universe by his unrestricted will opened an important front in the dispute with Leibniz, particularly through the correspondence with Clarke.¹¹⁷ The point here is, as I said, that God created the world without any restriction and, in virtue of his omnipresence, he can 'reform it at will'. In this context, Newton introduced another significant qualification to his comparison of active principles to laws of nature: that they are not necessary and therefore can 'vary'.

Since Space is divisible *in infinitum*, and Matter is not necessarily in all places, it may be also allow'd that God is able to create Particles of Matter of several Sizes and Figures, and in several Proportions to Space, and perhaps of different Densities and Forces, and therefore to vary the Laws of Nature, and make Worlds of several sorts in several Parts of the Universe. At least, I see nothing of Contradiction in all this.¹¹⁸

This aspect, with Barrowian undertones,¹¹⁹ relies on Newton's voluntarist theology, his conceptions of space and time and his criticism of Descartes' identification of matter with extension. The divisibility of space, that is, that space does not have minimum parts, implies that the nature of space does not impose any restriction on God's creative power. Seen the other way: had space indivisible parts, these would restrict God's possibilities of creating other kinds of matter; for example, God could not create matter composed of parts divisible *ad infinitum*, for it would imply that some parts of matter may be smaller than the indivisible *minima* of space and therefore they would be 'nowhere'.¹²⁰ Moreover, because matter—at least in this world—is not identified with space and thus 'is not necessarily in all places', God may create different kinds of matter: of different sizes and figures 'and in several proportions to space'. God could also create matter of varying densities and other forces and, therefore, establish other principles of motion ('to vary the laws of

¹¹⁶ Newton, *Optice*, 346.

¹¹⁷ Henry and McGuire, 'Voluntarism and Panentheism'; Connolly, 'Newton and God's Sensorium'; Koyré and Cohen, 'The Case of the Missing Tanquam: Leibniz, Newton & Clarke'; Vailati, *Leibniz and Clarke: A Study of Their Correspondence*.

¹¹⁸ Newton, *Opticks*, 403–4.

¹¹⁹ See section 6.4 and Malet, 'Isaac Barrow on the Mathematization of Nature'.

¹²⁰ McGuire, 'Existence, Actuality and Necessity'; McGuire, 'Space, Infinity and Indivisibility'.

nature'). The outcome of these variations may generate 'worlds of several sorts' that may be in different parts of the universe, given the infinite extension of space.

The infinite possibilities of worlds deriving from this theology does not imply that this world could not be explained with certainty. In a draft for the 1718 version of this query, Newton explained that 'the business of experimental philosophy is [only] to find out by experience and observation not how things were created but what is the present frame of nature'.¹²¹ Likewise, Newton concluded the 'Query' by drawing a parallel between the mathematical procedures of analysis and synthesis and the 'method' of investigation in natural philosophy (*physica*). The point is that both mathematics and natural philosophy explain causes by their effects and once these causes have been established they may be used to explain new effects.¹²² Causes cannot be known *a priori*—that would amount to understanding God's infinite will. However, they can be assumed as such after being 'deduced from phenomena', that is, after 'making experiments and observing phenomena' and then, from compound things to 'deduce' (*colligere*) by reasoning from the simple things. This is the 'method of analysis' by which we deduce 'motive forces from motions and, in general, causes from effects and from particular causes more general ones until we find the most general'. The synthesis consists in assuming 'the investigated and proved causes' as principles and from them to explain 'phenomena proceeding from them' and in this sense 'proving the explanations'.¹²³ These pronouncements are aligned with the redrawing of disciplinary boundaries as presented in the 1687 edition of the *Principia*: 'For the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces'.¹²⁴ In the conclusion of the query, Newton explains that if these methods are followed in natural philosophy it will be possible to reach a 'perfect science', restoring the lost knowledge and the uncorrupted religion—an expansion of the insight that motivated the *Principia*.¹²⁵

¹²¹ Newton, 'Hydrostatics, Optics, Sound and Heat', fol. 242v.

¹²² Guicciardini, *Newton on Mathematical Certainty*, 315.

¹²³ Guicciardini, 315–27; Guicciardini, 'Analysis and Synthesis in Newton's Mathematical Work'; Ducheyne, *The Main Business*.

¹²⁴ Newton, *The Principia*, 28.

¹²⁵ See Henry, 'Enlarging the Bounds of Moral Philosophy'.

Newton's strategy to defend his principle of gravitation as a universal law of nature did not work very well. When Newton was planning the second edition of the *Principia* in the early 1710s the dispute over the priority of the invention of calculus was reaching to its peak. Newton did not miss the chance to introduce some comments in his *magnum opus*, particularly the 'Scholium Generale' dealing again with topics similar to those of the Query 23. However, an important change of perspective here is that over the development of the priority dispute, Newton took shelter in the idea of 'experimental philosophy' as a reaction against Leibniz.¹²⁶ In the 1710 *Théodicée*, Leibniz insisted on claiming that gravity was essential to matter and that this opinion has its roots in Scholastic doctrines: 'remote operation has just been revived in England by the admirable Mr. Newton, who maintains that it is the nature of bodies to be attracted and gravitate one towards another'.¹²⁷ In his provocative style, Leibniz denied Newton's voluntarist consideration of laws, claiming that 'God gave such laws not without reason, for he chooses nothing from caprice and as though by chance or in pure indifference but the general reasons of good and of order, which have prompted him to the choice, may be overcome in some cases by stronger reasons of a superior order'.¹²⁸ A few years later, in 1712, some letters between Leibniz and Hartsoeker appeared in the *Memoirs de Literature*. This time, Leibniz attributed to Newton the view 'that all bodies attract each other is a law of nature that God has commanded when creating the bodies. Because, not providing anything causing this effect, and not admitting anything that God had made that could explain how this is done, they recourse to a miracle, that is, to the supernatural'. Claiming that gravity is a law of nature became, in Leibniz's hands, a recourse to supernaturalism. Newton drafted an anonymous letter to the editor of the journal criticising these views. Newton punched back claiming that from Leibniz's perspective, impenetrability, extension and duration of matter would be occult & miraculous because 'no man ever attempted to prove these qualities mechanically'. On the contrary, these were 'natural real reasonable manifest qualities of all bodies';

¹²⁶ Shapiro, 'Newton's "Experimental Philosophy"'; Feingold, "Experimental Philosophy"; Guicciardini, *Newton on Mathematical Certainty*, 315–27.

¹²⁷ Leibniz, *Theodicy. Essays on the Goodness of God, the Freedom of Man and the Origin of Evil*, 86.

¹²⁸ Leibniz, 75.

similarly, ‘bodies attract one another by a power whose cause is unknown’ a conclusion proved by experiments against the ‘fictions’ invented by Leibniz. The idea of laws of nature in connection with gravity became involved in the dispute, thanks to Newton’s *Optice*.¹²⁹

During the process of revision of the 1713 *Principia*, Cotes drew Newton’s attention to Leibniz’s letters in March 1712. Cotes took advantage of this situation to ask Newton if the word ‘*attractio*’ frequently used in the *Principia* did not imply a hypothesis to explain mutual attraction.¹³⁰ Newton assumed the letter as an occasion to instruct his curious editor in the subtleties of experimental philosophy that shall inform this new edition. ‘The difficulty you mention,—Newton replied—which lies in the words [Et cum *attractio* omnis *mutua* sit] is removed by considering that as in Geometry the word Hypothesis is not taken in so large a sense as to include the Axiomes & Postulates, so in experimental philosophy it is not to be taken in so a large sense as to include the first Principles or Axiomes which I call the laws of motion’. The laws of motion were deduced from phenomena and made general by induction, Newton added. Given that the proposition concerning the mutual attraction was ‘a branch of the third law’ (action and reaction), it is not a hypothesis but a ‘deduction’ from phenomena. In a draft of this letter, Newton made a longer argument involving the third rule for philosophising—also introduced for the 1713 edition—claiming that ‘if we break that rule, we cannot affirm any one general law of nature: we cannot so much as affirm that all matter is impenetrable’.¹³¹ Probably after checking Leibniz’s letters in the *Memoirs*, Newton wrote the version he finally sent to Cotes, summarising the way of proceeding in experimental philosophy and asking him to introduce some changes into the ‘*Scholium generale*’: ‘At the end of the last Paragraph but two now ready to be printed off I desire you to add after the words [nihil aliud est quam *Fatum et Natura*.] these words [Et haec de Deo: de quo utique ex *phænomenis* disserere, ad *Philosophiam experimentalem* pertinet.]’. This insertion, directed against Leibniz’s view of nature, makes more explicit that the

¹²⁹ Newton to the editor, 5 May 1712 Hall, *Correspondence*, 5:298–302.

¹³⁰ Koyré, *Newtonian Studies*, 273–82; Biener and Smeenk, ‘Cotes’s Queries. Newton’s Empiricism and Conceptions of Matter’.

¹³¹ Cohen, ‘Hypotheses in Newton’s Philosophy’.

foundation of the experimental approach had implications for theology. In addition, with the intention of preventing ‘exceptions against the use of the word hypothesis’, Newton asked Cotes to add this:

For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method. And it is enough that gravity really exists and acts according to the laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies and of our sea.¹³²

A former version of this paragraph that Newton probably changed after reading Leibniz’s papers of 1712, followed the strategy of the *Optice*:

From the phenomena it is very certain that gravity is given and acts on all bodies according to the laws described above in proportion to the distances, and suffices for all the motions of the Planets and Comets, and thus it is a law of nature although it has not yet been possible to understand the cause of this law from phenomena. For I avoid hypotheses [Nam hypotheses ... fugio], whether metaphysical or physical or mechanical or of occult qualities. They are harmful and do not engender science.¹³³

Newton reworded this paragraph and removed the claim that ‘gravity ... is a law of nature’ (not the law of gravity). As in 1706, gravity here is said to be a ‘law of nature’ but this version was discarded. The line of defence now was to claim the validity of universal gravitation as a legitimate product of the ‘experimental philosophy’, whose virtues were praised by Newton and his pupils. Accordingly, Cotes, guided by Bentley and Newton wrote the ‘Editoris Præfatio’ contrasting ‘this method of philosophising’ with the ‘Scholastic doctrines derived from Aristotle and the Peripatetics’ and with those ‘who speculate from hypotheses’.¹³⁴ Cotes addressed the question of the nature of gravity following Newton’s ideas to the editor of the *Memoirs de Literature*: explaining that gravity could not be counted among ‘the primary qualities of bodies’ but that ‘from phenomena this force really exists’ even if

¹³² Hall, *Correspondence*, 5:397.

¹³³ Hall and Hall, *Unpublished*, 350.

¹³⁴ Cohen, *Introduction to Newton’s Principia*, 227.

‘the cause of gravity is itself occult’. Cotes explained that the ‘province of true philosophy’ was ‘to derive the natures of things from causes that truly exist, and to seek those laws by which the supreme artificer willed to establish this most beautiful order of the world, not those laws by which he could have, had it pleased him’.¹³⁵ Making virtue of necessity, Cotes presented the lack of knowledge of the cause of gravity as a sign of Newton’s commitment to evidence and certainty and as an example of the rejection of hypotheses and speculations in experimental philosophy. This was a bold move in covering Newton’s *Principia* under the virtues of the ‘experimental philosophy’, although no further references to laws of nature appear in the work.¹³⁶

For the 1718 English edition of the *Opticks*, Newton introduced some additions explaining the virtues of experimental philosophy. The comparison between active principles and laws of nature remained, but it acquired a new qualification based on the approach presented in the 1713 *Principia*. Newton introduced, between [3] and [4], the idea that active principles were ‘manifest qualities, and their causes only are occult’. In contrast to the Aristotelians ‘that gave the name of occult qualities’, not to manifest qualities, but to such ‘qualities only as they supposed to lie hid in bodies, and to be the unknown causes of manifest effects’. What Aristotelians denominated occult ‘would be the causes of gravity, and of mangnetick and electrick Attractions, and of fermentations, if we should suppose that these forces or Actions arose from Qualities unknown to us and incapable of being discovered and made manifest’. He clarified that these ‘occult qualities’ were an obstacle to philosophy and therefore ‘have been rejected’.¹³⁷ Newton claimed that the *cause* of active principles might appear as occult, but that in truth they were causes of manifest qualities and thus there was no resemblance between his procedure and that of the Schoolmen. The methods of analysis and synthesis—now presented in experimental terminology—guaranteed the validity of his procedure.

¹³⁵ Newton, *The Principia*, 385–99.

¹³⁶ Shapiro, ‘Newton’s “Experimental Philosophy”’; Feingold, ““Experimental Philosophy””.

¹³⁷ Newton, *Opticks*, 403.

Newton did not suppress the reference to laws of nature, as he did in the draft of the ‘Scholium generale’, probably because by 1718 Leibniz had passed away and the dispute over the priority of calculus had come to an end. However, Newton did not miss the chance to align the idea that the cause of gravity was unknown with the intellectual virtues of the experimental philosophy. But the recourse to laws of nature introduced for the 1706 *Optice* that escalated the controversy with Leibniz and was connected to the experimental philosophy for the 1718 *Opticks*, did not make a way into the *Principia*. Similarly, the references to ‘experimental philosophy’ for the 1726 edition of the *Principia* were minimised. There is no evidence that Newton considered that natural philosophy or ‘rational mechanics’ were concerned with laws of nature or with the first creation. The entire task of ‘philosophy’ was ‘to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces’ by ‘exact laws which would allow of any easy calculation’.¹³⁸ These laws were mathematical correlations between forces—whose cause is unknown—and motions formulated to make nature intelligible, not secondary causes taking the place of God.

¹³⁸ Herivel, *Background*, 303.

Scholium

The idea that Descartes introduced ‘laws of nature’ into natural philosophy as an amalgamation of previous meanings and that ‘laws of nature’ remained roughly unaltered up to Newton, finds many difficulties in the light of the evidence examined throughout this thesis. That the ‘main sources for the later use of “laws of nature” as a concept to interpret the practice of science are to be found in the works of Descartes and then in Newton’ takes for granted that laws of nature operated in a similar way for both of them and that this use constitutes the key to understand contemporary science. This view assumes that for Descartes and Newton laws of nature were prescriptions laid down by God and therefore, ‘they are true, hold for the whole universe and are necessary in the sense of absolutely obligatory and independent of the beliefs of humans’.¹ Although it is correct that Descartes’ used ‘laws of nature’ in an innovative way, borrowing the term from mixed-mathematical sciences, the elucidation of previous uses does not explain, as it were, Descartes’ use of it. The meanings circulating before the *Principia Philosophiæ*, even those somehow referring to the natural world, could not prefigure or foreshadow Descartes’ reworking of ‘laws of nature’ as singular statements on matter and motion operating as causes. Similarly, examining Descartes’ laws does not explain Newton’s laws of motion and how they were conceived as universal or, at least, general.² Against this background, this thesis argued that ‘laws of nature’ and their immediate appropriation in England should be interpreted as functions of the redrawing of boundaries between mechanics, mathematics and natural philosophy. The transformation of the orders of knowledge, and mostly Descartes’ project of an *a priori* physics, constituted the specific problem that ‘laws of nature’ addressed.

The disciplinary approach set the background to ‘laws of nature’ in the antecedents to Descartes’ reform of physics as a science capable of *a priori* demonstrations, not in the previous uses of the expression. I traced this project back to the sixteenth-

¹ Giere, *Science without Laws*, 87.

² Bloor, Barnes, and Henry, *Scientific Knowledge: A Sociological Analysis*, 46–80; Kusch, ‘Meaning Finitism and Truth’.

century reshaping of the loose set of mechanical practices into a mixed-mathematical *scientia*. The newly emerged practitioners of this science developed mathematical tools to explain physical objects—machines—in terms of their configuration or design. As part of their attempts to codify mechanics in an axiomatic form, they developed a procedure to *reduce* complex machines to simple geometrical principles. Against the assumptions validating this procedure, distinguishing natural and violent motions, Galileo applied the *reductio* to the study of natural phenomena and rendered plausible a mathematical science of motion. Galileo saw the possibility of formulating a largescale explanation of the world based on the *reductio* if the physical postulate of the motion of the Earth was granted. In his view, our knowledge was possible by a piecemeal approach providing certainty to limited portions of our understanding of reality. Therefore, a full-scale explanation of the natural world relying on mathematics was the addition of all these findings. Another attempt to connect mathematics to natural philosophy was put forward by Kepler. In his view, although the major astronomical ‘hypotheses’ (Ptolemaic, Copernican, Thyconic) may be mathematically equivalent, they were physically contradictory. Therefore, he sketched a reform of astronomy based ‘upon causes’, in which mathematics and natural philosophy were connected in an archetypal cosmology that uncovered the blueprints of the divine creation of the world. These archetypes were instantiated in matter and stamped in human soul. They constituted formal causes and thus Kepler turned to mechanics and natural philosophy to determine efficient causes of natural phenomena, including the motion of planets. Neither Galileo nor Kepler used the expression ‘laws of nature’ in a way similar to Descartes. On the contrary, their reforms of natural philosophy by appealing to mathematics appealed to other resources connecting different disciplines and assembling resources coming from the new science of mechanics.

I showed that Descartes’ reform of an *a priori* physics was modelled after Kepler’s restructuring of astronomy as based upon causes. However, instead of an interaction of disciplines, Descartes conceived a more radical reduction of physics to mathematics in virtue of his conception of matter as (geometrical) extension. Descartes had access to Kepler by reading Beeckman’s *Journal* in an unexpected

visit in 1628. At that time, Beeckman was enthusiastically rewriting Kepler's astronomy in terms of his 'physico-mathematics', replacing Kepler's 'animistic' and 'occult' causes for corpuscles in motion. Although Descartes had been working in the solution of mechanical problems in a way resembling Galileo—a piecemeal approach and the use of *reductio*—after he became acquainted with Beeckman in 1618, the unplanned visit of 1628 gave him the new outlook of finding a definitive account of the effects of the natural world by their causes. From the correspondence with Mersenne from 1628 to 1630, I traced how Descartes construed this project and how the 'laws of nature' appeared as the principles explaining motion in nature. The *a priori* nature of Descartes' project entailed that these principles were not descriptions of matter in motion but, on the contrary, they made motion possible. The disciplinary perspective also revealed that Descartes shaped the account of specific phenomena based on these *a priori* 'laws of nature' after the model of enquiry of mechanicians. Descartes's moved away from the *reductio* and established a different connection with the sixteenth century mechanics as a physical rather than a mathematical discipline.

Descartes affirmed that mechanics in ancient times was a branch of natural philosophy that took temporarily shelter in mathematics during the 'confusing' age of Aristotle and that he had restored its foundations. Unconvinced by Descartes' genealogy, Barrow replied that Descartes' 'mechanical constructions, experiments concerning motions of projectiles, the ebb and flow of tides which I understand are all very elegantly explained by this [Cartesian] method' are part of 'Mathematics rather than Physics'.³ The English critics of 'laws of nature' considered that Descartes' principles explained the motion of bodies in collision with some success, but that his philosophy was insufficient to account for the activity in matter that collective experimental work had revealed. The disciplinary focus of this thesis revealed an intricate panorama in the English appropriation of Descartes' laws. English writers rejected, in general, the idea of a universe governed by immanent 'laws of nature' and the metaphysics that Descartes used to support his principles. However, they integrated the stipulation of these 'laws of nature' into their local

³ Barrow, 'Oratio', 85–86.

traditions accounting for the motion of bodies in mechanics and astronomy, while natural philosophy tried to illuminate the operations of nature based on active principles by experiments. In this way, they kept the expression laws of nature as a vague term to talk about order in the world, and reworked the Cartesian ‘laws of nature’ in laws of motion expressed in mathematical terms and ‘verified’ by experiments. This thesis showed how these laws of motion were integrated into the tradition of the ‘elliptical astronomy’ that in the hands of Wren and Hooke explained circular and conical motions in terms of reworked versions of Descartes’ first two laws and an attractive force or power towards the centre.

The relocation of ‘laws of nature’ from natural philosophy to mechanics occurred as mathematicians and scholars debated the nature of mathematics including the introduction of physical notions—such as motion—in mathematics and the nature of geometry and arithmetic. I explored two perspectives, Wallis’ and Barrow’s, that revealed divergent approaches, techniques and assumptions related to mechanics as a mathematical science. While for Wallis the laws of motion were arithmetical principles that could explain physical motions *via* geometry, Barrow postulated that motion was the principle generating geometrical figures and, therefore, the study of motion in geometry was the foundation of arithmetic, natural philosophy and any other science.

In line with some general features of the English appropriation of Descartes, the young Newton discussed the laws of motion as explaining the collision of bodies in mechanics. Similarly, when asked by Halley about the orbit of planets in 1684, Newton replied with a short piece of mechanics within the boundaries of the common practices in England. My thesis explained how Newton moved from this shared view of laws of motion in mechanics to the ‘axioms of laws of motion’ of the *Principia* which postulated a ‘rational mechanics’. I provided evidence showing that Newton was moved to redraw the disciplinary boundaries of mechanics, mathematics and natural philosophy by the realisation that the mathematical ‘hypotheses’ that he drafted in his first solution to Halley proved *a priori* the Copernican arrangement of the world. His historical and religious studies had revealed to him that this was the

true system of the world known to the ancients and lost by idolatry and corruption. Newton, then, transformed the ‘hypotheses’ into ‘axioms or laws of motion’ that could account for the motion of points but also for the path of planets and in general, for the motion of any particle in the universe as generated by forces. In this way, Newton modified a somewhat consolidated use of laws of motion that was restricted to mechanics as a mathematical science. The *Principia*, a ‘rational mechanics’, established three passive laws of motion—derived from the force of inertia—and uncovered one active law—derived from universal gravitation. Although this conclusion was acceptable in the eyes of most of his English fellows, Leibniz and other Continental philosophers criticised Newton’s misunderstanding of geometry, mechanics and natural philosophy and accused him of reintroducing occult qualities in establishing the law of gravity and the reality of its force. Newton’s only use in print of the expression laws of nature occurred in the *Optice* as part of his defence against Leibniz. When the reality of gravitation came under attack, Newton compared it with ‘universal laws of nature’ in a move that has puzzled historians. But Newton did not use the concept of laws of nature (as causes of motion) in natural philosophy, in line with other English philosophers, and in his place, he formulated ‘axioms or laws of motion’ that described the action of principles, sources of activity in the world and causes of motion. The *Principia* uncovered one law of activity in nature; the ‘Queries’ of the *Opticks* pointed to other laws describing other active principles, such as fermentation, cohesion of bodies and electricity as plausible candidates to exist in a world created by the free will of God.

Throughout this thesis, it became clear that ‘laws of nature’ did not emerge as a neutral label to qualify specific findings of sciences. On the contrary, they emerged as concrete achievements with an operative function within Descartes’ attempt to replace the Scholastic natural philosophy and consequently embedded within a network of assumptions, traditions and practices that were central to the appropriation of ‘laws of nature’. English natural philosophers and mathematicians reworked these ‘laws of nature’ within different disciplinary settings and put forward alternative ‘laws of motion’ in ways not previously noticed by scholars. The picture that emerges is not that of an amalgamation of previous meanings into a more

complex one that was subsequently disseminated. The dissemination model excludes the transformation of disciplines that constitutes the key to appreciate the complex appropriation of ‘laws of nature’, including their relocation to mechanics and Newton’s use of laws in the context of active and passive principles of motion. Instead of a unified concept of ‘laws of nature’, Descartes’ project triggered reactions framed within local traditions and therefore it is hard to claim that at the end of the seventeenth century there was any agreement on the meaning of ‘laws of nature’ or even laws of motion beyond the narrow circles that shared disciplinary commitments and values.

How Newton’s laws of motion and the law of gravity became laws of nature is another story to be told, emerging from the Scientific Revolution and making its way into the appropriation of Newton’s works. The name and achievements of Newton became historically associated to laws of nature. Pope’s epitaph stated that ‘Nature and nature’s laws lay hid in night: God said ‘Let Newton be!’ and all was light’. It is hard to know if Pope had in mind Newton’s specific laws of motion or—more probably—he was just going through the well-trodden path of using laws of nature as a general reference to the natural order. However, the idea that Newton’s laws were in a more defined way laws of nature became current in Newtonian circles at least since the early eighteenth century. In 1705 the Scottish mathematician John Keill set forth Newton’s laws of motion under the heading ‘De legibus naturæ’, explaining that laws of nature are such ‘as it is necessary that all natural bodies do obey’. Keill presented them in the ‘same order and in the very same words, as they were laid down by the *Illustrious Sir Isaac Newton*’. Later in 1721, Keill explained that Kepler ignored the reason of his third ‘law of nature’ that he had found by computation. It was not until ‘our great Newton (...) demonstrated that no other relationship could have place in the Universe, given the laws of nature’, that is, the three initial axioms of the *Principia*.⁴ Maybe more significant is that one of the main vehicles for the early dissemination of Newton’s natural philosophy was Samuel Clarke’s translation of Rohault’s *Traité de Physique* (1697). Rohault’s work was a popular textbook replacing the Aristotelian natural philosophy for the Cartesian theses of the *Principia*

⁴ Keill, *Lectiones Physicae*, 107–30; Keill, *Lectiones Astronomiae*, 37.

Philosophiæ. Clarke translated the work into Latin and presented Newtonian postulates in the form of extensive comments, antagonising with the Cartesian main text. In explaining ‘the continuation and cessation of motion’, for example, Rohault mentioned Descartes first ‘Law of Nature’. In the comments, Clarke explicitly elaborated Newtonian topics such as the collision of bodies, the action of elastic forces and gravitational attraction. This latter is explained as ‘the action of some immaterial cause which perpetually moves and governs matter by certain laws’, in contrast with the idea that matter can act by itself on other matter at a distance. For the untrained student, Newtonian laws of motion were woven with the Cartesian terminology.⁵ This bond of the Cartesian terminology of ‘laws of nature’ with Newton’s achievements dominated the views of the *Encyclopaedists* and largely, the subsequent developments in modern science during the eighteenth century.⁶ In fact, it was during the appropriation of Newton in the eighteenth century—to a large extent, through the *Opticks*—that his accomplishments and those of others honoured as his peers were linked as a succession of laws, inaugurating a foundational myth of the origins of modern science that reached up to the twentieth century.

⁵ Rohault, *Physica*, 51–53; See Schüller, ‘Samuel Clarke’s Annotations in Jacques Rohault’s *Traité de Physique*’.

⁶ Charrak, *Contingence et nécessité des lois de la nature au XVIIIe siècle*; Snobelen, ‘On Reading Isaac Newton’s *Principia* in the 18th Century’.

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